



# **Dynamic Simulation and Optimization using EMSO**

– Lecture 5 –

Simulation of tubular reactors.

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PEQ/COPPE-UFRJ January, 2013



## **Dynamic Modeling of Tubular Reactors**



Assumptions:

- Constant physical properties ( $\rho$ ,  $\mu$ ,  $c_{\rho}$ );
- Newtonian fluid;
- Angular symmetry;

• 
$$V_r = V_{\theta} = 0$$

#### Model classification based on physical-chemistry principles



#### **Microscopic Model**

$$(\nabla . \overline{v}) = 0 \longrightarrow \overline{v}_{z} = \overline{v}_{z}(r, t)$$

$$\frac{\partial \overline{C}_{i}}{\partial t} = -\overline{v}_{z}(r, t) \frac{\partial \overline{C}_{i}}{\partial z} - (\nabla \cdot \overline{C}_{i}' \overline{v}') + (\nabla \cdot \mathbf{D}^{(l)} \nabla \overline{C}_{i}) + R_{i}$$

$$\rho c_{p} \frac{\partial \overline{T}}{\partial t} = -\rho c_{p} \overline{v}_{z}(r, t) \frac{\partial \overline{T}}{\partial z} - \rho c_{p} (\nabla \cdot \overline{v'T'}) + (\nabla \cdot k^{(l)} \nabla \overline{T}) + (\mu^{(l)} \phi_{v}^{(l)} + \mu^{(t)} \phi_{v}^{(t)}) + \overline{S}_{r}$$

$$\rho \frac{\partial \overline{v}}{\partial t} = -\rho \overline{v} \cdot \nabla \overline{v} - \rho [\nabla \cdot \overline{v'v'}] - \nabla \overline{P} + \mu^{(l)} \nabla^{2} \overline{v} + \rho g$$

Turbulence Model (simple example):

$$\overline{v'c_i'} = J_i^{(t)} = -D^{(t)} \nabla \overline{C}_i$$

$$\rho c_p \overline{v'T'} = q^{(t)} = -k^{(t)} \nabla \overline{T}$$

$$\rho \overline{v'v'} = \tau^{(t)} = -\mu^{(t)} \nabla \overline{v}$$



Computational Fluid Dynamic (CFD)

## **Recommended Numerical Method**

#### Finite Volume:

Consist in carry out balances of properties in elementary volumes (finite volumes), or in an equivalent form in the integration over the elementary volume of the differential equation in the conservative form (or divergent form, where the fluxes appearing in the derivatives).

→ Use of CFD software (Computational Fluid Dynamics)

#### Simple example using finite-volume method:

Reaction-diffusion equations of a spherical catalytic particle

$$\begin{cases} \frac{\partial C}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial C}{\partial r} \right) - \Phi^2 C(r, t) & \text{First-order selections} \\ \frac{\partial C}{\partial r} \bigg|_{r=0} = 0 \ ; \ C(1, t) = 1 \\ C(r, 0) = C_0 \end{cases}$$

 $\Delta r_e$ 

 $dV = 4\pi r^2 dr$ 

r=1



W

w

 $\Delta r_w$ 

r=0

## Simple example using finite-volume method:

$$C_{p} = \frac{\int_{w}^{e} Cr^{2}dr}{\int_{w}^{e} r^{2}dr} = \frac{3}{\left(r_{e}^{2} - r_{w}^{2}\right)}\int_{w}^{e} Cr^{2}dr \qquad \text{mean value}$$

$$\frac{dC_p}{dt} = \frac{3}{\left(r_e^3 - r_w^3\right)} \left[r^2 \frac{\partial C}{\partial r}\right]_w^e - \Phi^2 C_p \qquad \qquad \frac{\partial C}{\partial r}\Big|_{r=r_e} = \frac{C_E - C_p}{\Delta r_e} \qquad \qquad \frac{\partial C}{\partial r}\Big|_{r=r_w} = \frac{C_p - C_W}{\Delta r_w}$$

$$\frac{dC_p}{dt} = A_W C_W + A_p C_p + A_E C_E - \Phi^2 C_p \qquad p = 2, ..., N-1$$

$$A_{W} = \frac{3r_{w}^{2}}{\left(r_{e}^{3} - r_{w}^{3}\right)\Delta r_{w}} \qquad A_{p} = \frac{-3}{\left(r_{e}^{3} - r_{w}^{3}\right)} \left(\frac{r_{e}^{2}}{\Delta r_{e}} + \frac{r_{w}^{2}}{\Delta r_{w}}\right) \qquad A_{E} = \frac{3r_{e}^{2}}{\left(r_{e}^{3} - r_{w}^{3}\right)\Delta r_{e}}$$

#### Simple example using finite-volume method:

**Boundary conditions** 





 $r_{\psi} = 0$ 

$$\left.\frac{\partial C}{\partial r}\right|_{r_w} = 0$$

 $r_e = 1$ 



Resulting systems:

$$\frac{d\underline{C}}{dt} = A\,\underline{C} + \underline{b}$$

where A is a tridiagonal matrix

Or a non-linear system for reaction of order  $\neq$  1:

$$\frac{d\underline{C}}{dt} = \underline{F}(\underline{C})$$

#### **Multiple Gradients Model**

$$\begin{split} \overline{\mathbf{D}} &= \mathbf{D}^{(t)} + \mathbf{D}^{(l)} \qquad \overline{k} = k^{(t)} + k^{(l)} \qquad \overline{\mu} = \mu^{(t)} + \mu^{(l)} \\ \frac{\partial \overline{C}_i}{\partial t} &= -\overline{v}_z(r,t) \frac{\partial \overline{C}_i}{\partial z} + (\nabla \cdot \overline{\mathbf{D}} \nabla \overline{C}_i) + R_i \\ \rho c_p \frac{\partial \overline{T}}{\partial t} &= -\rho c_p \overline{v}_z(r,t) \frac{\partial \overline{T}}{\partial z} + (\nabla \cdot \overline{k} \nabla \overline{T}) + \overline{\mu} \overline{\phi}_v + \overline{S}_r \\ \rho \frac{\partial \overline{v}_z}{\partial t} &= -\nabla \overline{P} + \overline{\mu} \nabla^2 \overline{v}_z + \rho g \end{split}$$

**Boundary Conditions:** 



#### **Component Mass Balance**

(removing time average notation)

$$\frac{\partial C_i}{\partial t} = \mathcal{D}_z(r,t) \frac{\partial^2 C_i}{\partial z^2} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \mathcal{D}_R(r,t) \frac{\partial C_i}{\partial r} \right) - v_z(r,t) \frac{\partial C_i}{\partial z} + R_i$$

$$v_{z}(r,t)C_{i0}(t) = v_{z}(r,t)C_{i}(0,r,t) - D_{z}(r,t)\frac{\partial C_{i}(0,r,t)}{\partial z}$$

$$\frac{\partial C_i}{\partial z}(L,r,t) = 0$$
 no reaction at exit

$$\frac{\partial C_i}{\partial r}(z,0,t) = 0$$

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Symmetry

 $\frac{\partial C_i}{\partial r}(z, R, t) = 0$ 

 $C_i(z, r, 0) = C_{i_{-}in}(z, r)$ 

Impermeable wall

**Initial condition** 

### **Energy Balance**

(removing time average notation)

$$\begin{split} \rho c_{p} \frac{\partial T}{\partial t} &= k_{z}(r,t) \frac{\partial^{2} T}{\partial z^{2}} + \frac{1}{r} \frac{\partial}{\partial r} \left( r k_{R}(r,t) \frac{\partial T}{\partial r} \right) - \rho c_{p} v_{z}(r,t) \frac{\partial T}{\partial z} + \Delta H_{r} R_{A} \\ v_{z}(r,t) T_{0}(t) &= v_{z}(r,t) T(0,r,t) - \frac{k_{z}(r,t)}{\rho c_{p}} \frac{\partial T(0,r,t)}{\partial z} \\ \frac{\partial T}{\partial z} (L,r,t) &= 0 \\ \frac{\partial T}{\partial r} (z,0,t) &= 0 \\ k_{R}(R,t) \frac{\partial T}{\partial r} (z,R,t) &= U \left[ T_{w} - T(z,R,t) \right] \\ Heat exchange with the wall \\ T(z,r,0) &= T_{in}(z,r) \end{split}$$

**Initial condition** 

#### **Momentum Balance**

(removing time average notation)

$$\rho \frac{\partial v_z}{\partial t} = -\frac{\partial P}{\partial z} + \mu \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_z}{\partial r} \right)$$

$$\frac{\partial v_z}{\partial r}(0,t) = 0$$
$$v_z(R,t) = 0$$
$$v_z(r,0) = v_{z_in}(r)$$

Symmetry

**Fixed wall** 

**Initial Condition** 

Additional assumptions:

- constant effective diffusive coefficients
- constant velocity

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## **Method of Lines**

- spatial discretization (finite differences, finite volumes, finite elements)
- time integration

Example: finite differences in axial direction and orthogonal collocation in radial

$$\frac{dC_{i,j,k}}{dt} = D_L \left( \frac{C_{i,j+1,k} - 2C_{i,j,k} + C_{i,j-1,k}}{\Delta z^2} \right) + 4D_R \left[ \sum_{m=0}^{n+1} \left( u_k B_{k,m} + A_{k,m} \right) C_{i,j,m} \right] - v_z \left( \frac{C_{i,j+1,k} - C_{i,j-1,k}}{2\Delta z} \right) + R_{i,j,k}$$

$$Dc_p \frac{dT_{j,k}}{dt} = k_L \left( \frac{T_{j+1,k} - 2T_{j,k} + T_{j-1,k}}{\Delta z^2} \right) + 4k_R \left[ \sum_{m=0}^{n+1} \left( u_k B_{k,m} + A_{k,m} \right) T_{j,m} \right] - \rho c_p v_z \left( \frac{T_{j+1,k} - T_{j-1,k}}{2\Delta z} \right) + \Delta H_r R_{A,j,k}$$
where:
$$u = r^2 \qquad l_m(u) = \prod_{\substack{p=0\\p\neq m}}^{n+1} \frac{u - u_p}{u_m - u_p} \qquad A_{k,m} = \frac{dl_m(u_k)}{du} \qquad B_{k,m} = \frac{d^2 l_m(u_k)}{du^2}$$

$$j = 1, 2, \dots, N \qquad k = 1, 2, \dots, n \qquad y(u) \cong P_{n+1}^{(\alpha,\beta)}(u) = \sum_{m=0}^{n+1} l_m(u) y_m$$

## **Method of Lines**

**Boundary conditions** 

$$v_z C_{i0}(t) = v_z C_{i,1,k} - D_L \left( \frac{C_{i,2,k} - C_{i0}(t)}{2\Delta z} \right)$$

 $C_{i,N+1,k} - C_{i,N,k} = 0$ 

$$\sum_{m=0}^{n+1} A_{0,m} C_{i,j,m} = 0$$

 $\sum_{m=0}^{n+1} A_{n+1,m} C_{i,j,m} = 0$ 

 $C_{i,j,k} = C_{i_i,j,k}$ j = 1, 2, ..., N k = 1, 2, ..., n



$$v_{z} T_{0}(t) = v_{z} T_{1,k} - \frac{k_{L}}{\rho c_{p}} \left( \frac{T_{2,k} - T_{0}(t)}{2\Delta z} \right)$$

 $T_{N+1,k} - T_{N,k} = 0$ 

$$\sum_{m=0}^{n+1} A_{0,m} T_{j,m} = 0$$

$$k_{R} \sum_{m=0}^{n+1} A_{n+1,m} T_{j,m} = U \left[ T_{w} - T_{j,n+1} \right]$$

 $T_{i,k} = T_{in, i,k}$ 

#### **Another Multiple Gradients Model**

(ignoring radial gradients) – PFR with axial dispersion

$$\frac{\partial C_{i}}{\partial t} = D_{L} \frac{\partial^{2} C_{i}}{\partial z^{2}} - v_{z} \frac{\partial C_{i}}{\partial z} + R_{i}$$

$$v_{z} C_{i0}(t) = v_{z} C_{i}(0,t) - D_{L} \frac{\partial C_{i}(0,t)}{\partial z}$$

$$\frac{\partial C_{i}}{\partial z}(L,t) = 0$$

$$C_{i}(z,0) = C_{i,im}(z)$$

$$\rho c_{p} \frac{\partial T}{\partial t} = k_{L} \frac{\partial^{2} T}{\partial z^{2}} - \rho c_{p} v_{z} \frac{\partial T}{\partial z} + \frac{2U}{R} (T_{w} - T) + \Delta H_{r} R_{A} \quad \longleftarrow \quad \overline{T}(z,t) = \frac{\int_{0}^{R} T(r,z,t) r \, dr}{\int_{0}^{R} r \, dr}$$

$$v_{z} T_{0}(t) = v_{z} T(0,t) - \frac{k_{L}}{\rho c_{p}} \frac{\partial T(0,t)}{\partial z}$$

$$\frac{\partial T}{\partial z}(L,t) = 0$$

$$T(z,0) = T_{im}(z)$$

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## **Method of Lines**

- spatial discretization (F.D., F.V., orthogonal collocation on finite elements)
- time integration

Example: finite differences in axial direction

$$\begin{aligned} \frac{dC_{i,j}}{dt} &= D_L \left( \frac{C_{i,j+1} - 2C_{i,j} + C_{i,j-1}}{\Delta z^2} \right) - v_z \left( \frac{C_{i,j+1} - C_{i,j-1}}{2\Delta z} \right) + R_{i,j} \\ p c_p \frac{dT_j}{dt} &= k_L \left( \frac{T_{j+1} - 2T_j + T_{j-1}}{\Delta z^2} \right) - \rho c_p v_z \left( \frac{T_{j+1} - T_{j-1}}{2\Delta z} \right) + \frac{2U}{R} \left( T_w - T_j \right) + \Delta H_r R_{A,j} \\ q_z C_{i0}(t) &= v_z C_{i,1} - D_L \left( \frac{C_{i,2} - C_{i0}(t)}{2\Delta z} \right) \\ C_{i,N+1} - C_{i,N} &= 0 \end{aligned}$$

$$\begin{aligned} T_{N+1} - T_N &= 0 \\ C_{i,j} &= C_{i_-in,j} \end{aligned}$$

## **Maximum Gradient Model**

(ignoring axial dispersion) – PFR without axial dispersion

$$\frac{\partial C_i}{\partial t} = -v_z \frac{\partial C_i}{\partial z} + R_i$$
$$C_i(0,t) = C_{i0}(t)$$
$$C_i(z,0) = C_{i\_in}(z)$$

$$\rho c_p \frac{\partial T}{\partial t} = -\rho c_p v_z \frac{\partial T}{\partial z} + \frac{2U}{R} (T_w - T) + \Delta H_r R_A$$
$$T(0,t) = T_0(t)$$
$$T(z,0) = T_{in}(z)$$

Results in an EDO system!

## **Macroscopic Model**

$$V \frac{dC_i}{dt} = C_{i0}(t) v_z S - C_i v_z S + \overline{R}_i V$$

$$C_i(0) = \overline{C}_{i\_in}$$

$$\overline{C}_i(0) = \overline{C}_{i\_in}$$

$$\overline{C}_i(0) = \overline{C}_i v_z S T_0(t) - \rho c_p v_z S T + U A_i (\overline{T_w - T}) + \Delta H_r \overline{R}_A V \quad \overline{T}(t) = \frac{\int_0^L T(z, t) S dz}{\int_0^L S dz}$$

$$T(0) = \overline{T}_{in}$$



# Method of Orthogonal Collocation with EMSO



DD as Plugin (Type="OCFEM", Boundary="BOTH", InternalPoints=5 alfa=1, beta=1)

Plugin: ocfem\_emso.dll

## Fixed-bed Reactor with Axial Dispersion (reaction of order *m*)

$$\frac{\partial y}{\partial \tau} + \frac{\partial y}{\partial x} = \frac{1}{Pe} \frac{\partial^2 y}{\partial x^2} - Da y^m$$

Boundary conditions:

$$-\frac{1}{Pe} \frac{\partial y}{\partial x}\Big|_{x=0} = 1 - y(\tau, 0) \quad \text{or} \quad y(\tau, 0) = 1$$
$$\frac{\partial y}{\partial x}\Big|_{x=1} = 0$$

Initial conditions:

y(0,x) = 0

## PDE

## Method of Lines: D.F. and Orthogonal Collocation

**Example:** add Plugin ocfem\_emso.dll and execute flowsheets of files FDM\_ss.mso, OCM\_ss.mso and OCFEM\_ss.mso, and compare results of discretizations. Repeat for the dynamic simulation in files FDM\_din.mso e OCM\_din.mso.



# **Comparing Results**



OCM by EMSO  $\alpha = 1$   $\beta = 1$ Number of internal points: 5

y(x=1) = 0.151475 (error of 0.038%)



Method of Finite Differences Number of internal points: 6000

y(x=1) = 0.15155 (error of 0.087%)

y(x=1) = 0.151418 (exact)

## **Case Study**

- Production of acetic anhydride in adiabatic PFR
  - Acetic anhydride is often produced by reacting acetic acid with ketene, obtained by heating acetone at 700-770°C.
  - A important step is the vapor phase cracking of acetone to ketene and methane:

 $CH_{3}COCH_{3} \rightarrow CH_{2}CO + CH_{4}$ 

- The second step is the reaction of ketene with acetic acid.

 $CH_2CO + CH_3COOH \rightarrow (CH_3CO)O$ 

Ref: G. V. Jeffreys, *A Problem in Chemical Engineering Design: The Manufacture of Acetic Anhydride*, 2<sup>nd</sup> ed. (London: Institution of Chemical Engineers, 1964)

## **Case Study**

## Problem Definition

 The first production step is carried out in a vapor phase reaction of acetone in an adiabatic PFR.



where A = acetone; B = ketene and C = methane

 $A \rightarrow B + C$ 

 The reaction is of 1<sup>a</sup> order in relation to acetone in the cracking reaction, with Arrhenius constant given by:

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• 
$$k - \text{seconds}^{-1}$$
  
•  $T - \text{Kelvin}$   $k = \exp\left(34.34 - \frac{34222}{T}\right)$ 

# **Case Study**

## **Process Description**

- Reactor geometry
  - adiabatic continuous tubular reactor;
  - bank of 1000 tubes of 1 in sch. 40 with cross section of 0.557 m<sup>2</sup>;
  - total length of 2.28 m;
- Operating conditions
  - feed temperature 762°C (1035 K);
  - operating pressure: 1.6 atm
  - feed flow rate of 8000 kg/h (137.9 kmol/h);
- Composition
  - acetone, ketene and methane
  - feed of pure acetone
- Kinetics
  - first order reaction,
  - pre-exponential factor ( $k_0$ ): 8.2 x 10<sup>14</sup> s<sup>-1</sup>
  - activation energy (E/R): 34222 K
  - heat of reaction: -80.77 kJ/mol

## **Case study** – Production of acetic anhydride –

**Example:** run FlowSheet in file PFR\_Adiabatico.mso and plot steady-state temperature and composition profiles. Show also the evolution of the temperature profile. Discuss the type and quality of discretization.



## **Exercise**

Solve the reaction-diffusion problem in a spherical catalytic particle, given by:

$$\frac{\partial y}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial y}{\partial r} \right) - \Phi^2 y^{1/2}$$

$$\frac{\partial y}{\partial r}\Big|_{r=0} = 0 \qquad y(t,r)\Big|_{r=1} = 1 \qquad y(t,r)\Big|_{t=0} = 0$$

 $\Phi = 2$  (Thiele modulus)

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#### **Special thanks to**

Prof. Rafael de Pelegrini Soares, D.Sc.Eng. Gerson Balbueno Bicca, M.Sc.Eng. Euclides Almeida Neto, D.Sc.Eng. Eduardo Moreira de Lemos, D.Sc.Eng. Marco Antônio Müller

## For helping in the preparation of this material





For supporting the ALSOC Project.

## **EP 2013**

### ... thank you for your attention!



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