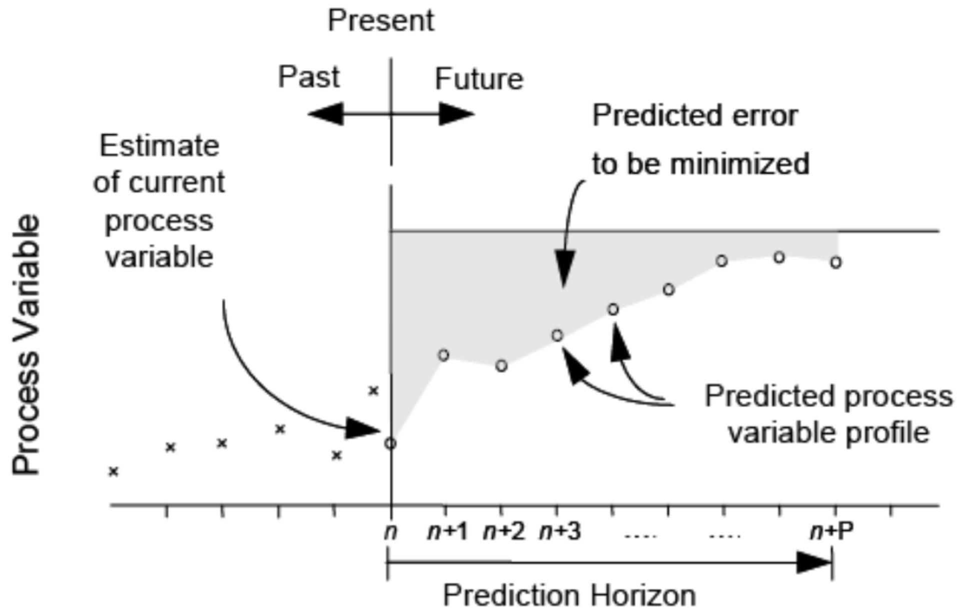
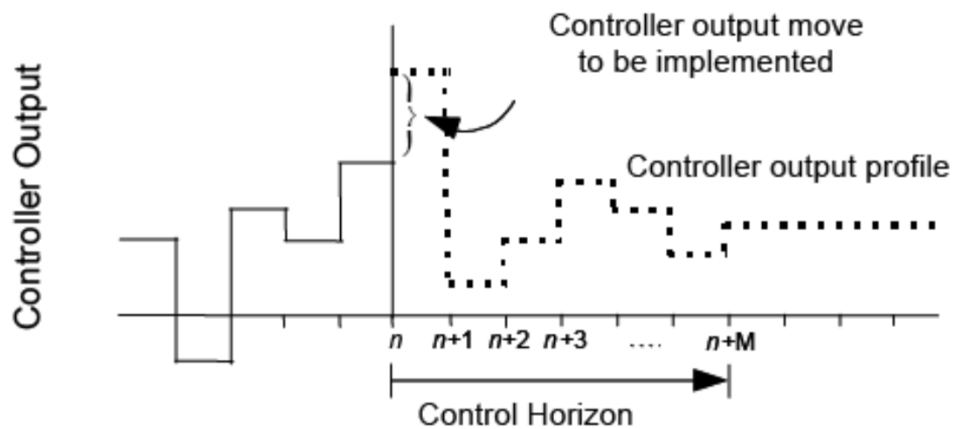
Controle de Processos

Professor: Argimiro

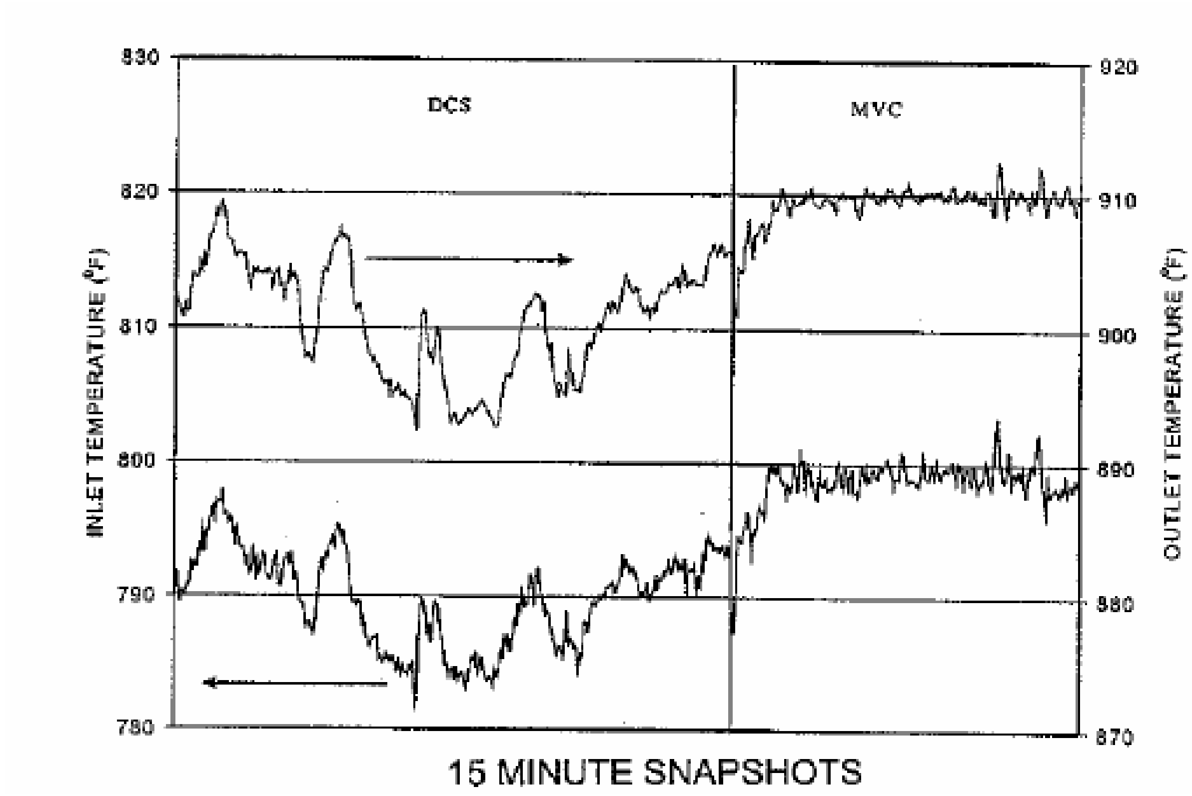
Facilitador: Perez

O que é o Controle Preditivo Multivariável



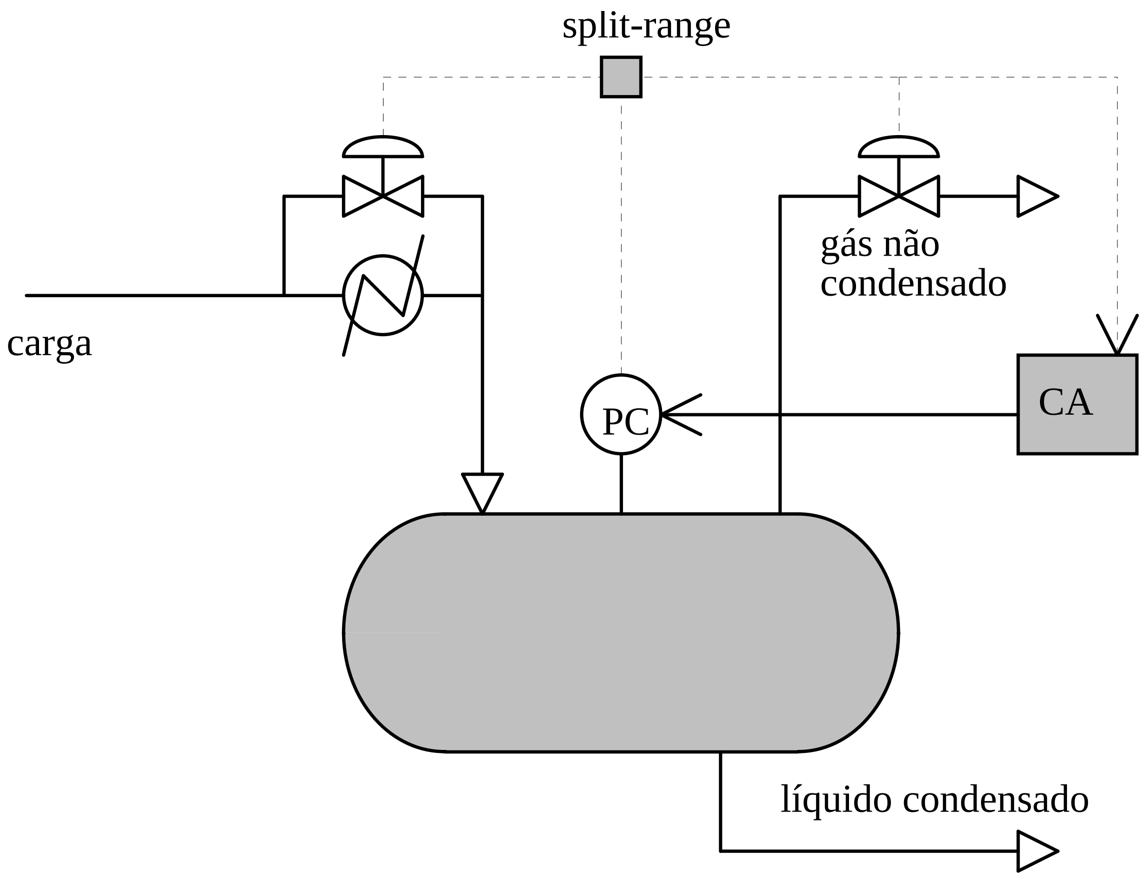


O que é o Controle Preditivo Multivariável



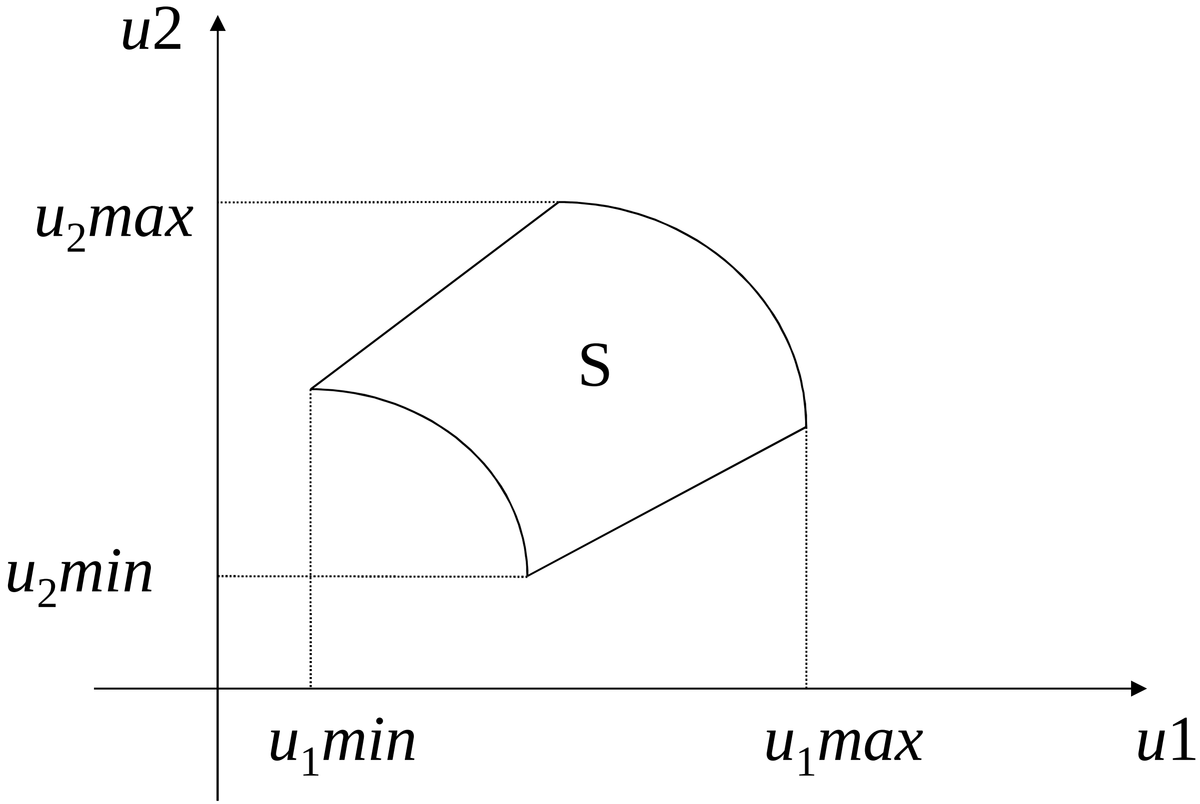
Odloak [1]

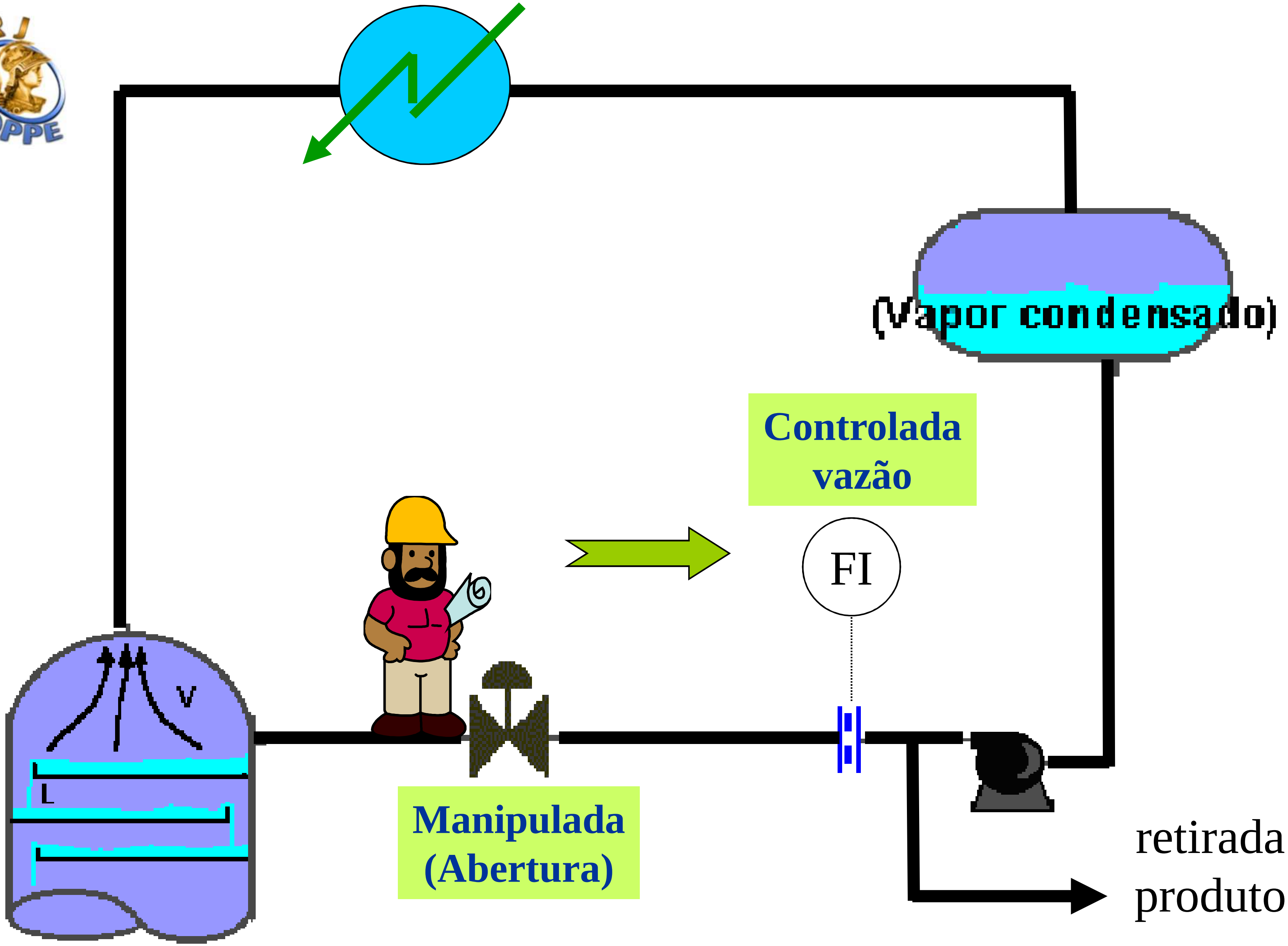
O que é o Controle Preditivo Multivariável

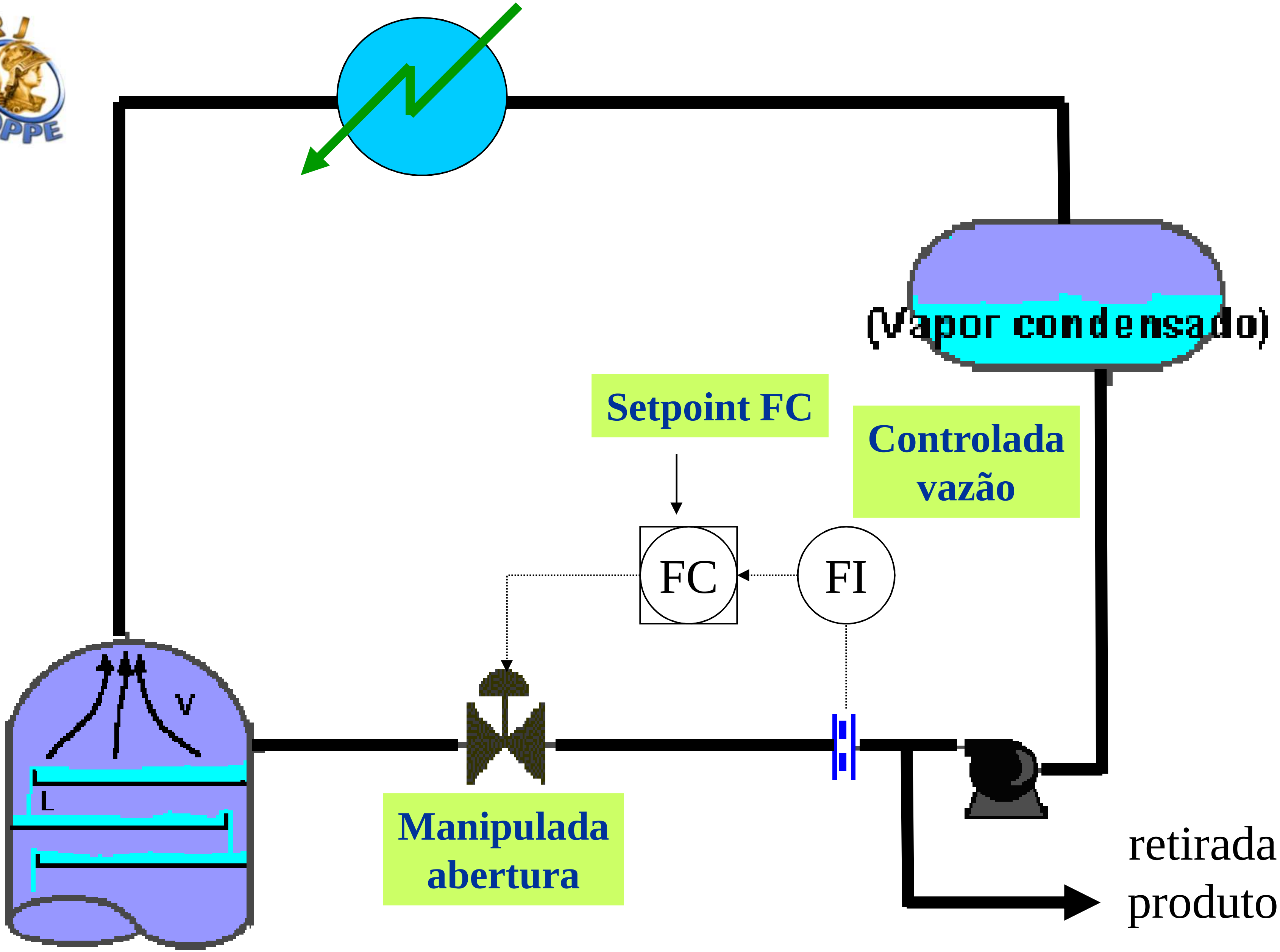


TriSolutions [2]

O que é o Controle Preditivo Multivariável




(Vapor condensado)
Controlada
vazão
FI
V
L
Manipulada
(Abertura)
retirada
produto



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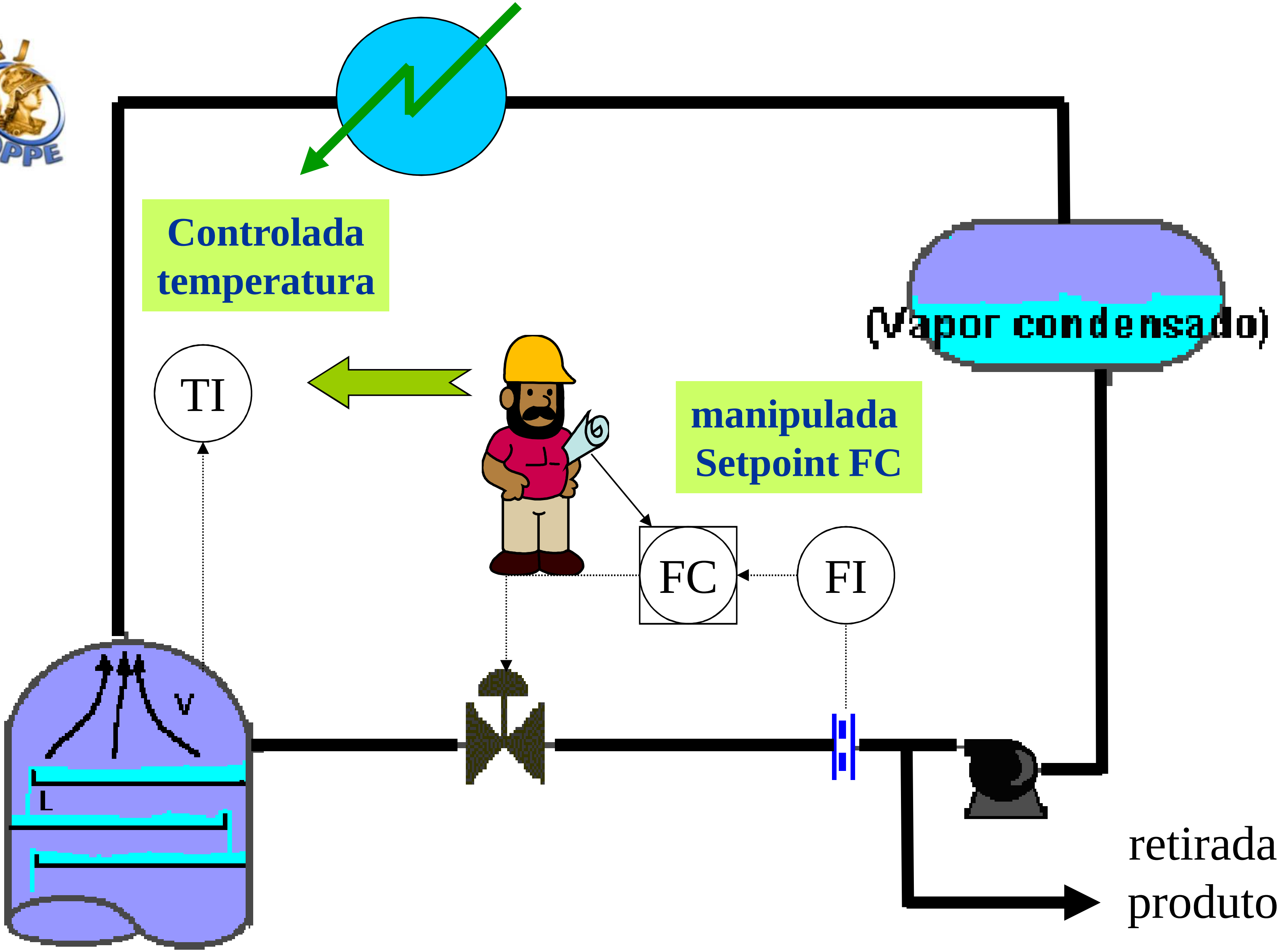

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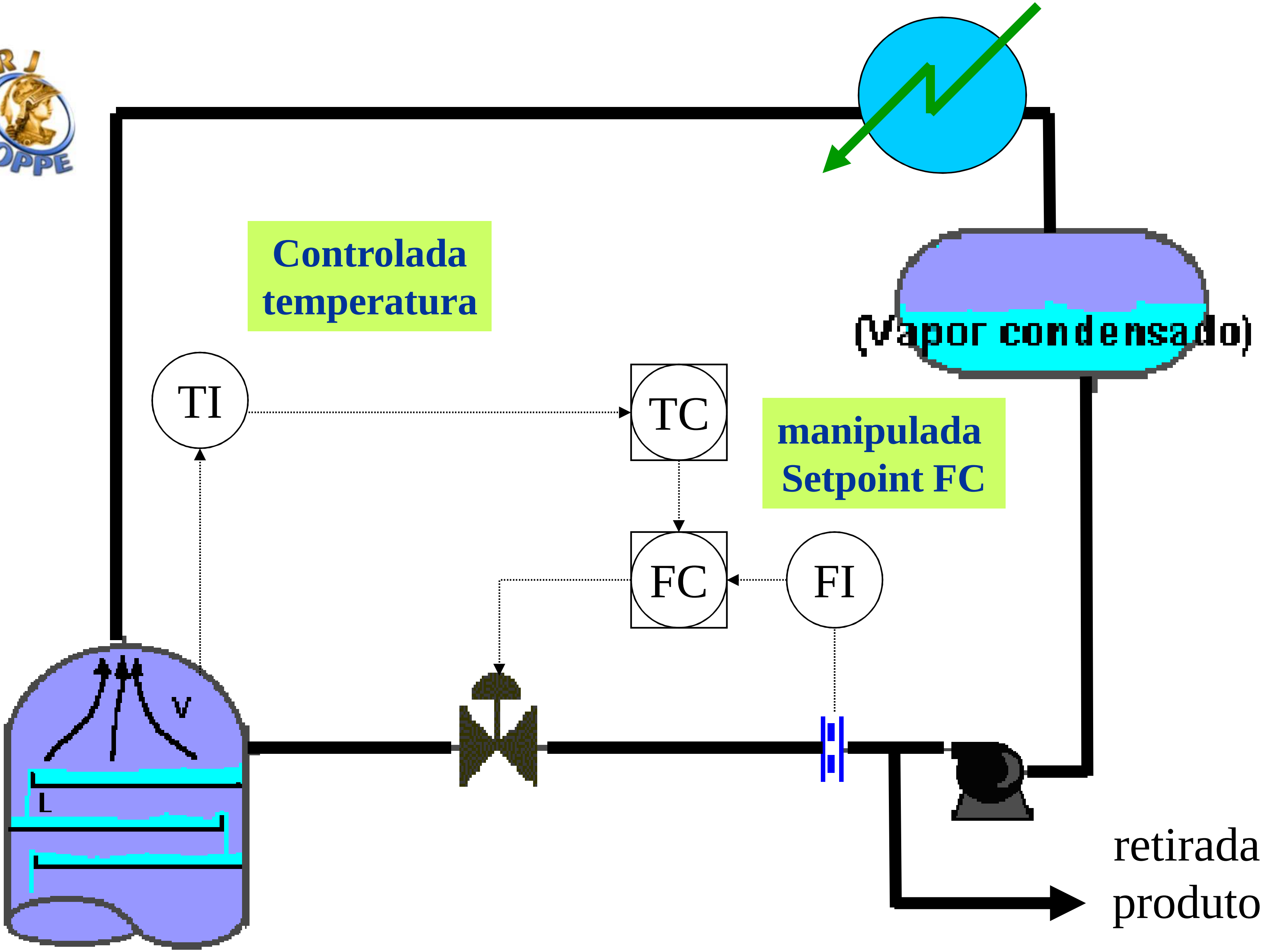

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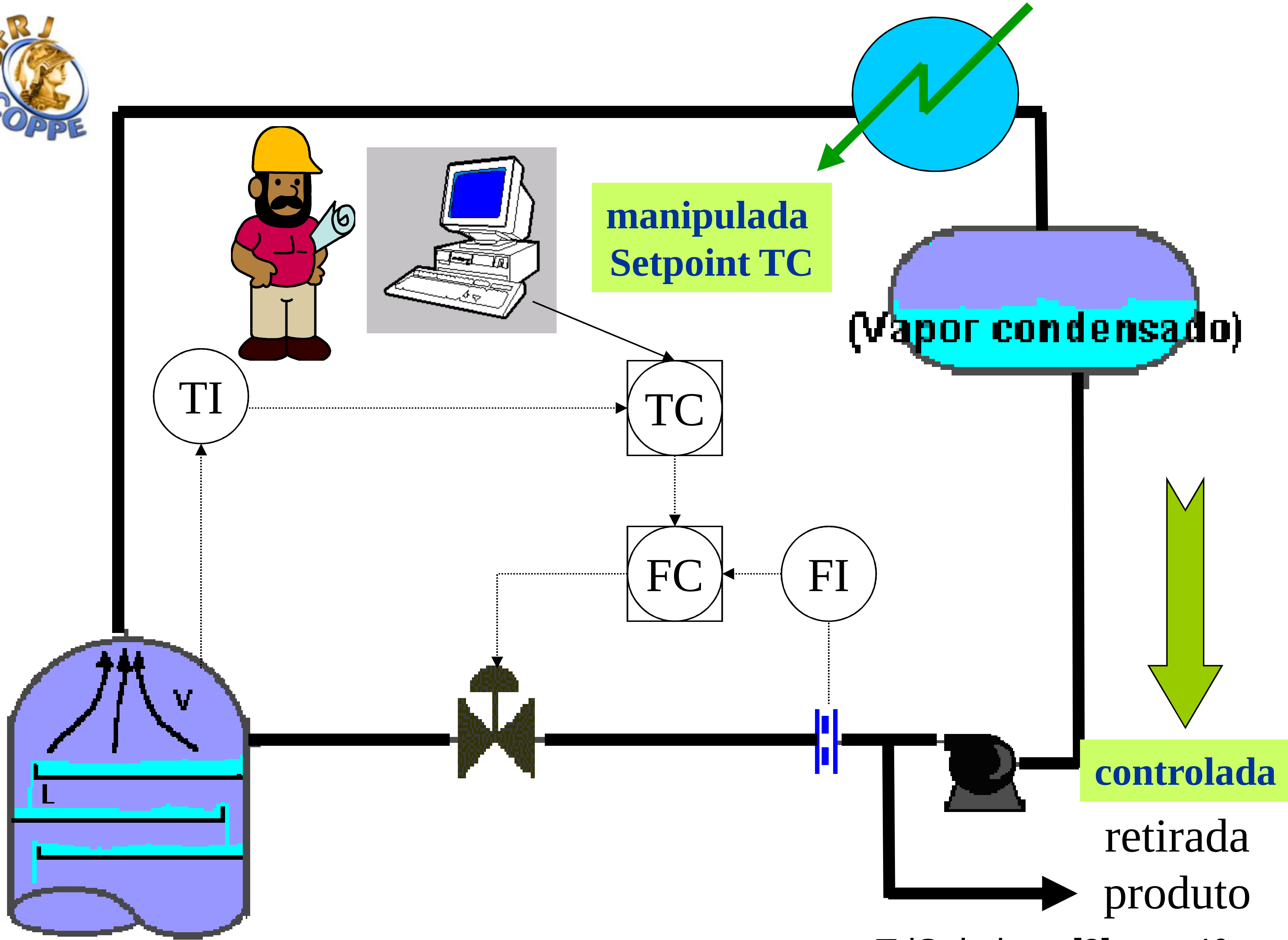
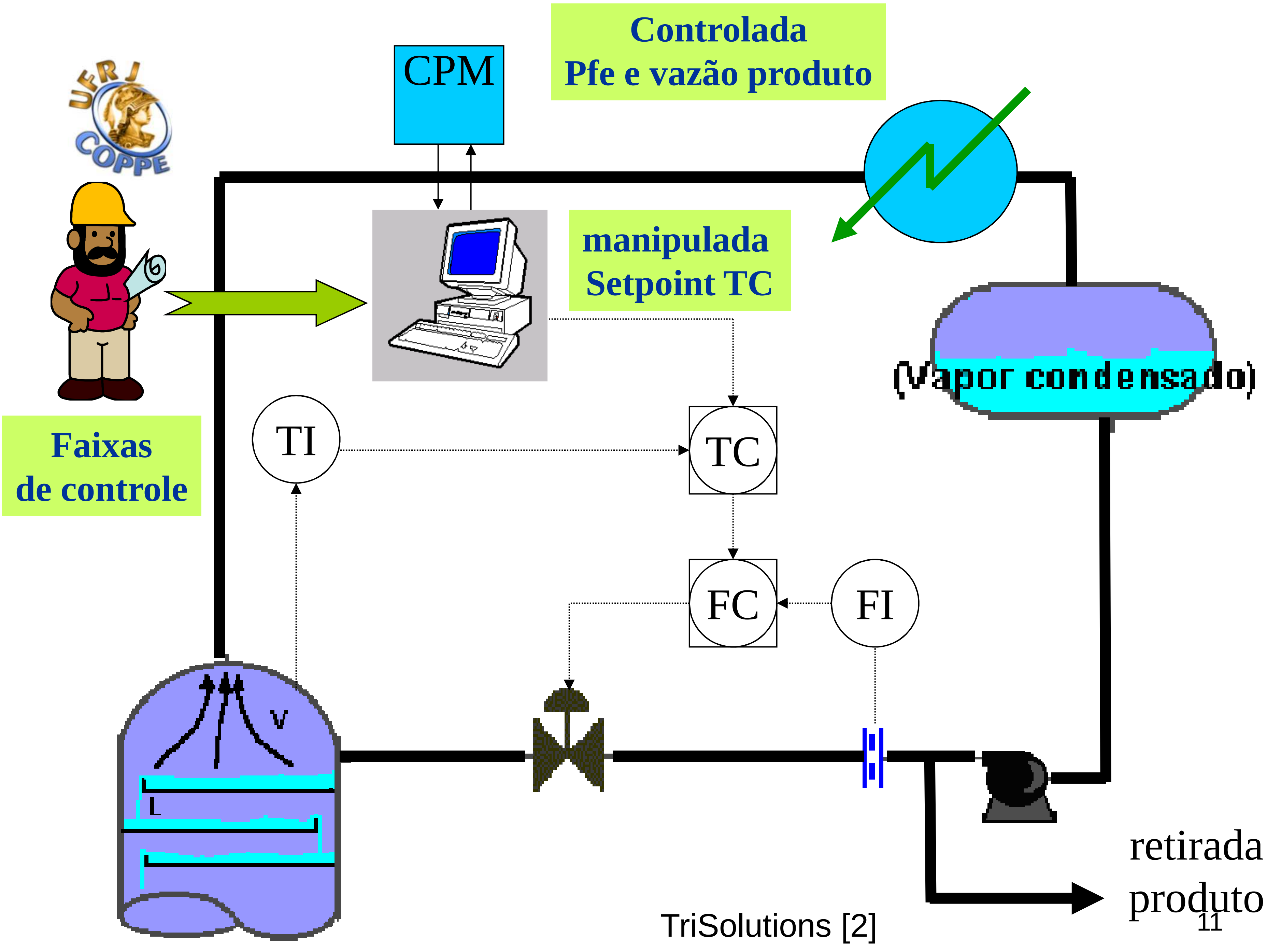


TriSolutions [2]


Controlada
Pfe e vazão produto
CPM
manipulada
Setpoint TC
Faixas
de controle
(Vapor condensado)
TI
TC
FC
FI
V
L
retirada
produto
TriSolutions [2]

OBJETIVOS

- Conceitos Básicos de Controle Preditivo
- Fases de Implantação
  - Elaboração do Projeto conceitual
  - Identificação de Processos
  - Ajustes dos modelos
  - Implantação
  - Sintonia

Conceitos Básicos de Otimização

Problema de Otimização

Elementos importantes na Otimização:

- Função Objetivo
- Variáveis
- Restrições
- Graus de Liberdade

Função Objetivo

- Indicador quantitativo da solução
- É um escalar
- Funções econômicas (lucros,custos)
- Critérios de desempenho
  - ✓somatório dos erros ao quadrado

Variáveis no problema de otimização

- Dimensões de equipamentos
- Condições de operação
- Saídas para o controle regulatório (*u*)
- As variáveis podem ser:
  - Variáveis Independentes ou de decisão ou de otimização
  - Variáveis Dependentes

Restrições no problema de otimização

- Relação entre variáveis
- Podem ser inequações ou equações
  - Balanços geram equações
  - Limites de Operação geram inequações

Graus de liberdade no problema de Otimização

- **Número de variáveis – Número de equações**

•Em uma simulação, o grau de liberdade é zero
• Em um problema de otimização, o grau de liberdade deve ser maior que zero.

Problema completo de Otimização

Min f(y,u,x)

sujeito a:

$h(y,u,x) = 0$

$g(y,u,x) \leq 0$

onde:

y: variáveis discretas

x: variáveis contínuas

u: variáveis de decisão

Resolução de um problema de Otimização

1. *Programação Linear (LP)*
2. *Programação Não Linear (NLP)*
3. *Programação Quadrática (QP)*
4. *Programação Mista Inteira Linear (MILP)*
5. *Programação Mista Inteira* Não *Linear (MINLP)*

Programação Linear LP

- ✓ Todas funções são lineares → f, g e h são lineares
- ✓ Não há variáveis discretas (y=0)

Programação Não Linear (NLP)

- ✓ Pelo menos uma função é não linear
  → f, g e/ou h não linear
- ✓ Não há variáveis discretas (y=0)

Programação Quadrática (QP)

- ✓ É um caso especial da NLP onde a função objetivo é do tipo quadrática

$$f(x) = C^T X + X^T A X$$

Quadrático (→ $X^T A X$)

- ✓ Não há variáveis discretas (y=0)

Otimização

$$X^T A X =$$

$$\begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} a_{11}x_1 + a_{12}x_2 \\ a_{21}x_1 + a_{22}x_2 \end{bmatrix}$$

$$= a_{11}x_1{}^2 + a_{12}x_1x_2 + a_{21}x_1x_2 + a_{22}x_2{}^2$$

Programação Mista Inteira Linear (MILP)

✓ Todas funções são lineares

→ f, g e h são lineares

✓ Há variáveis discretas ($y \neq 0$)

Programação Mista Inteira Não Linear (NMILP)

✓ Pelo menos uma função não é linear

→ f, g e/ou h não lineares

✓ Há variáveis discretas ($y \neq 0$)

Controladores Preditivos

- Histórico dos MPC's:
  - o MAC – Model Algorithmic Control – 1976
  - o DMC – Dynamic Matrix Control – Cutler, 1979
  - o LDMC – Linear Dynamic Matrix Control, 1983 - *utilizado no SICON da Petrobras*
  - o QDMC – Quadratic Dynamic Matrix Control – Morshedt, 1985

Algoritmo de Um MPC

1 - Através de um modelo implementado no controlador, o MPC é capaz de realizar a predição do comportamento da saída do processo, levando em consideração as entradas de controle atuais e futuras.

2 - Esta predição deve ser corrigida, a cada intervalo de instante, por uma leitura da planta. Um MPC opera, tipicamente, com intervalos de tempo na faixa de um minuto.

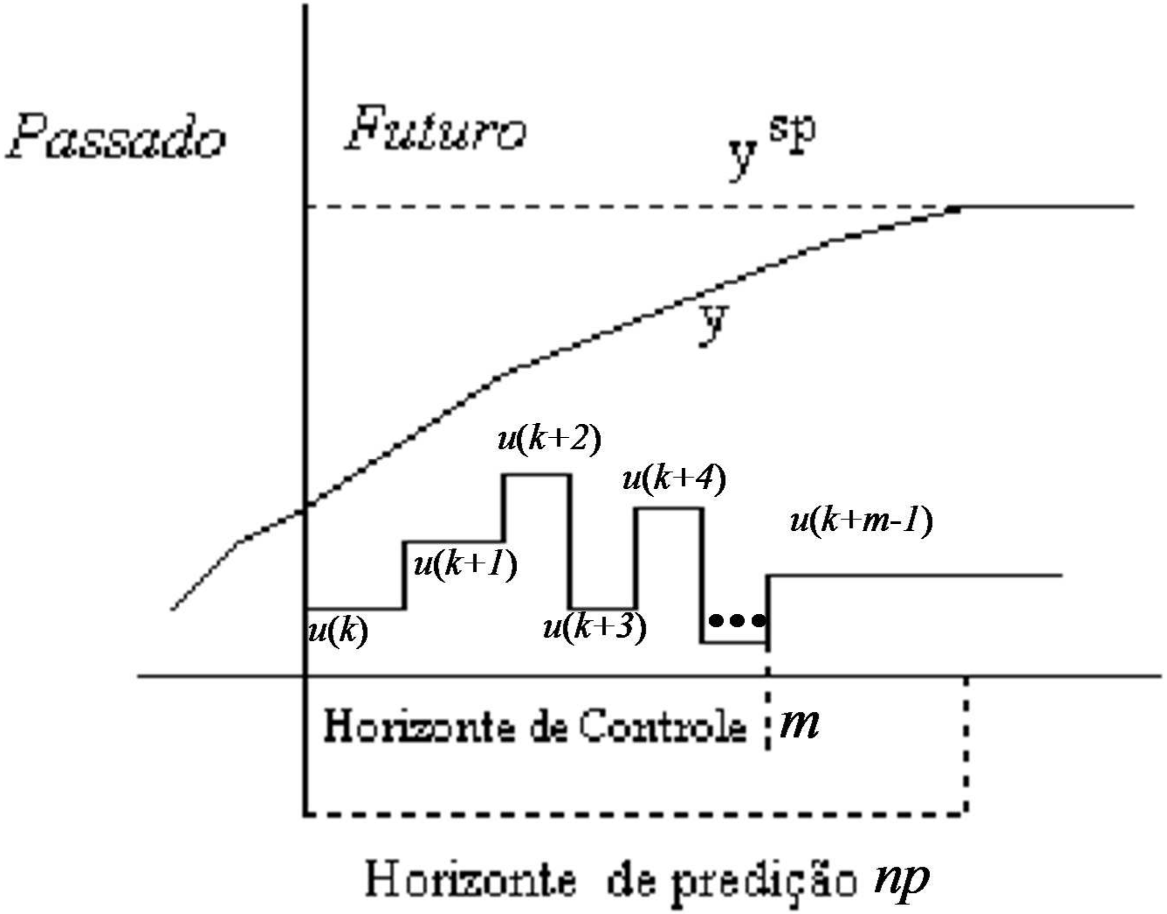
3 - Em cada iteração, o controlador calcula uma sequência de ações de controle que minimiza a função do erro das saídas previstas até um horizonte definido como horizonte de predição. O tamanho desta sequência é definido como horizonte de controle.

Algoritmo de Um MPC

4 - Após resolver o problema de otimização descrito no item 3, o controlador implementa na planta apenas a primeira ação de controle dentre a sequência de ações calculadas que vão do intervalo de instante atual até o intervalo correspondente ao horizonte de controle *m* ajustado no controlador.

5 - O controlador aguarda o próximo intervalo de tempo para retornar ao item 1.

Algoritmo de Um MPC



Definições do MPC

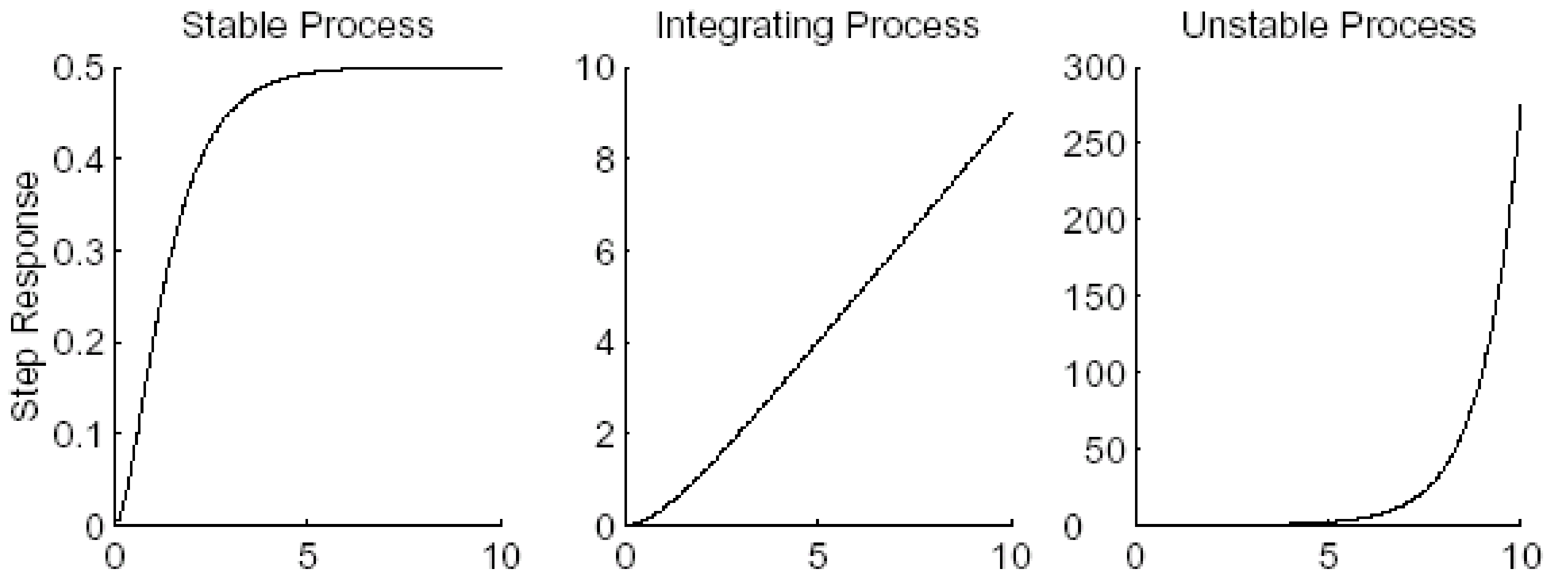
- ✓ Tempo de estabilização da planta (*N*): É o tempo que a planta estabiliza para uma perturbação em degrau aplicada à entrada.
- ✓ Horizonte de Predição (*np*): É o intervalo de tempo em que as variáveis de saída são preditas pelo controlador. Deve ser menor ou igual ao tempo de estabilização. Os controladores mais modernos trabalham com um horizonte de predição infinito.
- ✓ Horizonte de Controle (*m*): É o horizonte de cálculo das variáveis de entrada do processo. Normalmente o horizonte de controle é menor que o horizonte de predição.

Controladores Preditivos

- O MPC Linear é baseado em modelos lineares.
- Como representar o processo?
  - o Resposta ao impulso – Finite Impulse Response
  - o Resposta ao degrau
  - o Através de funções de transferência
    - Contínua $Y(s)/u(s) = G_P(s)=Q(s)/P(s)$
    - Discreta $Y(z)/u(z) = HG_P(z)=Q(z)/P(z)$
  - o Através de variáveis de estado (equações em espaço de estados)
    - $x(k+1)=Ax(k) + Bu(k)$
    - $y(k) = Cx(k) + Du(k)$ → *um sistema normalmente não responde imediatamente a entrada. Portanto, em um sistema real, D = 0.*

Controladores Preditivos

Tipos de processos e suas respostas ao degrau



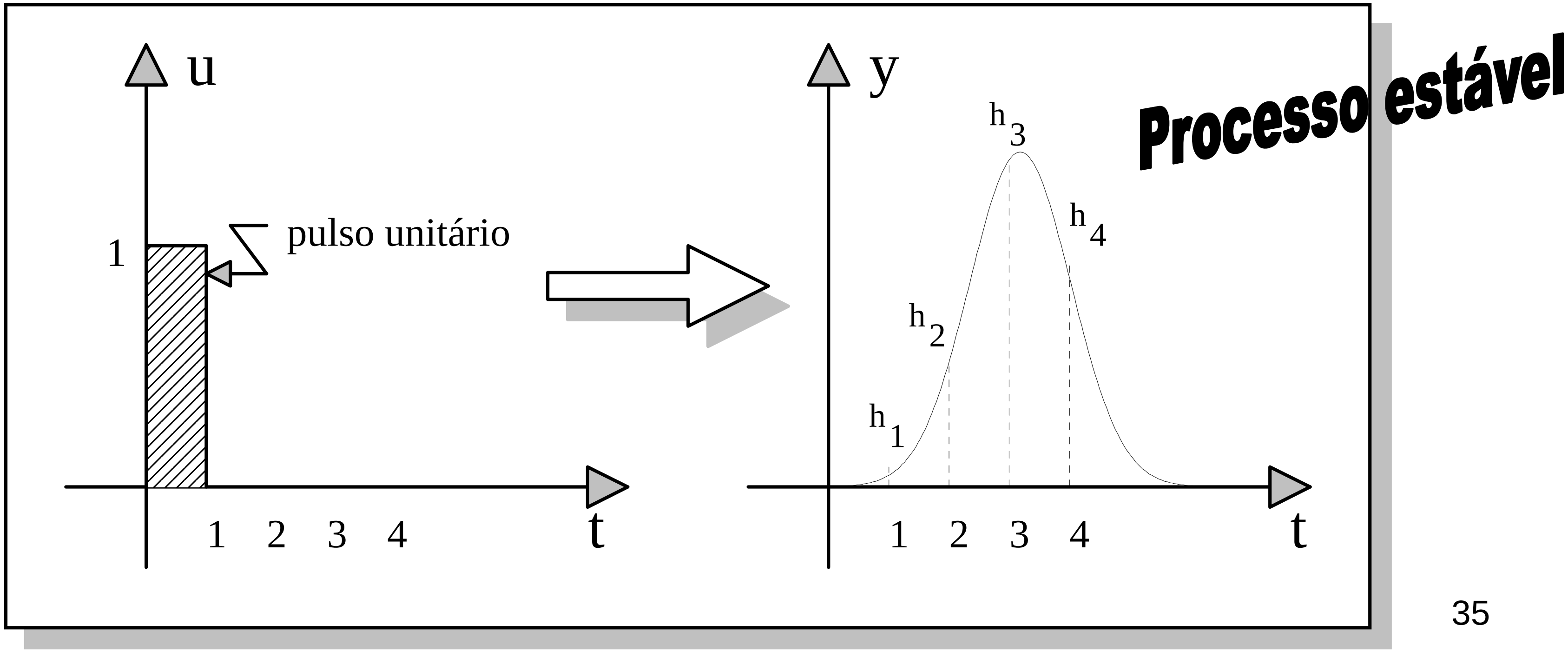
Resposta FIR

Resposta ao FIR

Modelo obtido a partir da resposta ao impulso (FIR: Finitive Impulse Response)

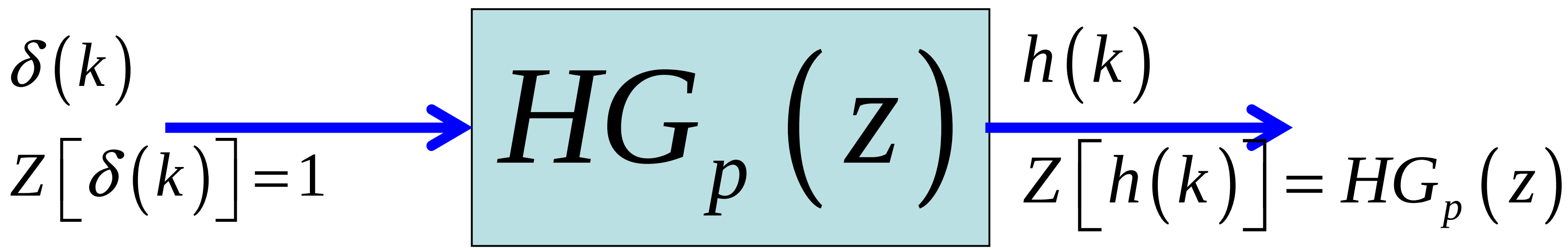
- ✓ Não tem sentido se falar em impulso em um sistema digital. Apenas em pulso unitário.

hi : Valor da saída no instante *i* após a aplicação do pulso – coeficientes da resposta ao impulso.



Resposta ao FIR

- Como a transformada Z do pulso de dirac é 1, as saídas do processo representam a sua própria função de transferência.



AÇÕES DE FUTURO E PASSADO

Resposta ao FIR

- Para um dado instante *i*:

$$y(k+i) = h_i u(k)$$

- Considerando um período de estabilização N e considerando a ação dos pulsos ocorridos no passado até esse instante N, a saída é uma soma das ações de cada um desses pulsos:

$$y(k+i) = \sum_{j=1}^{N} h_j u(k+i-j)$$

APENAS AÇÕES DE FUTURO

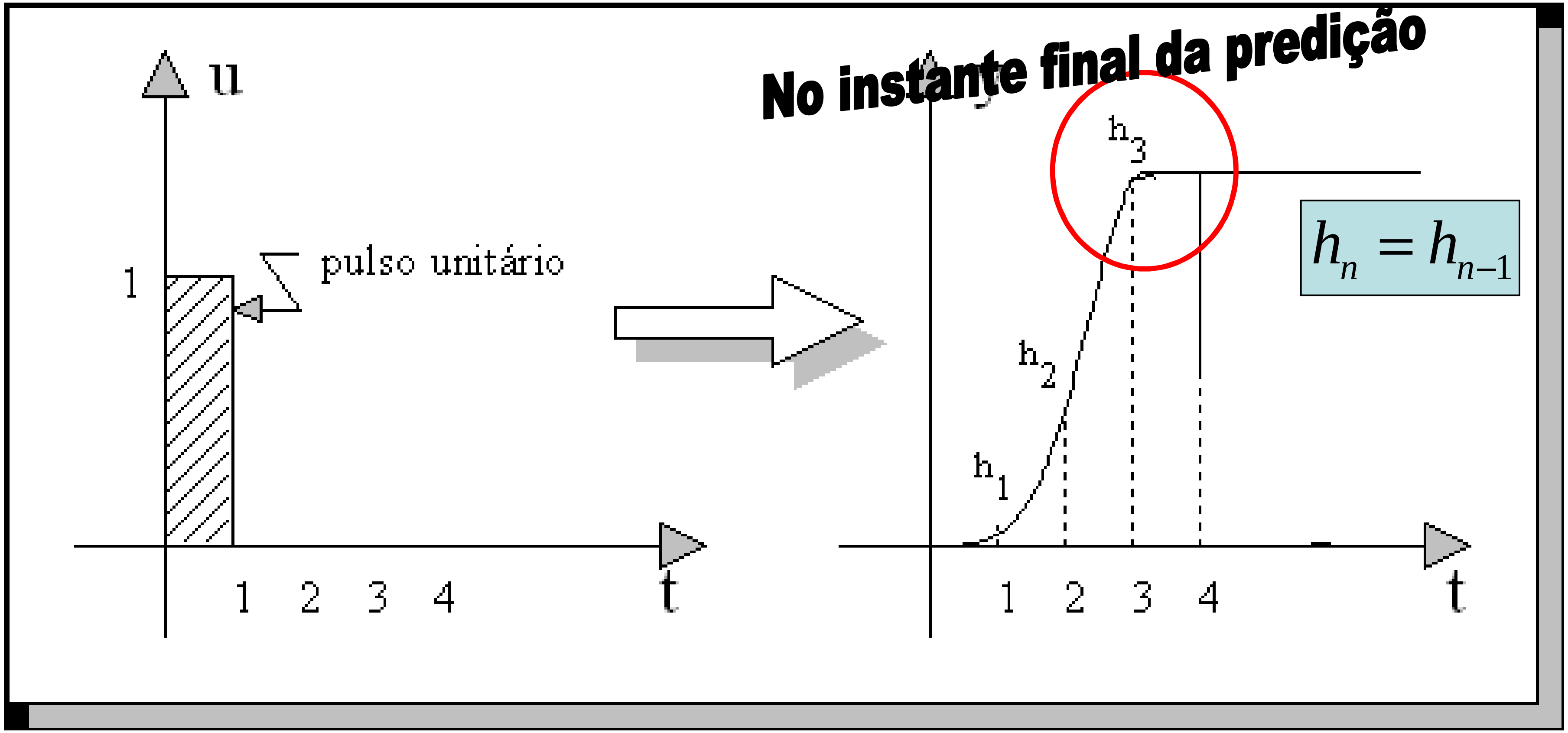
Resposta ao FIR

- ✓ Se considerarmos um instante *i* genérico, que no instante *k* o sistema estivesse em estado estacionário e que a cada novo instante *j*, o MPC aplica um pulso de valor u(*k*+*i*-*j*), valor da saída do sistema neste instante *i* será:

$$y(k+i) = h_1 u(k+i-1) + h_2 u(k+i-2) + h_3 u(k+i-3) + \ldots + h_i u(k)$$

$$y(k+i) = \sum_{j=1}^{i} h_j u(k+i-j)$$

Resposta ao FIR para sistemas integradores



Controladores Preditivos

- ✓ Para a obtenção do modelo incremental (a partir de Δu), ao invés do modelo posicional:

$$\hat{y}_{k+1} = \sum_{i=1}^{N} h_i u_{k+1-i}$$

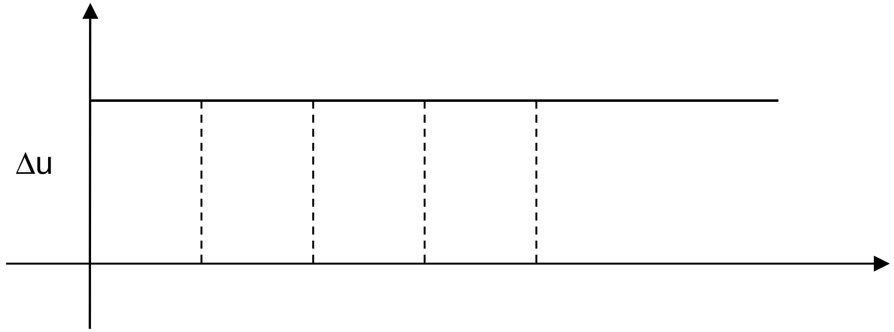
$$\hat{y}_{k} = \sum_{i=1}^{N} h_i u_{k-i}$$

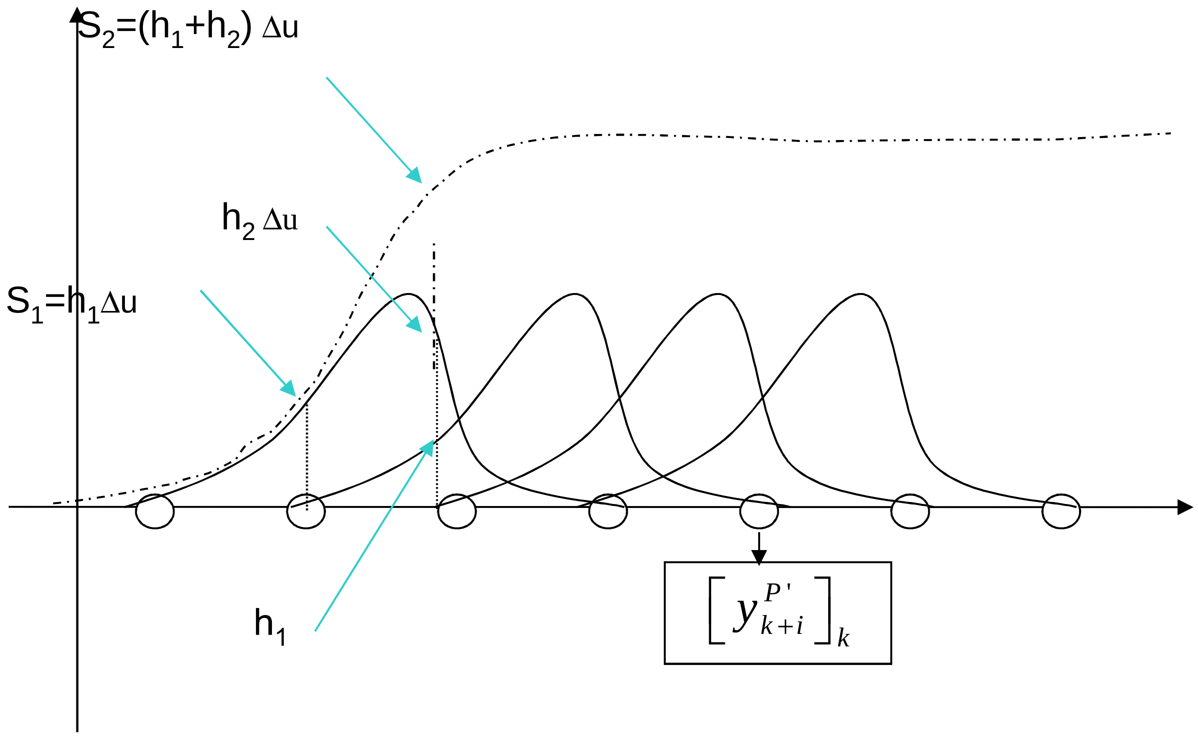
$$\Rightarrow \quad \hat{y}_{k+1} - \hat{y}_{k} = \sum_{i=1}^{N} h_i \Delta u_{k+1-i}$$

onde: $\Delta u_{k+1-i} = u_{k+1-i} - u_{k-i}$

Resposta ao Degrau

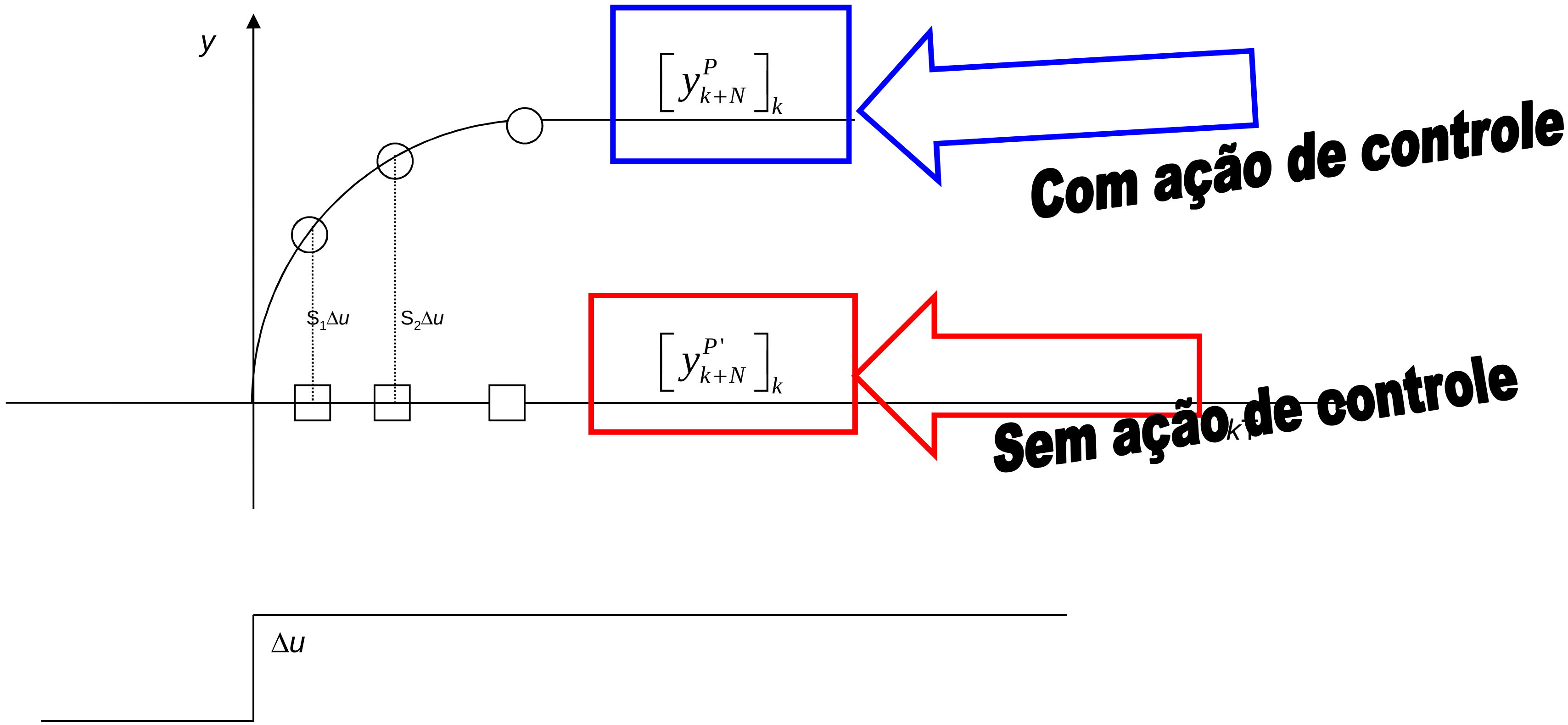
Resposta ao Degrau





$\left[y^{P'}_{k+i}\right]_k$

Trajetória da variável de controle



Simbologia

$\left[y_{k+i}^{P'} \right]_k$ - Predição da variável controlada *y* para o instante *k*+*i* feita no instante *k*, sem considerar nenhuma ação de controle.

$\left[y_{k+i}^{P} \right]_k$ - Predição da variável controlada *y* para o instante *k*+*i* feita no instante *k*, considerando as ações de controle.

$\left[y_{k+i}^{C} \right]_k$ - Predição da variável controlada *y* para o instante *k*+*i* feita no instante *k*, considerando as ações de controle e corrigida com o erro atual no instante *k.*

$\left[\hat{y}_{k+i}^{P} \right]_k$ - Variável de desvio da predição da variável *y* para o instante *k*+*i* feita no instante *k*

$\left[A \right]$ - Dimensão da matriz A:

PREDIÇÃO CASO SISO

Predição para um horizonte de controle igual a 1

K+1

$$\left[y^{P}_{k+1}\right]_k = h_1 \Delta u(k) + \left[y^{P'}_{k+1}\right]_k$$

K+2

$$\left[y^{P}_{k+2}\right]_k = h_1 \Delta u(k+1) + h_2 \Delta u(k) + \left[y^{P'}_{k+2}\right]_k$$

PENSANDO COMO PULSO
ΔU(K+1)= ΔU(K)

$$\Delta u(k+1) = \Delta u(k)$$

$$\left[y^{P}_{k+2}\right]_k = (h_1 + h_2) \Delta u(k) + \left[y^{P'}_{k+2}\right]_k$$

$$\left[y^{P}_{k+2}\right]_k = S_2 \Delta u(k) + \left[y^{P'}_{k+2}\right]_k$$

Predição para um horizonte de controle igual a 1

K+3

$$\left[y^{P}_{k+3}\right]_k = h_1\Delta u(k+2) + h_2\Delta u(k+1) + h_3\Delta u(k) + \left[y^{P'}_{k+3}\right]_k$$

$$\Delta u(k+2) = \Delta u(k+1) = \Delta u(k)$$

$$\left[y^{P}_{k+3}\right]_k = (h_1 + h_2 + h_3)\Delta u(k) + \left[y^{P'}_{k+3}\right]_k$$

$$\left[y^{P}_{k+3}\right]_k = S_3\Delta u(k) + \left[y^{P'}_{k+3}\right]_k$$

Predição para um horizonte de controle igual a 1

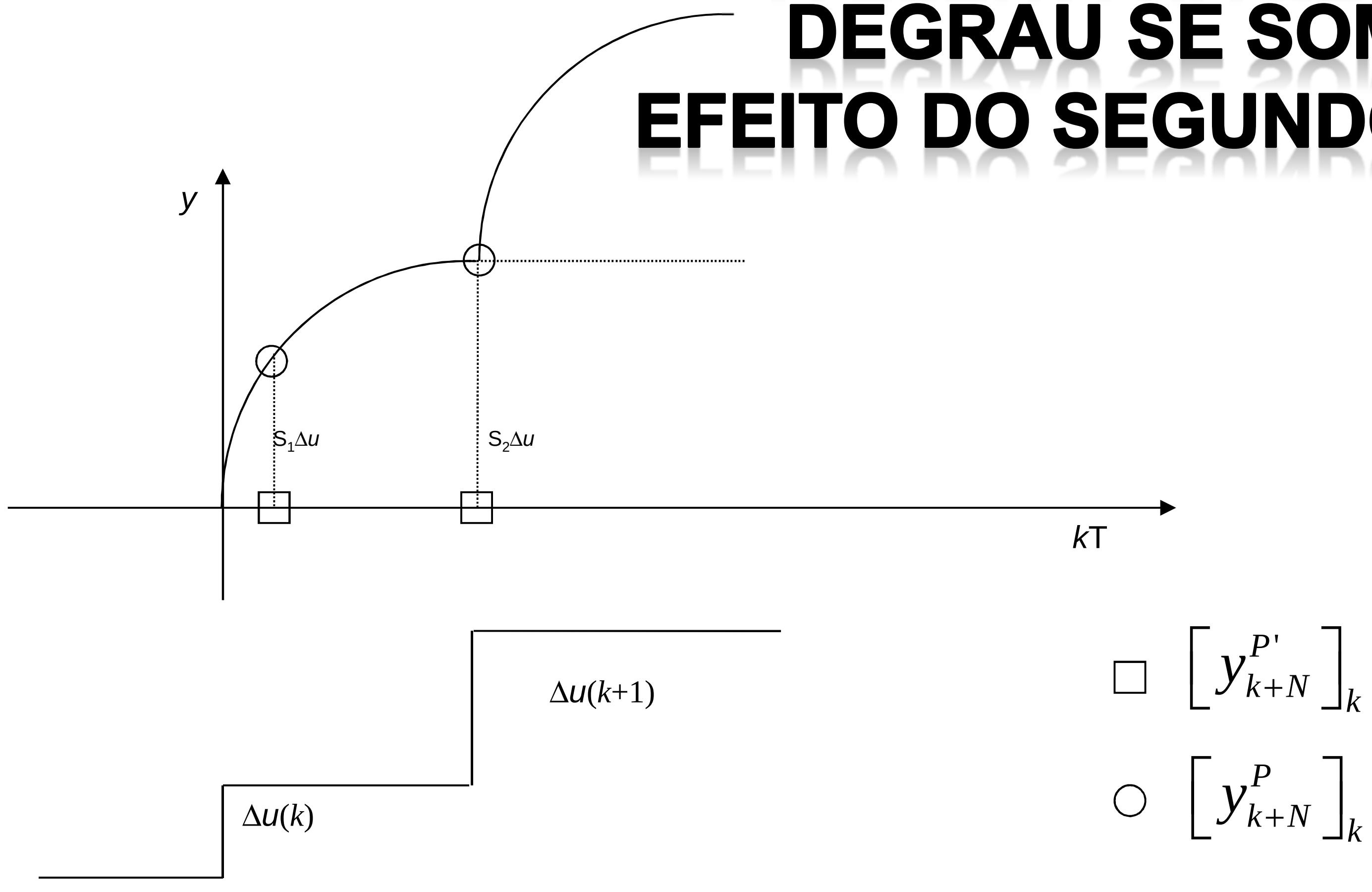
- ✓ Generalizando para o instante N a partir do instante 3:

$$\left[y^{P}_{k+3} \right]_k = S_3 \Delta u(k) + \left[y^{P'}_{k+3} \right]_k$$

$$\left[y^{P}_{k+N} \right]_k = S_N \Delta u(k) + \left[y^{P'}_{k+N} \right]_k$$

Predição para um horizonte de controle igual a 2

O EFEITO DO PRIMEIRO DEGRAU SE SOMA AO EFEITO DO SEGUNDO DEGRAU



Predição para um horizonte de controle igual a 2

K+1

$$\left[y^{P}_{k+1}\right]_k = S_1 \Delta u(k) + \left[y^{P'}_{k+1}\right]_k$$

K+2

$$\left[y^{P}_{k+2}\right]_k = S_1 \Delta u(k+1) + S_2 \Delta u(k) + \left[y^{P'}_{k+2}\right]_k$$

K+3

$$\left[y^{P}_{k+3}\right]_k = S_1 \Delta u(k+2) + S_2 \Delta u(k+1) + S_3 \Delta u(k) + \left[y^{P'}_{k+3}\right]_k$$

=0

Predição para um horizonte de controle igual a 3

K+1

$$\left[y^{P}_{k+1}\right]_k = S_1\Delta u(k) + \left[y^{P'}_{k+1}\right]_k$$

K+2

$$\left[y^{P}_{k+2}\right]_k = S_1\Delta u(k+1) + S_2\Delta u(k) + \left[y^{P'}_{k+2}\right]_k$$

K+3

$$\left[y^{P}_{k+3}\right]_k = S_1\Delta u(k+2) + S_2\Delta u(k+1) + S_3\Delta u(k) + \left[y^{P'}_{k+3}\right]_k$$

Predição para um horizonte de controle igual a *m*

$$\left[y^P_{k+3}\right]_k = S_1\Delta u(k+2) + S_2\Delta u(k+1) + S_3\Delta u(k) + \left[y^{P'}_{k+3}\right]_k$$

✓ Generalizando para o instante *m*

$$\left[y^P_{k+m}\right]_k = S_m\Delta u(k) + S_{m-1}\Delta u(k+1)$$

$$+\cdots S_2\Delta u(k+m-2) + S_1\Delta u(k+m-1) + \left[y^{P'}_{k+m}\right]_k$$

Predição para um horizonte de controle igual a *m*

- ✓ Após o instante *m*, as ações de controle do controlador são nulas:

$$\left[y_{k+m+1}^{P}\right]_k = S_{m+1}\Delta u(k) + S_m \Delta u(k+1) + \dots$$

$$+ S_2 \Delta u(k+m-1) + S_1 \Delta u(k+m) + \left[y_{k+m}^{P'}\right]_k$$

$\Delta u(k+m)$ → **ZERO**

Predição para um horizonte de controle igual a *m*

- ✓ Para o instante *N*, a predição será:

$$\left[y_{k+N}^{P} \right]_k = S_N \Delta u(k) + S_{N-1} \Delta u(k+1) + \ldots$$
$$+ S_{N-(m-2)} \Delta u(k+m-2) + S_{N-(m-1)} \Delta u(k+m-1) + \left[y_{k+m}^{P'} \right]_k$$

Predição em notação matricial

$$\begin{bmatrix} y^P_{k+1} \\ y^P_{k+2} \\ \vdots \\ y^P_{k+N} \end{bmatrix}_k = \underbrace{\begin{bmatrix} S_1 & 0 & \dots & 0 \\ S_2 & S_1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ S_N & S_{N-1} & \dots & S_{N-m+1} \end{bmatrix}}_{\text{Matriz Dinamica}} \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \vdots \\ \Delta u(k+m-1) \end{bmatrix} + \begin{bmatrix} y^{P'}_{k+1} \\ y^{P'}_{k+2} \\ \vdots \\ y^{P'}_{k+N} \end{bmatrix}$$

$$\underline{y}^P = \underline{\underline{S}}\Delta\underline{u} + \underline{y}^{P'}$$

$\left[\underline{y}^P\right] = N \times 1$ $\left[\underline{y}^{P'}\right] = N \times 1$ $\left[\underline{\underline{S}}\right] = N \times m$ $\left[\Delta\underline{u}\right] = m \times 1$

Função Objetivo

$$\min_{\Delta u} J = e_1^2 + e_2^2 + e_3^2 + \ldots + e_{np}^2$$

onde

e_i: erro entre *setpoint* e predição no instante *k+i*

np: horizonte de predição

Notação vetorial

$$\min J = \underline{e}^T \underline{e}$$

$$[\underline{e}] = np \times 1$$

DMC CUTLER CASO SISO

DMC Caso SISO

$$\left[y_{k+1}^{P}\right]_k = h_1 u(k) + h_2 u(k-1) + h_3 u(k-2) + \ldots + \left[y_{k+1}^{P'}\right]_k$$

VARIÁVEL DE DESVIO

$$\left[\hat{y}_{k+1}^{P}\right]_k = \left[y_{k+1}^{P}\right]_k - \left[y_{k+1}^{P'}\right]_k$$

$$\left[\hat{y}_{k+1}^{P}\right]_k = h_1 u(k) + h_2 u(k-1) + h_3 u(k-2) + \ldots$$

DMC Caso SISO

$$\left[\hat{y}^P_{k+1}\right]_k = \sum_{i=1}^{N} h_i u(k+1-i)$$

$$\left[\hat{y}^P_{k}\right]_k = \sum_{i=1}^{N} h_i u(k-i)$$

$$\left[\hat{y}^P_{k+1}\right]_k - \left[\hat{y}^P_{k}\right]_k = \sum_{i=1}^{N} h_i u(k+1-i) - \sum_{i=1}^{N} h_i u(k-i)$$

$$\left[\hat{y}^P_{k+1}\right]_k - \left[\hat{y}^P_{k}\right]_k = \sum_{i=1}^{N} h_i \left[u(k+1-i) - u(k-i)\right]$$

$$\left[\hat{y}^P_{k+1}\right]_k = \left[\hat{y}^P_{k}\right]_k + \sum_{i=1}^{N} h_i \Delta u(k+1-i)$$

DMC Caso SISO

Para um instante *j* qualquer

$$\left[\hat{y}^{P}_{k+1}\right]_k = \left[\hat{y}^{P}_{k}\right]_k + \sum_{i=1}^{N} h_i \Delta u(k+1-i)$$

$$\left[\hat{y}^{P}_{k+j}\right]_k = \left[\hat{y}^{P}_{k+j-1}\right]_k + \sum_{i=1}^{N} h_i \Delta u(k+j-i) \quad (4)$$

$$\left[\hat{y}^{P}_{k+j}\right]_k = \left[\hat{y}^{P}_{k+j-1}\right]_k + h_1 \Delta u(k+j-1) + h_2 \Delta u(k+j-2) + \ldots$$
$$+ h_N \Delta u(k+j-N)$$

Predição clássica X Predição Cutler

$$\underline{y}^{P} = \underline{\underline{S}}\Delta\underline{u} + \underline{y}^{P'}$$

PREDIÇÃO SEM AÇÃO DE CONTROLE CONSIDERADA EM TODOS INSTANTES

DMC CUTLER – APENAS A PREDIÇÃO DO INSTANTE ANTERIOR

$$\left[\hat{y}^{P}_{k+j}\right]_k = \left[\hat{y}^{P}_{k+j-1}\right]_k + \sum_{i=1}^{N} h_i \Delta u(k+j-i)$$

DMC Caso SISO
Incluindo a realimentação

**EM QUALQUER INSTANTE
O ERRO DE PREDIÇÃO
É O MESMO**

$$\left[\hat{y}^{C}_{k+j}\right]_k - \left[\hat{y}^{P}_{k+j}\right]_k = \left[\hat{y}^{C}_{k+j-1}\right]_k - \left[\hat{y}^{P}_{k+j-1}\right]_k$$

$$\left[\hat{y}^{C}_{k+j}\right]_k = \left[\hat{y}^{C}_{k+j-1}\right]_k + \left[\hat{y}^{P}_{k+j}\right]_k - \left[\hat{y}^{P}_{k+j-1}\right]_k$$

DMC Caso SISO
Incluindo a realimentação

$$\left[\hat{y}^{C}_{k+j}\right]_k = \left[\hat{y}^{C}_{k+j-1}\right]_k + \left[\hat{y}^{P}_{k+j}\right]_k - \left[\hat{y}^{P}_{k+j-1}\right]_k$$

$$\left[\hat{y}^{P}_{k+j}\right]_k = \left[\hat{y}^{P}_{k+j-1}\right]_k + \sum_{i=1}^{N} h_i \Delta u\left(k+j-i\right)$$

$$\left[\hat{y}^{C}_{k+j}\right]_k = \left[\hat{y}^{C}_{k+j-1}\right]_k + \sum_{i=1}^{N} h_i \Delta u\left(k+j-i\right)$$

Predição do DMC Caso SISO

$$\left[\hat{y}^{C}_{k+j}\right]_k = \left[\hat{y}^{C}_{k+j-1}\right]_k + \sum_{i=1}^{N} h_i \Delta u\left(k+j-i\right)$$

J=1

$$\left[\hat{y}^{C}_{k+1}\right]_k = \left[\hat{y}^{C}_{k}\right]_k + \sum_{i=1}^{N} h_i \Delta u\left(k+1-i\right)$$

$$\left[\hat{y}^{C}_{k+1}\right]_k = \hat{y}_k + h_1 \Delta u\left(k\right) + h_2 \Delta u\left(k-1\right) + h_3 \Delta u\left(k-2\right) + \ldots$$

LEITURA DA PLANTA

Predição do DMC Caso SISO

$$\left[\hat{y}^{C}_{k+1}\right]_k = \hat{y}_k + h_1\Delta u(k) + h_2\Delta u(k-1) + h_3\Delta u(k-2) + \ldots$$

$$\left[\hat{y}^{C}_{k+1}\right]_k = \hat{y}_k + h_1\Delta u(k) + \underbrace{\sum_{i=2}^{N} h_i\Delta u(k+1-i)}_{\phi_1}$$

$$\left[\hat{y}^{C}_{k+1}\right]_k = \hat{y}_k + S_1\Delta u(k) + \phi_1 \quad (6)$$

P1

Predição do DMC Caso SISO

$$\left[\hat{y}^{C}_{k+j}\right]_k = \left[\hat{y}^{C}_{k+j-1}\right]_k + \sum_{i=1}^{N} h_i \Delta u(k+j-i)$$

J=2

$$\left[\hat{y}^{C}_{k+2}\right]_k = \left[\hat{y}^{C}_{k+1}\right]_k + \sum_{i=1}^{N} h_i \Delta u(k+2-i)$$

$$\left[\hat{y}^{C}_{k+2}\right]_k = \left[\hat{y}^{C}_{k+1}\right]_k + h_1 \underbrace{\Delta u(k+1)}_{\text{desconhecido}} + h_2 \underbrace{\Delta u(k)}_{\text{desconhecido}} + h_3 \underbrace{\Delta u(k-1)}_{\text{conhecido}} + h_4 \underbrace{\Delta u(k-2)}_{\text{conhecido}} + \ldots$$

Predição do DMC Caso SISO

$$\left[\hat{y}^{C}_{k+2}\right]_k = \left[\hat{y}^{C}_{k+1}\right]_k + h_1 \underbrace{\Delta u(k+1)}_{\text{desconhecido}} + h_2 \underbrace{\Delta u(k)}_{\text{desconhecido}} + h_3 \underbrace{\Delta u(k-1)}_{\text{conhecido}} + h_4 \underbrace{\Delta u(k-2)}_{\text{conhecido}} + \ldots$$

$$\left[\hat{y}^{C}_{k+2}\right]_k = \left[\hat{y}^{C}_{k+1}\right]_k + h_1 \Delta u(k+1) + h_2 \Delta u(k) + \underbrace{\sum_{i=3}^{N} h_i \Delta u(k+2-i)}_{\phi_2}$$

$$\left[\hat{y}^{C}_{k+1}\right]_k = \hat{y}_k + S_1 \Delta u(k) + \phi_1$$

$$\left[\hat{y}^{C}_{k+2}\right]_k = \hat{y}_k + S_1 \Delta u(k) + \phi_1 + h_1 \Delta u(k+1) + h_2 \Delta u(k) + \phi_2$$

Predição do DMC Caso SISO

$$\left[\hat{y}^{C}_{k+2}\right]_k = \hat{y}_k + S_1\Delta u(k) + \phi_1 + h_1\Delta u(k+1) + h_2\Delta u(k) + \phi_2$$

$$\left[\hat{y}^{C}_{k+2}\right]_k = \hat{y}_k + h_1\Delta u(k+1) + (S_1 + h_2)\Delta u(k) + \phi_1 + \phi_2$$

$$\left[\hat{y}^{C}_{k+2}\right]_k = \hat{y}_k + S_2\Delta u(k) + S_1\Delta u(k+1) + \phi_1 + \phi_2 \quad (7)$$

P2

Predição do DMC Caso SISO

$$\left[\hat{y}^{C}_{k+j}\right]_k = \left[\hat{y}^{C}_{k+j-1}\right]_k + \sum_{i=1}^{N} h_i \Delta u(k+j-i)$$

J=3

$$\left[\hat{y}^{C}_{k+3}\right]_k = \left[\hat{y}^{C}_{k+2}\right]_k + \sum_{i=1}^{N} h_i \Delta u(k+3-i)$$

$$\left[\hat{y}^{C}_{k+3}\right]_k = \left[\hat{y}^{C}_{k+2}\right]_k + h_1 \underbrace{\Delta u(k+2)}_{\text{desconhecido}} + h_2 \underbrace{\Delta u(k+1)}_{\text{desconhecido}} + h_3 \underbrace{\Delta u(k)}_{\text{desconhecido}} + h_4 \underbrace{\Delta u(k-1)}_{\text{conhecido}} + \ldots$$

Predição do DMC Caso SISO

$$\left[\hat{y}^{C}_{k+3}\right]_k = \left[\hat{y}^{C}_{k+2}\right]_k + h_1 \underbrace{\Delta u(k+2)}_{\text{desconhecido}} + h_2 \underbrace{\Delta u(k+1)}_{\text{desconhecido}} + h_3 \underbrace{\Delta u(k)}_{\text{desconhecido}} + h_4 \underbrace{\Delta u(k-1)}_{\text{conhecido}} + \ldots$$

$$\left[\hat{y}^{C}_{k+3}\right]_k = \left[\hat{y}^{C}_{k+2}\right]_k + h_1 \Delta u(k+2) + h_2 \Delta u(k+1) + h_3 \Delta u(k) + \underbrace{\sum_{i=4}^{N} h_i \Delta u(k+3-i)}_{\phi_3}$$

Predição do DMC Caso SISO

$$\left[\hat{y}^{C}_{k+3}\right]_k = \left[\hat{y}^{C}_{k+2}\right]_k + h_1\Delta u(k+2) + h_2\Delta u(k+1) + h_3\Delta u(k) + \phi_3$$

$$\left[\hat{y}^{C}_{k+2}\right]_k = \hat{y}_k + S_2\Delta u(k) + S_1\Delta u(k+1) + \phi_1 + \phi_2$$

$$\left[\hat{y}^{C}_{k+3}\right]_k = \hat{y}_k + S_1\Delta u(k+1) + S_2\Delta u(k) + \phi_1 + \phi_2 + h_1\Delta u(k+2) + h_2\Delta u(k+1) + h_3\Delta u(k) + \phi_3$$

$$\left[\hat{y}^{C}_{k+3}\right]_k = \hat{y}_k + S_3\Delta u(k) + S_2\Delta u(k+1) + S_1\Delta u(k+2) + \phi_1 + \phi_2 + \phi_3 \quad (8)$$

Predição do DMC Caso SISO

$$P_j = \sum_{n=1}^{j} \phi_n$$

$$\left[\hat{y}^{C}_{k+1} \right]_k = \hat{y}_k + S_1 \Delta u(k) + P_1$$

$$\left[\hat{y}^{C}_{k+2} \right]_k = \hat{y}_k + S_2 \Delta u(k) + S_1 \Delta u(k+1) + P_2$$

$$\left[\hat{y}^{C}_{k+3} \right]_k = \hat{y}_k + S_3 \Delta u(k) + S_2 \Delta u(k+1) + S_1 \Delta u(k+2) + P_3$$

$$\left[\hat{y}^{C}_{k+j} \right]_k = \hat{y}_k + S_j \Delta u(k) + S_{j-1} \Delta u(k+1) + S_{j-2} \Delta u(k+2) + \ldots + S_1 \Delta u(k+j-1) + P_j$$

Predição do DMC Caso SISO

$$
\begin{bmatrix} y^C_{k+1} \\ y^C_{k+2} \\ y^C_{k+3} \\ \vdots \\ y^C_{k+np-1} \\ y^C_{k+np} \end{bmatrix} = \begin{bmatrix} S_1 & 0 & 0 & \dots & 0 & 0 \\ S_2 & S_1 & 0 & \dots & 0 & 0 \\ S_3 & S_2 & S_1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ S_{np-1} & S_{np-2} & S_{np-3} & \dots & S_1 & 0 \\ S_{np} & S_{np-1} & S_{np-2} & \dots & S_2 & S_1 \end{bmatrix} \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \Delta u(k+2) \\ \vdots \\ \Delta u(k+np-2) \\ \Delta u(k+np-1) \end{bmatrix}_k + \begin{bmatrix} \hat{y}_k + P_1 \\ \hat{y}_k + P_2 \\ \hat{y}_k + P_3 \\ \vdots \\ \hat{y}_k + P_{np-1} \\ \hat{y}_k + P_{np} \end{bmatrix} \quad (9)
$$

NA FORMA MATRICIAL **ATÉ NP**

$$
\underline{y}^C = \underline{\underline{S}}\Delta\underline{u} + \underline{y}_k + \underline{P}
$$

Formulação da Predição de erro
Caso SISO

$$\begin{bmatrix} y^C_{k+1} \\ y^C_{k+2} \\ y^C_{k+3} \\ \vdots \\ y^C_{k+np-1} \\ y^C_{k+np} \end{bmatrix} = \begin{bmatrix} S_1 & 0 & 0 & \dots & 0 & 0 \\ S_2 & S_1 & 0 & \dots & 0 & 0 \\ S_3 & S_2 & S_1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ S_{np-1} & S_{np-2} & S_{np-3} & \dots & S_1 & 0 \\ S_{np} & S_{np-1} & S_{np-2} & \dots & S_2 & S_1 \end{bmatrix} \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \Delta u(k+2) \\ \vdots \\ \Delta u(k+np-2) \\ \Delta u(k+np-1) \end{bmatrix}_k + \begin{bmatrix} \hat{y}_k + P_1 \\ \hat{y}_k + P_2 \\ \hat{y}_k + P_3 \\ \vdots \\ \hat{y}_k + P_{np-1} \\ \hat{y}_k + P_{np} \end{bmatrix}$$

SUBTRAINDO O SET POINT EM AMBOS OS LADOS DA EQUAÇÃO

$$y^{SP} - \begin{bmatrix} y^C_{k+1} \\ y^C_{k+2} \\ y^C_{k+3} \\ \vdots \\ y^C_{k+np-1} \\ y^C_{k+np} \end{bmatrix} = y^{SP} - \begin{bmatrix} S_1 & 0 & 0 & \dots & 0 & 0 \\ S_2 & S_1 & 0 & \dots & 0 & 0 \\ S_3 & S_2 & S_1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ S_{np-1} & S_{np-2} & S_{np-3} & \dots & S_1 & 0 \\ S_{np} & S_{np-1} & S_{np-2} & \dots & S_2 & S_1 \end{bmatrix} \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \Delta u(k+2) \\ \vdots \\ \Delta u(k+np-2) \\ \Delta u(k+np-1) \end{bmatrix}_k - \begin{bmatrix} \hat{y}_k + P_1 \\ \hat{y}_k + P_2 \\ \hat{y}_k + P_3 \\ \vdots \\ \hat{y}_k + P_{np-1} \\ \hat{y}_k + P_{np} \end{bmatrix}$$

Formulação da Predição de erro Caso SISO

$$y^{SP}-\begin{bmatrix} y^C_{k+1}\\ y^C_{k+2}\\ y^C_{k+3}\\ \vdots\\ y^C_{k+np-1}\\ y^C_{k+np}\end{bmatrix} = y^{SP}-\begin{bmatrix} S_1 & 0 & 0 & \dots & 0 & 0\\ S_2 & S_1 & 0 & \dots & 0 & 0\\ S_3 & S_2 & S_1 & \dots & 0 & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ S_{np-1} & S_{np-2} & S_{np-3} & \dots & S_1 & 0\\ S_{np} & S_{np-1} & S_{np-2} & \dots & S_2 & S_1\end{bmatrix}\begin{bmatrix}\Delta u(k)\\ \Delta u(k+1)\\ \Delta u(k+2)\\ \vdots\\ \Delta u(k+np-2)\\ \Delta u(k+np-1)\end{bmatrix}_k - \begin{bmatrix}\hat{y}_k+P_1\\ \hat{y}_k+P_2\\ \hat{y}_k+P_3\\ \vdots\\ \hat{y}_k+P_{np-1}\\ \hat{y}_k+P_{np}\end{bmatrix}$$

$$\begin{bmatrix} y^{SP}-y^C_{k+1}\\ y^{SP}-y^C_{k+2}\\ y^{SP}-y^C_{k+3}\\ \vdots\\ y^{SP}-y^C_{k+np-1}\\ y^{SP}-y^C_{k+np}\end{bmatrix} = -\begin{bmatrix} S_1 & 0 & 0 & \dots & 0 & 0\\ S_2 & S_1 & 0 & \dots & 0 & 0\\ S_3 & S_2 & S_1 & \dots & 0 & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ S_{np-1} & S_{np-2} & S_{np-3} & \dots & S_1 & 0\\ S_{np} & S_{np-1} & S_{np-2} & \dots & S_2 & S_1\end{bmatrix}\begin{bmatrix}\Delta u(k)\\ \Delta u(k+1)\\ \Delta u(k+2)\\ \vdots\\ \Delta u(k+np-2)\\ \Delta u(k+np-1)\end{bmatrix}_k + \begin{bmatrix}y^{SP}-\hat{y}_k-P_1\\ y^{SP}-\hat{y}_k-P_2\\ y^{SP}-\hat{y}_k-P_3\\ \vdots\\ y^{SP}-\hat{y}_k-P_{np-1}\\ y^{SP}-\hat{y}_k-P_{np}\end{bmatrix}$$

Formulação da Predição de erro
Caso SISO

NA FORMA MATRICIAL

$$
\begin{bmatrix} e_{k+1} \\ e_{k+2} \\ e_{k+3} \\ \vdots \\ e_{k+np-1} \\ e_{k+np} \end{bmatrix}
= -
\begin{bmatrix}
S_1 & 0 & 0 & \dots & 0 & 0 \\
S_2 & S_1 & 0 & \dots & 0 & 0 \\
S_3 & S_2 & S_1 & \dots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
S_{np-1} & S_{np-2} & S_{np-3} & \dots & S_1 & 0 \\
S_{np} & S_{np-1} & S_{np-2} & \dots & S_2 & S_1
\end{bmatrix}
\begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \Delta u(k+2) \\ \vdots \\ \Delta u(k+np-2) \\ \Delta u(k+np-1) \end{bmatrix}_k
+
\begin{bmatrix} e_k - P_1 \\ e_k - P_2 \\ e_k - P_3 \\ \vdots \\ e_k - P_{np-1} \\ e_k - P_{np} \end{bmatrix}
$$

$$\underline{e} = -\underline{\underline{S}}\Delta\underline{u} + \underline{e}_k - \underline{P}$$

Definindo $\underline{e}' = \underline{e}_k - \underline{P}$

$$\underline{e} = -\underline{\underline{S}}\Delta\underline{u} + \underline{e}' \qquad (10)$$

LIMITANDO AS AÇÕES DE CONTROLE ATÉ O INSTANTE M

Formulação da Predição de erro
Caso SISO

$$\begin{bmatrix} e_{k+1} \\ e_{k+2} \\ e_{k+3} \\ \vdots \\ e_{k+np-1} \\ e_{k+np} \end{bmatrix} = -\begin{bmatrix} S_1 & 0 & 0 & \dots & 0 & 0 \\ S_2 & S_1 & 0 & \dots & 0 & 0 \\ S_3 & S_2 & S_1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ S_{np-1} & S_{np-2} & S_{np-3} & \dots & S_{np-m+1} & 0 \\ S_{np} & S_{np-1} & S_{np-2} & \dots & S_{np-m+2} & S_{np-m+1} \end{bmatrix} \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \Delta u(k+2) \\ \vdots \\ \Delta u(k+m-2) \\ \Delta u(k+m-1) \end{bmatrix}_k + \begin{bmatrix} e_k - P_1 \\ e_k - P_2 \\ e_k - P_3 \\ \vdots \\ e_k - P_{np-1} \\ e_k - P_{np} \end{bmatrix}$$

$$[\underline{e}'] = np \times 1 \qquad [\underline{e}] = np \times 1$$

$$[\underline{\underline{S}}] = np \times m \qquad [\Delta \underline{u}] = m \times 1$$

ATÉ O INSTANTE M

Função Objetivo

$$J = (e)^T (e)$$

$$J = \left(-\underline{\underline{S}}\Delta\underline{u} + \underline{e}'\right)^T \left(-\underline{\underline{S}}\Delta\underline{u} + \underline{e}'\right)$$

$$\underline{e} = -\underline{\underline{S}}\Delta\underline{u} + \underline{e}'$$

$$J = \left(-\Delta\underline{u}^T \underline{\underline{S}}^T + \underline{e}'^T\right)\left(-\underline{\underline{S}}\Delta\underline{u} + \underline{e}'\right)$$

$$J = \Delta\underline{u}^T \underline{\underline{S}}^T \underline{\underline{S}}\Delta\underline{u} - \Delta\underline{u}^T \underline{\underline{S}}^T \underline{e}' - \underline{e}'^T \underline{\underline{S}}\Delta\underline{u} + \underline{e}'^T \underline{e}'$$

Ponto ótimo da Função Objetivo

$$\frac{\partial J}{\partial u} = 2\underline{\underline{S}}^T \underline{\underline{S}} \Delta \underline{u} - \underline{\underline{S}}^T \underline{e}' - \underline{e}'^T \underline{\underline{S}} + 0 = 0$$

$$2\underline{\underline{S}}^T \underline{\underline{S}} \Delta \underline{u} = 2\underline{\underline{S}}^T \underline{e}'$$

Solução do problema sem restrições

$$\Delta \underline{u} = \left(\underline{\underline{S}}^T \underline{\underline{S}} \right)^{-1} \underline{\underline{S}}^T \underline{e}'$$

$$\left[\underline{\underline{S}} \right] = np \times m \qquad \left[\underline{e}' \right] = np \times 1 \qquad \left[\Delta \underline{u} \right] = m \times 1$$

IMPORTÂNCIA NA VARIAÇÃO DAS VARIÁVEIS MANIPULADAS

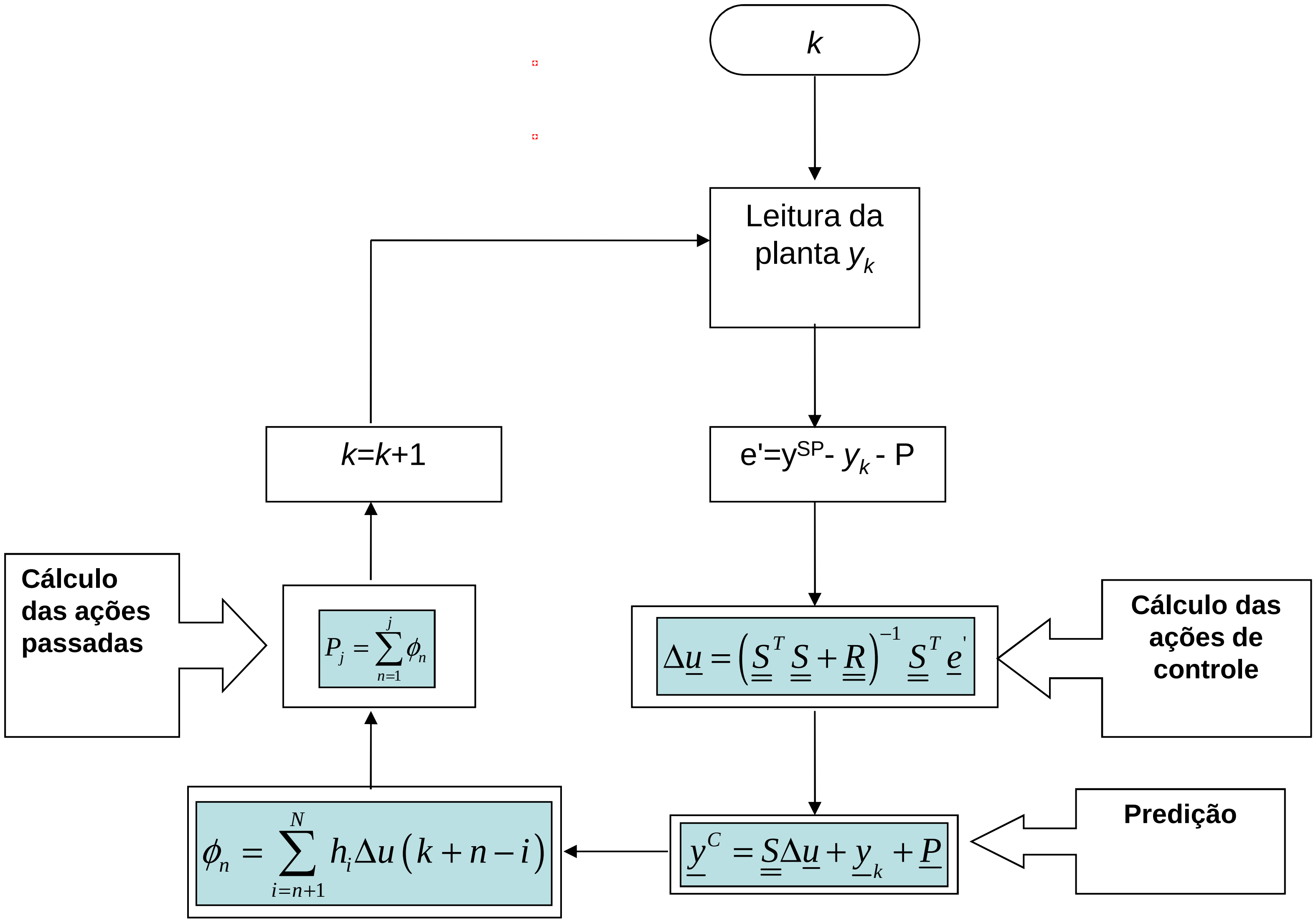
Função Objetivo Modificada

$$J = \underline{e}^T \underline{e} + \Delta \underline{u}^T \underline{\underline{R}} \Delta \underline{u}$$

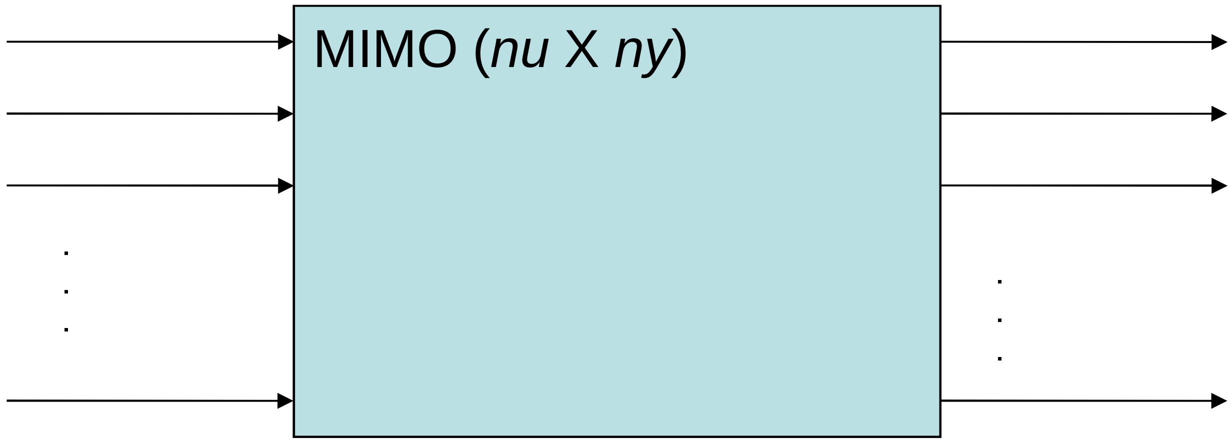
$$\underline{\underline{R}} = \begin{bmatrix} R_1 & & \cdots & 0 \\ 0 & R_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & R_m \end{bmatrix}$$

$$\Delta \underline{u} = \left(\underline{\underline{S}}^T \underline{\underline{S}} + \underline{\underline{R}} \right)^{-1} \underline{\underline{S}}^T \underline{e}'$$

Fluxograma DMC



DMC Caso MIMO



$$\underline{y}^T = \begin{bmatrix} y_1 & y_2 & \dots & y_{ny} \end{bmatrix}^T$$

$$\underline{u}^T = \begin{bmatrix} u_1 & u_2 & \dots & u_{nu} \end{bmatrix}^T$$

DMC Caso MIMO

PREDIÇÃO PAR SISO

$$\left[\hat{y}_k^P\right]_{k-1} = \sum_{i=1}^{N} h_i u(k-i)$$

PREDIÇÃO MISO DA VARIÁVEL J

$$\left[\hat{y}_{j,k}^P\right]_{k-1} = \sum_{i=1}^{N} \sum_{l=1}^{nu} h_{j,l,i} u_l(k-i)$$

DMC Caso MIMO

PARA A VARIÁVEL CONTROLADA J=1

$$\left[\hat{y}^{P}_{j,k}\right]_{k-1} = \sum_{i=1}^{N}\sum_{l=1}^{nu} h_{j,l,i}\, u_l\,(k-i)$$

$$\left[\hat{y}^{P}_{1,k}\right]_{k-1} = h_{1,1,1}u_1(k-1) + h_{1,2,1}u_2(k-1) + \ldots + h_{1,nu,1}u_{nu}(k-1) +$$
$$h_{1,1,2}u_1(k-2) + \ldots + h_{1,nu,2}u_{nu}(k-2) + \ldots$$
$$h_{1,1,N}u_1(k-N) + h_{1,2,N}u_2(k-N) + \ldots + h_{1,nu,N}u_{nu}(k-N)$$

DMC Caso MIMO

VARREDURA DE TODAS VARIÁVEIS MANIPULADAS

$$\left[\hat{y}^{P}_{j,k}\right]_{k-1} = \sum_{i=1}^{N}\sum_{l=1}^{nu} h_{j,l,i}\, u_{l}(k-i)$$

$$\begin{aligned}\left[\hat{y}^{P}_{1,k}\right]_{k-1} = {} & h_{1,1,1}u_{1}(k-1) + h_{1,2,1}u_{2}(k-1) + \ldots + h_{1,nu,1}u_{nu}(k-1) + \\ & h_{1,1,2}u_{1}(k-2) + \ldots + h_{1,nu,2}u_{nu}(k-2) + \ldots \\ & h_{1,1,N}u_{1}(k-N) + h_{1,2,N}u_{2}(k-N) + \ldots + h_{1,nu,N}u_{nu}(k-N)\end{aligned}$$

DMC Caso MIMO

VARREDURA DO INSTANTE IMEDIATAMENTE ANTERIOR ATÉ O INSTANTE DE ESTABILIZAÇÃO

$$\left[\hat{y}^{P}_{j,k}\right]_{k-1} = \sum_{i=1}^{N}\sum_{l=1}^{nu} h_{j,l,i}\,u_l\left(k-i\right)$$

$$\left[\hat{y}^{P}_{1,k}\right]_{k-1} = h_{1,1,1}u_1\left(k-1\right) + h_{1,2,1}u_2\left(k-1\right) + \ldots + h_{1,nu,1}u_{nu}\left(k-1\right) +$$
$$h_{1,1,2}u_1\left(k-2\right) + \ldots + h_{1,nu,2}u_{nu}\left(k-2\right) + \ldots$$
$$h_{1,1,N}u_1\left(k-N\right) + h_{1,2,N}u_2\left(k-N\right) + \ldots + h_{1,nu,N}u_{nu}\left(k-N\right)$$

DMC Caso MIMO

PARA CADA INSTANTE I:

$$\underline{\underline{H_i}} = \begin{bmatrix} h_{11,i} & h_{12,i} & \cdots & h_{1nu,i} \\ h_{21,i} & h_{22,i} & \cdots & h_{2nu,i} \\ \vdots & \vdots & \ddots & \vdots \\ h_{ny1,i} & h_{ny2,i} & \cdots & h_{nynu,i} \end{bmatrix}$$

DMC Caso MIMO

CADA SAÍDA CORRESPONDE A UMA LINHA:

$$\underline{\underline{H_i}} = \begin{bmatrix} h_{11,i} & h_{12,i} & \cdots & h_{1nu,i} \\ h_{21,i} & h_{22,i} & \cdots & h_{2nu,i} \\ \vdots & \vdots & \ddots & \vdots \\ h_{ny1,i} & h_{ny2,i} & \cdots & h_{nynu,i} \end{bmatrix}$$

DMC Caso MIMO

E CADA COLUNA CORRESPONDE A UMA POSSÍVEL ENTRADA:

$$\underline{\underline{H}}_i = \begin{bmatrix} h_{11,i} & h_{12,i} & \cdots & h_{1nu,i} \\ h_{21,i} & h_{22,i} & \cdots & h_{2nu,i} \\ \vdots & \vdots & \ddots & \vdots \\ h_{ny1,i} & h_{ny2,i} & \cdots & h_{nynu,i} \end{bmatrix}$$

$$\left[\underline{\underline{H}}_i\right] = ny \times nu$$

DMC Caso MIMO

$$\left[\hat{y}^{P}_{j,k}\right]_{k-1} = \sum_{i=1}^{N}\sum_{l=1}^{nu} h_{j,l,i}\,u_{l}\left(k-i\right)$$

$$\left[\hat{\underline{y}}^{P}_{k}\right]_{k-1} = \sum_{i=1}^{N} \underline{\underline{H}}_{i}\,\underline{u}\left(k-i\right)$$

Predição para todas variáveis controladas

DMC Caso MIMO

$$\left[\hat{\underline{y}}_{k}^{P}\right]_{k-1} = \sum_{i=1}^{N} \underline{\underline{H}}_i \, \underline{u}(k-i)$$

PREDIÇÃO PARA O INSTANTE K+1

$$\left[\hat{\underline{y}}_{k+1}^{P}\right]_{k} = \sum_{i=1}^{N} \underline{\underline{H}}_i \, \underline{u}(k+1-i)$$

$$\left[\hat{\underline{y}}_{k+1}^{P}\right]_{k} - \left[\hat{\underline{y}}_{k}^{P}\right]_{k-1} = \sum_{i=1}^{N} \underline{\underline{H}}_i \, \underline{u}(k+1-i) - \sum_{i=1}^{N} \underline{\underline{H}}_i \, \underline{u}(k-i) = \sum_{i=1}^{N} \underline{\underline{H}}_i \, \Delta\underline{u}(k+1-i)$$

DMC Caso MIMO

$$\left[\underline{\hat{y}}^{P}_{k+1}\right]_k = \left[\underline{\hat{y}}^{P}_{k}\right]_{k-1} + \sum_{i=1}^{N} \underline{\underline{H}}_i \Delta \underline{u}(k+1-i)$$

GENERALIZANDO A PREDIÇÃO PARA O INSTANTE K+J

$$\left[\underline{\hat{y}}^{P}_{k+j}\right]_k = \left[\underline{\hat{y}}^{P}_{k}\right]_{k+j-1} + \sum_{i=1}^{N} \underline{\underline{H}}_i \Delta \underline{u}(k+j-i)$$

Correção da predição

O MESMO ERRO EM TODA PREDIÇÃO

DMC Caso MIMO

$$\left[\hat{\underline{y}}^{C}_{k+j}\right]_k - \left[\hat{\underline{y}}^{P}_{k+j}\right]_k = \left[\hat{\underline{y}}^{C}_{k+j-1}\right]_k - \left[\hat{\underline{y}}^{P}_{k+j-1}\right]_k$$

$$\left[\hat{\underline{y}}^{C}_{k+j}\right]_k = \left[\hat{\underline{y}}^{C}_{k+j-1}\right]_k + \left[\hat{\underline{y}}^{P}_{k+j}\right]_k - \left[\hat{\underline{y}}^{P}_{k+j-1}\right]_k$$

$$\left[\hat{\underline{y}}^{P}_{k+j}\right]_k = \left[\hat{\underline{y}}^{P}_{k+j-1}\right]_k + \sum_{i=1}^{N} \underline{\underline{H}}_i \Delta \underline{u}(k+j-i)$$

$$\left[\hat{\underline{y}}^{C}_{k+j}\right]_k = \left[\hat{\underline{y}}^{C}_{k+j-1}\right]_k + \sum_{i=1}^{N} \underline{\underline{H}}_i \Delta \underline{u}(k+j-i)$$

DMC Caso MIMO

$$\left[\underline{\hat{y}}^{C}_{k+j}\right]_k = \left[\underline{\hat{y}}^{C}_{k+j-1}\right]_k + \sum_{i=1}^{N} \underline{\underline{H}}_i \, \Delta \underline{u}(k+j-i)$$

J=1

$$\left[\underline{\hat{y}}^{C}_{k+1}\right]_k = \underbrace{\left[\underline{\hat{y}}^{C}_{k}\right]_k}_{\text{Valor Atual lido da Planta}} + \sum_{i=1}^{N} \underline{\underline{H}}_i \, \Delta \underline{u}(k+1-i)$$

DMC Caso MIMO

$$\left[\underline{\hat{y}}^{C}_{k+1}\right]_k = \underbrace{\left[\underline{\hat{y}}^{C}_{k}\right]_k}_{\text{Valor Atual lido da Planta}} + \sum_{i=1}^{N} \underline{\underline{H}}_i \Delta \underline{u}(k+1-i)$$

$$\left[\underline{\hat{y}}^{C}_{k+1}\right]_k = \underline{\hat{y}}_k + \underline{\underline{H}}_1 \underbrace{\Delta \underline{u}(k)}_{\text{Valor desconhecido}} + \underbrace{\underline{\underline{H}}_2 \Delta \underline{u}(k-1) + \underline{\underline{H}}_3 \Delta \underline{u}(k-2) + \ldots}_{\text{Passado}}$$

$$\left[\underline{\hat{y}}^{C}_{k+1}\right]_k = \underline{\hat{y}}_k + \underline{\underline{H}}_1 \Delta \underline{u}(k) + \underbrace{\sum_{i=2}^{N} \underline{\underline{H}}_i \Delta \underline{u}(k+1-i)}_{\phi_1}$$

DMC Caso MIMO

$$\left[\hat{\underline{y}}^{C}_{k+1}\right]_k = \hat{\underline{y}}_k + \underline{\underline{H}}_1 \Delta \underline{u}(k) + \underbrace{\sum_{i=2}^{N} \underline{\underline{H}}_i \Delta \underline{u}(k+1-i)}_{\phi_1}$$

$$\left[\hat{\underline{y}}^{C}_{k+1}\right]_k = \hat{\underline{y}}_k + \underline{\underline{S}}_1 \Delta \underline{u}(k) + \underbrace{\phi_1}_{P_1}$$

Resposta do processo conforme modelo e ações aplicados no passado

$$\left[\underline{\underline{S}}_1\right] = ny \times nu$$

DMC Caso MIMO

$$\left[\underline{\hat{y}}^{C}_{k+j}\right]_k = \left[\underline{\hat{y}}^{C}_{k+j-1}\right]_k + \sum_{i=1}^{N} \underline{\underline{H}}_i \, \Delta \underline{u}\left(k+j-i\right)$$

J=2

$$\left[\underline{\hat{y}}^{C}_{k+2}\right]_k = \left[\underline{\hat{y}}^{C}_{k+1}\right]_k + \sum_{i=1}^{N} \underline{\underline{H}}_i \, \Delta \underline{u}\left(k+2-i\right)$$

$$\left[\underline{\hat{y}}^{C}_{k+2}\right]_k = \underline{\hat{y}}_k + \underline{\underline{S}}_1 \Delta \underline{u}\left(k\right) + \phi_1 + \sum_{i=1}^{N} \underline{\underline{H}}_i \, \Delta \underline{u}\left(k+2-i\right)$$

DMC Caso MIMO

$$\left[\underline{\hat{y}}^{C}_{k+2}\right]_k = \underline{\hat{y}}_k + \underline{\underline{S}}_1 \Delta \underline{u}(k) + \phi_1 + \sum_{i=1}^{N} \underline{\underline{H}}_i \Delta \underline{u}(k+2-i)$$

$$\left[\underline{\hat{y}}^{C}_{k+2}\right]_k = \underline{\hat{y}}_k + \underline{\underline{S}}_1 \Delta \underline{u}(k) + \phi_1 + \underline{\underline{H}}_1 \Delta \underline{u}(k+1) + \underline{\underline{H}}_2 \Delta \underline{u}(k) + \underline{\underline{H}}_3 \Delta \underline{u}(k-1) + \ldots$$

$$\left[\underline{\hat{y}}^{C}_{k+2}\right]_k = \underline{\hat{y}}_k + \underline{\underline{H}}_1 \Delta \underline{u}(k+1) + \left(\underline{\underline{H}}_1 + \underline{\underline{H}}_2\right) \Delta \underline{u}(k) + \phi_1 + \underline{\underline{H}}_3 \Delta \underline{u}(k-1) + \ldots$$

DMC Caso MIMO

$$\left[\underline{\hat{y}}^{C}_{k+2}\right]_k = \underline{\hat{y}}_k + \underline{\underline{H}}_1 \Delta \underline{u}(k+1) + \left(\underline{\underline{H}}_1 + \underline{\underline{H}}_2\right)\Delta \underline{u}(k) + \phi_1 + \underline{\underline{H}}_3 \Delta \underline{u}(k-1) + \ldots$$

$$\left[\underline{\hat{y}}^{C}_{k+2}\right]_k = \underline{\hat{y}}_k + \underline{\underline{S}}_1 \Delta \underline{u}(k+1) + \underline{\underline{S}}_2 \Delta \underline{u}(k) + \phi_1 + \underbrace{\sum_{i=3}^{N} \underline{\underline{H}}_i \Delta \underline{u}(k+2-i)}_{\phi_2}$$

$$\left[\underline{\hat{y}}^{C}_{k+2}\right]_k = \underline{\hat{y}}_k + \underline{\underline{S}}_1 \Delta \underline{u}(k+1) + \underline{\underline{S}}_2 \Delta \underline{u}(k) + \underbrace{\phi_1 + \phi_2}_{P_2}$$

Resposta do processo conforme modelo e ações aplicados no passado

DMC Caso MIMO

$$\left[\underline{\hat{y}}^{C}_{k+j}\right]_k = \left[\underline{\hat{y}}^{C}_{k+j-1}\right]_k + \sum_{i=1}^{N} \underline{\underline{H}}_i \Delta \underline{u}(k+j-i)$$

J=3

$$\left[\underline{\hat{y}}^{C}_{k+3}\right]_k = \left[\underline{\hat{y}}^{C}_{k+2}\right]_k + \sum_{i=1}^{N} \underline{\underline{H}}_i \Delta \underline{u}(k+3-i)$$

$$\left[\underline{\hat{y}}^{C}_{k+2}\right]_k = \underline{\hat{y}}_k + \underline{\underline{S}}_1 \Delta \underline{u}(k+1) + \underline{\underline{S}}_2 \Delta \underline{u}(k) + \underbrace{\phi_1 + \phi_2}_{P_2}$$

$$\left[\underline{\hat{y}}^{C}_{k+3}\right]_k = \underline{\hat{y}}_k + \underline{\underline{S}}_1 \Delta \underline{u}(k+1) + \underline{\underline{S}}_2 \Delta \underline{u}(k) + \phi_1 + \phi_2 + \sum_{i=1} \underline{\underline{H}}_i \Delta \underline{u}(k+3-i)$$

DMC Caso MIMO

$$\left[\hat{\underline{y}}^{C}_{k+3}\right]_k = \hat{\underline{y}}_k + \underline{\underline{S}}_1 \Delta \underline{u}(k+1) + \underline{\underline{S}}_2 \Delta \underline{u}(k) + \phi_1 + \phi_2 + \sum_{i=1}^{N} \underline{\underline{H}}_i \Delta \underline{u}(k+3-i)$$

$$\left[\hat{\underline{y}}^{C}_{k+3}\right]_k = \hat{\underline{y}}_k + \underline{\underline{S}}_1 \Delta \underline{u}(k+1) + \underline{\underline{S}}_2 \Delta \underline{u}(k) + \phi_1 + \phi_2 + \underline{\underline{H}}_1 \Delta \underline{u}(k+2) + \underline{\underline{H}}_2 \Delta \underline{u}(k+1) + \underline{\underline{H}}_3 \Delta \underline{u}(k) + \underline{\underline{H}}_4 \Delta \underline{u}(k-1) + \ldots$$

$$\left[\hat{\underline{y}}^{C}_{k+3}\right]_k = \hat{\underline{y}}_k + \underline{\underline{S}}_1 \Delta \underline{u}(k+2) + \underline{\underline{S}}_2 \Delta \underline{u}(k+1) + \underline{\underline{S}}_3 \Delta \underline{u}(k) + \phi_1 + \phi_2 + \underline{\underline{H}}_4 \Delta \underline{u}(k-1) + \ldots$$

DMC Caso MIMO

$$\left[\underline{\hat{y}}^{C}_{k+3}\right]_k = \underline{\hat{y}}_k + \underline{\underline{S}}_1 \Delta \underline{u}(k+2) + \underline{\underline{S}}_2 \Delta \underline{u}(k+1) + \underline{\underline{S}}_3 \Delta \underline{u}(k) + \phi_1 + \phi_2 + \underline{\underline{H}}_4 \Delta \underline{u}(k-1) + \ldots$$

$$\left[\underline{\hat{y}}^{C}_{k+3}\right]_k = \underline{\hat{y}}_k + \underline{\underline{S}}_1 \Delta \underline{u}(k+2) + \underline{\underline{S}}_2 \Delta \underline{u}(k+1) + \underline{\underline{S}}_3 \Delta \underline{u}(k) + \phi_1 + \phi_2 + \underbrace{\sum_{i=4}^{N} \underline{\underline{H}}_i \Delta \underline{u}(k+3-i)}_{\phi_3}$$

$$\left[\underline{\hat{y}}^{C}_{k+3}\right]_k = \underline{\hat{y}}_k + \underline{\underline{S}}_1 \Delta \underline{u}(k+2) + \underline{\underline{S}}_2 \Delta \underline{u}(k+1) + \underline{\underline{S}}_3 \Delta \underline{u}(k) + \underbrace{\phi_1 + \phi_2 + \phi_3}_{P_3}$$

Resposta do processo conforme modelo e ações aplicados no passado

DMC Caso MIMO

$$\left[\underline{\hat{y}}^{C}_{k+3}\right]_k = \underline{\hat{y}}_k + \underline{\underline{S}}_1 \Delta \underline{u}(k+2) + \underline{\underline{S}}_2 \Delta \underline{u}(k+1) + \underline{\underline{S}}_3 \Delta \underline{u}(k) + \underbrace{\phi_1 + \phi_2 + \phi_3}_{P_3}$$

PARA UM INSTANTE GENÉRICO J

$$\left[\underline{\hat{y}}^{C}_{k+j}\right]_k = \underline{\hat{y}}_k + \underline{\underline{S}}_1 \Delta \underline{u}(k+j-1) + \underline{\underline{S}}_2 \Delta \underline{u}(k+j-2) + \underline{\underline{S}}_3 \Delta \underline{u}(k+j-3) + \ldots$$

$$+ \underline{\underline{S}}_i \Delta \underline{u}(k+j-i) + \ldots + \underline{\underline{S}}_j \Delta \underline{u}(k) + \ldots + \underbrace{\phi_1 + \phi_2 + \phi_3 + \ldots \phi_j}_{P_j}$$

Resposta do processo conforme modelo e ações aplicados no passado

DMC Caso MIMO

- Montando a predição em modelo de matriz

$$\begin{bmatrix} \underline{\hat{y}}^C_{k+1} \\ \underline{\hat{y}}^C_{k+2} \\ \underline{\hat{y}}^C_{k+3} \\ \vdots \\ \underline{\hat{y}}^C_{k+np} \end{bmatrix} = \begin{bmatrix} \underline{\underline{S}}_1 & 0_{ny\times nu} & 0_{ny\times nu} & \cdots & 0_{ny\times nu} \\ \underline{\underline{S}}_2 & \underline{\underline{S}}_1 & 0_{ny\times nu} & \cdots & 0_{ny\times nu} \\ \underline{\underline{S}}_3 & \underline{\underline{S}}_2 & \underline{\underline{S}}_1 & \cdots & 0_{ny\times nu} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \underline{\underline{S}}_{np} & \underline{\underline{S}}_{np-1} & \underline{\underline{S}}_{np-2} & \cdots & \underline{\underline{S}}_{np-(m-1)} \end{bmatrix} \begin{bmatrix} \Delta\underline{u}(k) \\ \Delta\underline{u}(k+1) \\ \Delta\underline{u}(k+2) \\ \vdots \\ \Delta\underline{u}(k+m-1) \end{bmatrix} + \begin{bmatrix} \underline{\hat{y}}_k \\ \underline{\hat{y}}_k \\ \underline{\hat{y}}_k \\ \vdots \\ \underline{\hat{y}}_k \end{bmatrix} + \begin{bmatrix} \underline{P}_1 \\ \underline{P}_2 \\ \underline{P}_3 \\ \vdots \\ \underline{P}_{np} \end{bmatrix}$$

$$\underline{y}^C = \underline{\underline{S}}\,\Delta\underline{u} + \overline{\underline{y_k}} + \overline{\underline{P}}$$

$$\underline{P}_i = \sum_{n=1}^{i} \underline{\phi}_n$$

$$\underline{\phi}_n = \sum_{j=n+1}^{N} H_l \Delta\underline{u}(k+n-j)$$

$$\left[\underline{\underline{S}}_1\right] = ny \times nu$$

DMC Caso MIMO

- Cada matriz S_k

$$\underline{\underline{S_2}} = \underline{\underline{H_1}} + \underline{\underline{H_2}}$$

$$\underline{\underline{S_k}} = \sum_{l=1}^{k} \underline{\underline{H_l}}$$

$$\underline{\underline{H_l}} = \begin{bmatrix} h_{11,l} & h_{12,l} & \cdots & h_{1nu,l} \\ h_{21,l} & h_{22,l} & \cdots & h_{2nu,l} \\ \vdots & \vdots & \ddots & \vdots \\ h_{ny1,l} & h_{ny2,l} & \cdots & h_{nynu,l} \end{bmatrix}$$

DMC Caso MIMO

- Subtraindo o set point para gera o vetor de erros

$$\underline{y}^{C} = \underline{\underline{S}}\,\Delta \underline{u} + \overline{\underline{y_k}} + \overline{\underline{P}}$$

$$\underline{y}^{SP} - \underline{y}^{C} = \underline{y}^{SP} - \underline{\underline{S}}\,\Delta \underline{u} - \overline{\underline{y_k}} - \overline{\underline{P}}$$

$$\underline{e} = -\underline{\underline{S}}\,\Delta \underline{u} + e_k - \overline{\underline{P}} \qquad \underline{e} = -\underline{\underline{S}}\,\Delta \underline{u} + \underline{e}'$$

Erro atual com a resposta do processo conforme modelo e ações aplicados no passado

DMC Caso MIMO

- Função Objetivo

$$J = \left(-\underline{\underline{S}}\Delta\underline{u} + \underline{e}'\right)^T \left(-\underline{\underline{S}}\Delta\underline{u} + \underline{e}'\right) + \Delta\underline{u}^T \underline{\underline{R}}\,\Delta\underline{u}$$

$$J = \Delta\underline{u}^T \underline{\underline{S}}^T \underline{\underline{S}}\Delta\underline{u} - \Delta\underline{u}^T \underline{\underline{S}}^T \underline{e}' - \underline{e}'^T \underline{\underline{S}}\Delta\underline{u} + \underline{e}'^T \underline{e}' + \Delta\underline{u}^T \underline{\underline{R}}\,\Delta\underline{u}$$

DMC Caso MIMO

- Ótimo sem restrições

$$J = \Delta\underline{u}^T \underline{\underline{S}}^T \underline{\underline{S}}\Delta\underline{u} - \Delta\underline{u}^T \underline{\underline{S}}^T \underline{e}' - \underline{e}'^T \underline{\underline{S}}\Delta\underline{u} + \underline{e}'^T \underline{e}' + \Delta\underline{u}^T \underline{\underline{R}}\Delta\underline{u}$$

$$\frac{\partial J}{\partial u} = 2\underline{\underline{S}}^T \underline{\underline{S}}\Delta\underline{u} - \underline{\underline{S}}^T \underline{e}' - \underline{e}'^T \underline{\underline{S}} + +2\underline{\underline{R}}\Delta\underline{u} = 0$$

$$\Delta\underline{u} = \left(\underline{\underline{S}}^T \underline{\underline{S}} + \underline{\underline{R}}\right)^{-1} \underline{\underline{S}}^T \underline{e}'$$

DMC Caso MIMO

- Vetor de soluções

$$\Delta \underline{u} = \begin{bmatrix} \Delta \underline{u}_k \\ \Delta \underline{u}_{k+1} \\ \vdots \\ \Delta \underline{u}_{k+j} \\ \vdots \\ \Delta \underline{u}_{k+m-1} \end{bmatrix}$$

$$\Delta \underline{u}_{k+j} = \begin{bmatrix} \Delta u_{1,k+j} \\ \Delta u_{2,k+j} \\ \vdots \\ \Delta u_{nu,k+j} \end{bmatrix}, \; j = 0,1\ldots m-1$$

DMC Caso MIMO

- Matriz R

$$\underline{\underline{R}}_{k+j} = \begin{bmatrix} r_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & r_{nu} \end{bmatrix}_{k+j} \quad , \; j = 0,1\dots m-1$$

$$\underline{\underline{R}} = \begin{bmatrix} \underline{\underline{R}}_k & 0_{nu\times nu} & & & & \\ 0_{nu\times nu} & \underline{\underline{R}}_{k+1} & & & & \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0_{nu\times nu} & 0_{nu\times nu} & \dots & \underline{\underline{R}}_{k+j} & \dots & 0_{nu\times nu} \\ \vdots & \vdots & \dots & \vdots & \ddots & \vdots \\ 0_{nu\times nu} & 0_{nu\times nu} & \dots & 0_{nu\times nu} & \dots & \underline{\underline{R}}_{k+m-1} \end{bmatrix}$$

DMC Caso MIMO

- Refinamento na função objetivo – priorização das variáveis controladas

$$J = \underline{e}^T \underline{\underline{W}}^T \underline{\underline{W}}\, \underline{e} + \Delta \underline{u}^T \underline{\underline{R}}\, \Delta \underline{u}$$

$$\frac{\partial J}{\partial u} = 0$$

$$\Delta \underline{u} = \left(\underline{\underline{S}}^T \underline{\underline{W}}^T \underline{\underline{W}}\, \underline{\underline{S}} + \underline{\underline{R}} \right)^{-1} \underline{\underline{S}}^T \underline{\underline{W}}^T \underline{\underline{W}}\, \underline{e}'$$

DMC Caso MIMO

$$J = \underline{e}^T \underline{\underline{W}}^T \underline{\underline{W}}\, \underline{e}$$

$$\underline{\underline{W}}^{k+j} = \begin{bmatrix} W_1 & 0 & \cdots & 0 \\ 0 & W_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & W_{ny} \end{bmatrix}$$

$$\underline{\underline{W}} = \begin{bmatrix} \underline{\underline{W}}^{k+1} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & & \\ 0_{ny\times ny} & \underline{\underline{W}}^{k+2} & \cdots & 0_{ny\times ny} & \cdots & 0_{ny\times ny} \\ \vdots & \vdots & \ddots & \vdots & \cdots & \vdots \\ 0_{ny\times ny} & 0_{ny\times ny} & \cdots & \underline{\underline{W}}^{k+j} & \cdots & 0_{ny\times ny} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & \cdots & \underline{\underline{W}}^{k+np} \end{bmatrix}$$

QDMC
MIMO

$$\underline{e} = -\underline{\underline{S}}\,\Delta\underline{u} + \underline{e}'$$

$$J = \underline{e}^T\,\underline{\underline{W}}^T\,\underline{\underline{W}}\,\underline{e} + \Delta\underline{u}^T\,\underline{\underline{R}}\,\Delta\underline{u}$$

$$J = \Delta\underline{u}^T\left(\underline{\underline{S}}^T\,\underline{\underline{W}}^T\,\underline{\underline{W}}\,\underline{\underline{S}} + \underline{\underline{R}}\right)\Delta\underline{u} - \underline{e}'^T\,\underline{\underline{W}}^T\,\underline{\underline{W}}\,\underline{\underline{S}}\,\Delta\underline{u} -$$
$$\Delta\underline{u}^T\,\underline{\underline{S}}^T\,\underline{\underline{W}}^T\,\underline{\underline{W}}\,\underline{e}' + \underline{e}'^T\,\underline{\underline{W}}^T\,\underline{\underline{W}}\,\underline{e}'$$

QDMC

$$J = \Delta\underline{u}^T \left(\underline{\underline{S}}^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{\underline{S}} + \underline{\underline{R}}\right)\Delta\underline{u} - \underline{e}'^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{\underline{S}}\,\Delta\underline{u} - \Delta\underline{u}^T \underline{\underline{S}}^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{e}' + \underline{e}'^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{e}'$$

$$J = \Delta\underline{u}^T \left(\underline{\underline{S}}^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{\underline{S}} + \underline{\underline{R}}\right)\Delta\underline{u} - 2\underline{e}'^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{\underline{S}}\,\Delta\underline{u} + \underline{e}'^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{e}'$$

$$J = \Delta\underline{u}^T \underbrace{\left(\underline{\underline{S}}^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{\underline{S}} + \underline{\underline{R}}\right)}_{\underline{\underline{H}}}\Delta\underline{u} - 2\underbrace{\underline{e}'^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{\underline{S}}}_{C_f^T}\,\Delta\underline{u}$$

Problema QDMC

$$J = \Delta \underline{u}^T \underline{\underline{H}} \Delta \underline{u} + 2 C_f^T \Delta \underline{u}$$

$$\underline{\underline{H}} = \underline{\underline{S}}^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{\underline{S}} + \underline{\underline{R}}$$

$$C_f^T = -\underline{e}'^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{\underline{S}}$$

s.a

$$\underline{y}_{\min} \le \underline{y}^C \le \underline{y}_{\max}$$

$$-\Delta \underline{u}_{\max} \le \Delta \underline{u} \le \Delta \underline{u}_{\max}$$

$$\underline{u}_{\min} \le \underline{u} \le \underline{u}_{\max}$$

Problema QP
Forma Clássica

$$\min \frac{1}{2}\underline{x}^T \underline{\underline{H}}\,\underline{x} + C_f^T \underline{x}$$

$$s.a.\ \underline{\underline{A}}\,\underline{x} - \underline{b} \leq 0$$

Problema QP
Forma Clássica

- Manipulação algébrica para colocar as restrições da forma requerida no solver:

$$s.a.\ \underline{\underline{A}}\,\underline{x} - \underline{b} \leq 0$$

$$-\Delta \underline{u}_{\max} \leq \Delta \underline{u} \leq \Delta \underline{u}_{\max} \rightarrow \underline{\underline{I}}_{nu} \Delta \underline{u} - \Delta \underline{u}_{\max} \leq 0$$

A **B**

Problema QP
Forma Clássica

- Manipulação algébrica para colocar as restrições da forma requerida no solver:

$$s.a. \; \underline{\underline{A}}\,\underline{x} - \underline{b} \le 0$$

$$-\Delta \underline{u}_{\max} \le \Delta \underline{u} \le \Delta \underline{u}_{\max} \quad \rightarrow \quad -\underline{\underline{I}}_{nu} \Delta \underline{u} - \Delta \underline{u}_{\max} \le 0$$

A **B**

Problema QP Forma Clássica

- Manipulação algébrica para colocar as restrições da forma requerida no solver:

$$s.a.\ \underline{\underline{A}}\,\underline{x} - \underline{b} \le 0$$

$$\underline{u}_{\min} \le \underline{u} \le \underline{u}_{\max}$$

$$\underline{\underline{N}}\Delta\underline{u} - \left(\underline{u}_{\max} - \bar{\underline{u}}(k-1)\right) \le 0$$

$$\underline{u} = \begin{bmatrix} \underline{u}(k) \\ \underline{u}(k+1) \\ \vdots \\ \underline{u}(k+m-1) \end{bmatrix}$$

A

$$\underline{u} = \underbrace{\begin{bmatrix} \underline{u}(k-1) \\ \underline{u}(k-1) \\ \vdots \\ \underline{u}(k-1) \end{bmatrix}}_{\bar{\underline{u}}(k-1)} + \underbrace{\begin{bmatrix} \underline{\underline{I}}_{nu} & 0 & \cdots & 0 \\ \underline{\underline{I}}_{nu} & \underline{\underline{I}}_{nu} & 0 & \cdots & 0 \\ \underline{\underline{I}}_{nu} & \underline{\underline{I}}_{nu} & \underline{\underline{I}}_{nu} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \underline{\underline{I}}_{nu} & \underline{\underline{I}}_{nu} & \underline{\underline{I}}_{nu} & \cdots & \underline{\underline{I}}_{nu} \end{bmatrix}}_{\underline{\underline{N}}} \underbrace{\begin{bmatrix} \Delta\underline{u}(k) \\ \Delta\underline{u}(k+1) \\ \vdots \\ \Delta\underline{u}(k+m-1) \end{bmatrix}}_{\Delta\underline{u}} = \bar{\underline{u}}(k-1) + \underline{\underline{N}}\Delta\underline{u}$$

B

Problema QP
Forma Clássica

- Manipulação algébrica para colocar as restrições da forma requerida no solver:

$$s.a.\ \underline{\underline{A}}\,\underline{x} - \underline{b} \leq 0$$

$$\underline{u}_{\min} \leq \underline{u} \leq \underline{u}_{\max} \rightarrow \left(-\underline{\underline{N}}\right)\Delta\underline{u} - \left[\bar{\underline{u}}\left(k-1\right) - \underline{u}_{\min}\right] \leq 0$$

$$\underline{u} = \begin{bmatrix} \underline{u}(k) \\ \underline{u}(k+1) \\ \vdots \\ \underline{u}(k+m-1) \end{bmatrix}$$

A

B

$$\underline{u} = \underbrace{\begin{bmatrix} \underline{u}(k-1) \\ \underline{u}(k-1) \\ \vdots \\ \underline{u}(k-1) \end{bmatrix}}_{\bar{\underline{u}}(k-1)} + \underbrace{\begin{bmatrix} \underline{\underline{I}}_{nu} & 0 & \cdots & 0 \\ \underline{\underline{I}}_{nu} & \underline{\underline{I}}_{nu} & 0 & \cdots & 0 \\ \underline{\underline{I}}_{nu} & \underline{\underline{I}}_{nu} & \underline{\underline{I}}_{nu} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \underline{\underline{I}}_{nu} & \underline{\underline{I}}_{nu} & \underline{\underline{I}}_{nu} & \cdots & \underline{\underline{I}}_{nu} \end{bmatrix}}_{\underline{\underline{N}}} \underbrace{\begin{bmatrix} \Delta\underline{u}(k) \\ \Delta\underline{u}(k+1) \\ \vdots \\ \Delta\underline{u}(k+m-1) \end{bmatrix}}_{\Delta\underline{u}} = \bar{\underline{u}}(k-1) + \underline{\underline{N}}\Delta\underline{u}$$

Problema QP
Forma Clássica

- Manipulação algébrica para colocar as restrições da forma requerida no solver:

$$s.a.\ \underline{\underline{A}}\,\underline{x} - \underline{b} \le 0$$

$$\underline{y}_{min} \le \underline{y}^{C} \le \underline{y}_{max}$$

$$\underline{\underline{S}}\,\Delta \underline{u} - \left(\underline{y}_{max} - \overline{\underline{y}_k} - \overline{\underline{P}} \right) \le 0$$

A

B

$$\underline{y}^{C} = \underline{\underline{S}}\,\Delta \underline{u} + \overline{\underline{y}_k} + \overline{\underline{P}}$$

Problema QP
Forma Clássica

- Manipulação algébrica para colocar as restrições da forma requerida no solver:

$$s.a.\ \underline{\underline{A}}\,\underline{x} - \underline{b} \le 0$$

$$\underline{y}_{\min} \le \underline{y}^{C} \le \underline{y}_{\max}$$

$$-\underline{\underline{S}}\,\Delta \underline{u} - \left(\overline{\underline{y}_k} + \overline{\underline{P}} - \underline{y}_{\min} \right) \le 0$$

A **B**

$$\underline{y}^{C} = \underline{\underline{S}}\,\Delta \underline{u} + \overline{\underline{y}_k} + \overline{\underline{P}}$$

Problema QDMC
Reunindo todas restrições

$$J = \frac{1}{2}\Delta \underline{u}^T \underline{\underline{H}} \Delta \underline{u} + C_f^T \Delta \underline{u}$$

$$\underline{\underline{H}} = \underline{\underline{S}}^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{\underline{S}} + \underline{\underline{R}}$$

$$C_f^T = -\underline{e}'^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{\underline{S}}$$

s.a

$$\begin{bmatrix} \underline{\underline{I}}_{nu} \\ -\underline{\underline{I}}_{nu} \\ \underline{\underline{N}} \\ -\underline{\underline{N}} \\ \underline{\underline{S}} \\ -\underline{\underline{S}} \end{bmatrix} \Delta \underline{u} - \begin{bmatrix} \Delta \underline{u}_{\max} \\ \Delta \underline{u}_{\max} \\ \underline{u}_{\max} - \overline{\underline{u}}(k-1) \\ \overline{\underline{u}}(k-1) - \underline{u}_{\min} \\ \underline{y}_{\max} - \overline{\underline{y}_k} - \overline{\underline{P}} \\ \overline{\underline{y}_k} + \overline{\underline{P}} - \underline{y}_{\min} \end{bmatrix} \leq 0$$

DMC POR REALINHAMENTO

Ações passadas

$$\underline{y}^{C} = \underline{\underline{S}}\,\Delta\underline{u} + \underline{\overline{y_k}} + \underline{\overline{P}}$$

$$\underline{\overline{P}}_i = \sum_{n=1}^{i} \phi_n$$

$$\phi_n = \sum_{j=n+1}^{N} \underline{\underline{H_j}}\,\Delta\underline{u}\left(k+n-j\right)$$

ESFORÇO COMPUTACIONAL ELEVADO
DIFICULDADE DE REALIZAÇÃO DO CONTROLE EM TEMPO REAL

DMC por realinhamento

$$\begin{bmatrix} \underline{\hat{y}}^{C}_{k+1} \\ \underline{\hat{y}}^{C}_{k+2} \\ \underline{\hat{y}}^{C}_{k+3} \\ \vdots \\ \underline{\hat{y}}^{C}_{k+np} \end{bmatrix} = \begin{bmatrix} \underline{\underline{S}}_1 & 0_{ny\times nu} & 0_{ny\times nu} & \cdots & 0_{ny\times nu} \\ \underline{\underline{S}}_2 & \underline{\underline{S}}_1 & 0_{ny\times nu} & \cdots & 0_{ny\times nu} \\ \underline{\underline{S}}_3 & \underline{\underline{S}}_2 & \underline{\underline{S}}_1 & \cdots & 0_{ny\times nu} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \underline{\underline{S}}_{np} & \underline{\underline{S}}_{np-1} & \underline{\underline{S}}_{np-2} & \cdots & \underline{\underline{S}}_{np-m+1} \end{bmatrix} \begin{bmatrix} \Delta\underline{u}(k) \\ \Delta\underline{u}(k+1) \\ \Delta\underline{u}(k+2) \\ \vdots \\ \Delta\underline{u}(k+m-1) \end{bmatrix} + \begin{bmatrix} \underline{\hat{y}}_k + \underline{P}_1 \\ \underline{\hat{y}}_k + \underline{P}_2 \\ \underline{\hat{y}}_k + \underline{P}_3 \\ \vdots \\ \underline{\hat{y}}_k + \underline{P}_{np} \end{bmatrix}$$

DMC por realinhamento

- Cuttler propôs o realinhamento

$$\left[\hat{y}^{P}_{k+1}\right]_{k+1} = \left[\hat{y}^{P}_{k+2}\right]_{k}$$

A PREDIÇÃO DE K+1 NO INSTANTE K+1 SERÁ A PREDIÇÃO DE K+2 NO INSTANTE K

INSERINDO M=1 AÇÃO DE CONTROLE NO DMC DE REALINHAMENTO

DMC por realinhamento

- Inserindo o efeito da ação de controle aplicada no instante *k*:

A AÇÃO DAS ENTRADAS NO INSTANTE K-1 SÃO CONSIDERADAS, DE CERTA FORMA, QUANDO SE FAZ O DESLOCAMENTO DA PREDIÇÃO

O DEGRAU APLICADO EM K TEM O EFEITO S2 NO INSTANTE K+2

$$\left[\hat{y}^{P}_{k+1}\right]_{k+1} = \left[\hat{y}^{P}_{k+2}\right]_{k}$$

$$\left[\underline{\hat{y}}^{P}_{k+1}\right]_{k+1} = \left[\underline{\hat{y}}^{P}_{k+2}\right]_{k} + \underline{\underline{S}}_{2}\Delta\underline{u}(k)$$

DMC por realinhamento

- A predição para *k*+2 nesse instante *k*+1 será, analogamente:

$$\left[\hat{y}^{P}_{k+2}\right]_{k+1} = \left[\hat{y}^{P}_{k+3}\right]_{k}$$

O DEGRAU APLICADO EM K TEM O EFEITO S3 NO INSTANTE K+3

$$\left[\underline{\hat{y}}^{P}_{k+2}\right]_{k+1} = \left[\underline{\hat{y}}^{P}_{k+3}\right]_{k} + \underline{\underline{S}}_3 \Delta \underline{u}(k)$$

DMC por realinhamento

- A predição em *k*+1 para um instante genérico j

$$\left[\hat{\underline{y}}^{P}_{k+1}\right]_{k+1} = \left[\hat{\underline{y}}^{P}_{k+2}\right]_{k} + \underline{\underline{S}}_{2}\,\Delta\underline{u}(k)$$

$$\left[\hat{\underline{y}}^{P}_{k+j}\right]_{k+1} = \left[\hat{\underline{y}}^{P}_{k+j+1}\right]_{k} + \underline{\underline{S}}_{j+1}\,\Delta\underline{u}(k)$$

DMC por realinhamento
Predição até o horizonte NP

$$
\begin{bmatrix} \underline{\hat{y}}^{P}_{k+1} \\ \underline{\hat{y}}^{P}_{k+2} \\ \underline{\hat{y}}^{P}_{k+3} \\ \vdots \\ \underline{\hat{y}}^{P}_{k+np} \end{bmatrix}_{k+1}
=
\underbrace{\begin{bmatrix}
0_{ny\times ny} & I_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & I_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny}
\end{bmatrix}}_{\underline{\underline{M}}}
\begin{bmatrix} \underline{\hat{y}}^{P}_{k+1} \\ \underline{\hat{y}}^{P}_{k+2} \\ \underline{\hat{y}}^{P}_{k+3} \\ \vdots \\ \underline{\hat{y}}^{P}_{k+np} \end{bmatrix}_{k}
+
\underbrace{\begin{bmatrix}
0_{ny\times ny} & I_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & I_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny}
\end{bmatrix}}_{\underline{\underline{M}}}
\underbrace{\begin{bmatrix} \underline{\underline{S}}_{1} \\ \underline{\underline{S}}_{2} \\ \underline{\underline{S}}_{3} \\ \vdots \\ \underline{\underline{S}}_{np} \end{bmatrix}}_{\underline{\underline{S}}}
\Delta \underline{u}(k)
$$

DMC por realinhamento
Predição até o horizonte NP

$$
\begin{bmatrix} \hat{\underline{y}}^{P}_{k+1} \\ \hat{\underline{y}}^{P}_{k+2} \\ \hat{\underline{y}}^{P}_{k+3} \\ \vdots \\ \hat{\underline{y}}^{P}_{k+np} \end{bmatrix}_{k+1}
=
\underbrace{\begin{bmatrix}
0_{ny\times ny} & I_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & I_{ny\times ny} & \cdot & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny}
\end{bmatrix}}_{\underline{\underline{M}}}
\begin{bmatrix} \hat{\underline{y}}^{P}_{k+1} \\ \hat{\underline{y}}^{P}_{k+2} \\ \hat{\underline{y}}^{P}_{k+3} \\ \vdots \\ \hat{\underline{y}}^{P}_{k+np} \end{bmatrix}_{k}
+
$$

$$
\underbrace{\begin{bmatrix}
0_{ny\times ny} & I_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & I_{ny\times ny} & \cdot & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny}
\end{bmatrix}}_{\underline{\underline{M}}}
\underbrace{\begin{bmatrix} \underline{\underline{S}}_{1} \\ \underline{\underline{S}}_{2} \\ \underline{\underline{S}}_{3} \\ \vdots \\ \underline{\underline{S}}_{np} \end{bmatrix}}_{\underline{\underline{S}}}
\Delta \underline{u}(k)
$$

$$
\begin{bmatrix} \hat{\underline{y}}^{P}_{k+1} \\ \hat{\underline{y}}^{P}_{k+2} \\ \vdots \\ \hat{\underline{y}}^{P}_{k+np-1} \\ \hat{\underline{y}}^{P}_{k+np} \end{bmatrix}_{k+1}
=
\underbrace{\begin{bmatrix}
0_{ny\times ny} & I_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & I_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny}
\end{bmatrix}}_{\underline{\underline{M}}}
\begin{bmatrix} \hat{\underline{y}}^{P}_{k+1} \\ \hat{\underline{y}}^{P}_{k+2} \\ \hat{\underline{y}}^{P}_{k+3} \\ \vdots \\ \hat{\underline{y}}^{P}_{k+np} \end{bmatrix}_{k}
+
\underbrace{\begin{bmatrix} \vdots \\ \underline{\underline{S}}_{np-1} \\ \underline{\underline{S}}_{np} \end{bmatrix}}_{\underline{\underline{S}}}
\Delta \underline{u}(k)
$$

$$\underline{\underline{S}}_{np} \approx \underline{\underline{S}}_{np-1}$$

VÁLIDO PARA SISTEMAS ESTÁVEIS

$$
\left[\hat{\underline{y}}^{P}_{k+np}\right]_{k+1} = \left[\hat{\underline{y}}^{P}_{k+np-1}\right]_{k+1} = \left[\hat{\underline{y}}^{P}_{k+np}\right]_{k}
$$

M MATRIZ DE DESLOCAMENTO

$$
\begin{bmatrix} \hat{\underline{y}}^{P}_{k+1} \\ \hat{\underline{y}}^{P}_{k+2} \\ \hat{\underline{y}}^{P}_{k+3} \\ \vdots \\ \hat{\underline{y}}^{P}_{k+np} \end{bmatrix}_{k+1} =
\begin{bmatrix}
0_{ny\times ny} & I_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & I_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny}
\end{bmatrix}
\begin{bmatrix} \hat{\underline{y}}^{P}_{k+1} \\ \hat{\underline{y}}^{P}_{k+2} \\ \hat{\underline{y}}^{P}_{k+3} \\ \vdots \\ \hat{\underline{y}}^{P}_{k+np} \end{bmatrix}_{k} +
\begin{bmatrix}
0_{ny\times ny} & I_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & I_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny}
\end{bmatrix}
\begin{bmatrix} \underline{\underline{S}}_{1} \\ \underline{\underline{S}}_{2} \\ \underline{\underline{S}}_{3} \\ \vdots \\ \underline{\underline{S}}_{np} \end{bmatrix} \Delta u(k)
$$

APENAS UMA AÇÃO DE CONTROLE

$$
\hat{\underline{y}}^{P}_{k+1} = \underline{\underline{M}}\,\hat{\underline{y}}^{P}_{k} + \underline{\underline{M}}\,\underline{\underline{S}}\,\Delta \underline{u}(k)
$$

DMC por realinhamento

- Predição até o horizonte *np*

$$\underline{\hat{y}}^{P}_{k+1} = \underline{\underline{M}}\,\underline{\hat{y}}^{P}_{k} + \underline{\underline{M}}\,\underline{\underline{\bar{S}}}\Delta \underline{u}(k)$$

$$\left[\underline{\underline{M}}\right] = np.ny \times np.ny$$

$$\left[\underline{\underline{\bar{S}}}\right] = np.ny \times nu$$

AS AÇÕES FEITAS NO PASSADO SÃO TRANSFERIDAS NO DESLOCAMENTO DA PREDIÇÃO

Correção da Predição DMC por realinhamento

- Predição para o instante k+1 em k+1

$$\left[\hat{y}^{P}_{k+1}\right]_{k+1} = \left[\hat{y}^{C}_{k+2}\right]_{k}$$

- Correção da Predição

$$d_{k+1} = \hat{\underline{y}}_{k+1} - \left\{\left[\hat{\underline{y}}^{P}_{k+1}\right]_{k} + \underline{\underline{S}}_1 \Delta \underline{u}(k)\right\}$$

Correção da Predição DMC por realinhamento

- Predição corrigida para o instante *np*

$$\underline{\hat{y}}^{C}_{k+1} = \underline{\hat{y}}^{P}_{k+1} + \underline{d}(k+1)$$

$$\underline{\hat{y}}^{P}_{k+1} = \underline{\underline{M}}\,\underline{\hat{y}}^{C}_{k} + \underline{\underline{M}}\,\underline{\underline{\bar{S}}}\Delta\underline{u}(k)$$

$$\underline{\hat{y}}^{C}_{k+1} = \underline{\underline{M}}\left\{\underline{\hat{y}}^{C}_{k} + \underline{\underline{\bar{S}}}\Delta\underline{u}(k)\right\} + \underline{d}(k+1)$$

DMC com correção

$$\begin{bmatrix} \underline{\hat{y}}^{C}_{k+1} \\ \underline{\hat{y}}^{C}_{k+2} \\ \underline{\hat{y}}^{C}_{k+3} \\ \vdots \\ \underline{\hat{y}}^{C}_{k+np} \end{bmatrix}_{k+1} = \begin{bmatrix} 0_{ny\times ny} & I_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\ 0_{ny\times ny} & 0_{ny\times ny} & I_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\ 0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \\ 0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \end{bmatrix} \left\{ \begin{bmatrix} \underline{\hat{y}}^{C}_{k+1} \\ \underline{\hat{y}}^{C}_{k+2} \\ \underline{\hat{y}}^{C}_{k+3} \\ \vdots \\ \underline{\hat{y}}^{C}_{k+np} \end{bmatrix}_{k} + \begin{bmatrix} \underline{\underline{S}}_{1} \\ \underline{\underline{S}}_{2} \\ \underline{\underline{S}}_{3} \\ \vdots \\ \underline{\underline{S}}_{np} \end{bmatrix} \Delta \underline{u}(k) \right\} + \begin{bmatrix} \underline{d}_{k+1} \\ \underline{d}_{k+1} \\ \underline{d}_{k+1} \\ \vdots \\ \underline{d}_{k+1} \end{bmatrix}$$

VETOR DE CORREÇÃO

$$\underline{d}_{k+1} = \underline{\hat{y}}_{k+1} - \left\{ \left[\underline{\hat{y}}^{P}_{k+1} \right]_{k} + \underline{\underline{S}}_{1} \Delta \underline{u}(k) \right\}$$

INSERINDO M=2 AÇÕES DE CONTROLE NO DMC DE REALINHAMENTO

DMC por realinhamento

- Predição de *k*+1 feita no instante *k*+1:

A AÇÃO DAS ENTRADAS NO INSTANTE K-1 SÃO CONSIDERADAS, DE CERTA FORMA, QUANDO SE FAZ O DESLOCAMENTO DA PREDIÇÃO

$$\left[\hat{y}^{P}_{k+1}\right]_{k+1} = \left[\hat{y}^{C}_{k+2}\right]_{k}$$

O DEGRAU APLICADO EM K TEM O EFEITO S2 NO INSTANTE K+2

O DEGRAU APLICADO EM K+1 TEM O EFEITO S1 NO INSTANTE K+2

$$\left[\hat{\underline{y}}^{P}_{k+1}\right]_{k+1} = \left[\hat{\underline{y}}^{C}_{k+2}\right]_{k} + \underline{\underline{S}}_2 \Delta \underline{u}(k) + \underline{\underline{S}}_1 \Delta \underline{u}(k+1)$$

DMC por realinhamento

- A predição para *k*+2 nesse instante *k*+1 será, analogamente:

$$\left[\hat{y}^{P}_{k+2}\right]_{k+1} = \left[\hat{y}^{C}_{k+3}\right]_{k}$$

O DEGRAU APLICADO EM K TEM O EFEITO S3 NO INSTANTE K+3 E O DEGRAU APLICADO EM K+1 DE S2.
NÃO HOUVE DEGRAU EM K+2 PORQUE SÓ EXISTEM DUAS AÇÕES DE CONTROLE.

$$\left[\underline{\hat{y}}^{P}_{k+2}\right]_{k+1} = \left[\underline{\hat{y}}^{C}_{k+3}\right]_{k} + \underline{\underline{S}}_3 \Delta \underline{u}(k) + \underline{\underline{S}}_2 \Delta \underline{u}(k+1)$$

DMC por realinhamento

- Montando a equação matricial dessas duas ações de controle para predição até o horizonte *np*:

$$
\begin{bmatrix} \hat{\underline{y}}^{C}_{k+1} \\ \hat{\underline{y}}^{C}_{k+2} \\ \hat{\underline{y}}^{C}_{k+3} \\ \vdots \\ \hat{\underline{y}}^{C}_{k+np} \end{bmatrix}_{k+1}
=
\begin{bmatrix}
0_{ny\times ny} & I_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & I_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny}
\end{bmatrix}
\begin{bmatrix} \hat{\underline{y}}^{C}_{k+1} \\ \hat{\underline{y}}^{C}_{k+2} \\ \hat{\underline{y}}^{C}_{k+3} \\ \vdots \\ \hat{\underline{y}}^{C}_{k+np} \end{bmatrix}_{k}
+
\begin{bmatrix}
0_{ny\times ny} & I_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & I_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny}
\end{bmatrix}
\begin{bmatrix} \underline{\underline{S}}_{1} \\ \underline{\underline{S}}_{2} \\ \underline{\underline{S}}_{3} \\ \vdots \\ \underline{\underline{S}}_{np} \end{bmatrix}
\Delta \underline{u}(k)
$$

$$
+
\begin{bmatrix}
0_{ny\times ny} & I_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & I_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny}
\end{bmatrix}
\begin{bmatrix} 0_{ny\times nu} \\ \underline{\underline{S}}_{1} \\ \underline{\underline{S}}_{2} \\ \vdots \\ \underline{\underline{S}}_{np-1} \end{bmatrix}
\Delta \underline{u}(k+1)
+
\begin{bmatrix} \underline{d}_{k+1} \\ \underline{d}_{k+1} \\ \underline{d}_{k+1} \\ \vdots \\ \underline{d}_{k+1} \end{bmatrix}
$$

DMC por realinhamento

- Montando a equação matricial dessas duas ações de controle para predição até o horizonte *np*:

$$
\begin{bmatrix} \hat{\underline{y}}^{C}_{k+1} \\ \hat{\underline{y}}^{C}_{k+2} \\ \hat{\underline{y}}^{C}_{k+3} \\ \vdots \\ \hat{\underline{y}}^{C}_{k+np} \end{bmatrix}_{k+1}
=
\begin{bmatrix}
0_{ny\times ny} & I_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & I_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny}
\end{bmatrix}
\begin{bmatrix} \hat{\underline{y}}^{C}_{k+1} \\ \hat{\underline{y}}^{C}_{k+2} \\ \hat{\underline{y}}^{C}_{k+3} \\ \vdots \\ \hat{\underline{y}}^{C}_{k+np} \end{bmatrix}_{k}
+
\begin{bmatrix}
0_{ny\times ny} & I_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & I_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny}
\end{bmatrix}
\begin{bmatrix} \underline{\underline{S}}_{1} \\ \underline{\underline{S}}_{2} \\ \underline{\underline{S}}_{3} \\ \vdots \\ \underline{\underline{S}}_{np} \end{bmatrix}
\Delta \underline{u}(k)
$$

$$
+
\begin{bmatrix}
0_{ny\times ny} & I_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & I_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny}
\end{bmatrix}
\begin{bmatrix} 0_{ny\times nu} \\ \underline{\underline{S}}_{1} \\ \underline{\underline{S}}_{2} \\ \vdots \\ \underline{\underline{S}}_{np-1} \end{bmatrix}
\Delta \underline{u}(k+1)
+
\begin{bmatrix} \underline{d}_{k+1} \\ \underline{d}_{k+1} \\ \underline{d}_{k+1} \\ \vdots \\ \underline{d}_{k+1} \end{bmatrix}
$$

INSERINDO M=3 AÇÕES DE CONTROLE NO DMC DE REALINHAMENTO

DMC por realinhamento

- A predição para *k*+2 nesse instante *k*+1 será, analogamente:

$$\left[\hat{y}^{P}_{k+2}\right]_{k+1} = \left[\hat{y}^{P}_{k+3}\right]_{k}$$

O DEGRAU APLICADO EM K TEM O EFEITO S3 NO INSTANTE K+3. DE K+1 S2 E DE K+2 S1

$$\left[\underline{\hat{y}}^{P}_{k+2}\right]_{k+1} = \left[\underline{\hat{y}}^{C}_{k+3}\right]_{k} + \underline{\underline{S}}_{3}\Delta\underline{u}(k) + \underline{\underline{S}}_{2}\Delta\underline{u}(k+1) + \underline{\underline{S}}_{1}\Delta\underline{u}(k+2) + \underline{d}_{k+1}$$

DEFINE A AÇÃO DU(K+J) QUE O DEGRAU VALE S1

DMC por realinhamento

$$
\begin{bmatrix} \hat{\underline{y}}^{C}_{k+1} \\ \hat{\underline{y}}^{C}_{k+2} \\ \hat{\underline{y}}^{C}_{k+3} \\ \vdots \\ \hat{\underline{y}}^{C}_{k+np} \end{bmatrix}_{k+1}
=
\begin{bmatrix}
0_{ny\times ny} & I_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & I_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny}
\end{bmatrix}
\begin{bmatrix} \hat{\underline{y}}^{C}_{k+1} \\ \hat{\underline{y}}^{C}_{k+2} \\ \hat{\underline{y}}^{C}_{k+3} \\ \vdots \\ \hat{\underline{y}}^{C}_{k+np} \end{bmatrix}_{k}
+
\begin{bmatrix}
0_{ny\times ny} & I_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & I_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny}
\end{bmatrix}
\begin{bmatrix} \underline{\underline{S}}_{1} \\ \underline{\underline{S}}_{2} \\ \underline{\underline{S}}_{3} \\ \vdots \\ \underline{\underline{S}}_{np} \end{bmatrix}
\Delta\underline{u}(k)
$$

$$
+
\begin{bmatrix}
0_{ny\times ny} & I_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & I_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny}
\end{bmatrix}
\begin{bmatrix} 0_{ny\times nu} \\ \underline{\underline{S}}_{1} \\ \underline{\underline{S}}_{2} \\ \vdots \\ \underline{\underline{S}}_{np-1} \end{bmatrix}
\Delta\underline{u}(k+1)
+
\begin{bmatrix}
0_{ny\times ny} & I_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & I_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny}
\end{bmatrix}
\begin{bmatrix} 0_{ny\times nu} \\ 0_{ny\times nu} \\ \underline{\underline{S}}_{1} \\ \vdots \\ \underline{\underline{S}}_{np-2} \end{bmatrix}
\Delta\underline{u}(k+2)
$$

$$
+
\begin{bmatrix} \underline{d}_{k+1} \\ \underline{d}_{k+1} \\ \underline{d}_{k+1} \\ \vdots \\ \underline{d}_{k+1} \end{bmatrix}
$$

DMC por realinhamento

$$
\begin{bmatrix} \hat{\underline{y}}^{C}_{k+1} \\ \hat{\underline{y}}^{C}_{k+2} \\ \hat{\underline{y}}^{C}_{k+3} \\ \vdots \\ \hat{\underline{y}}^{C}_{k+np} \end{bmatrix}_{k+1} =
\begin{bmatrix}
0_{ny\times ny} & I_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & I_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny}
\end{bmatrix}
\begin{bmatrix} \hat{\underline{y}}^{C}_{k+1} \\ \hat{\underline{y}}^{C}_{k+2} \\ \hat{\underline{y}}^{C}_{k+3} \\ \vdots \\ \hat{\underline{y}}^{C}_{k+np} \end{bmatrix}_{k} +
\begin{bmatrix}
0_{ny\times ny} & I_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & I_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny}
\end{bmatrix}
\begin{bmatrix} \underline{\underline{S}}_{1} \\ \underline{\underline{S}}_{2} \\ \underline{\underline{S}}_{3} \\ \vdots \\ \underline{\underline{S}}_{np} \end{bmatrix} \Delta\underline{u}(k)
$$

$$
+ \begin{bmatrix}
0_{ny\times ny} & I_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & I_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny}
\end{bmatrix}
\begin{bmatrix} 0_{ny\times nu} \\ \underline{\underline{S}}_{1} \\ \underline{\underline{S}}_{2} \\ \vdots \\ \underline{\underline{S}}_{np-1} \end{bmatrix} \Delta\underline{u}(k+1) +
\begin{bmatrix}
0_{ny\times ny} & I_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & I_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny}
\end{bmatrix}
\begin{bmatrix} 0_{ny\times nu} \\ 0_{ny\times nu} \\ \underline{\underline{S}}_{1} \\ \vdots \\ \underline{\underline{S}}_{np-2} \end{bmatrix} \Delta\underline{u}(k+2)
$$

$$
+ \begin{bmatrix} \underline{d}_{k+1} \\ \underline{d}_{k+1} \\ \underline{d}_{k+1} \\ \vdots \\ \underline{d}_{k+1} \end{bmatrix}
$$

DMC por realinhamento

- A predição em *k*+1 para um instante *m*-1 e horizonte de controle *m*

$$\left[\underline{\hat{y}}^{C}_{k+m-1}\right]_{k+1} = \left[\underline{\hat{y}}^{C}_{k+m}\right]_{k} + \underline{\underline{S}}_{m}\Delta\underline{u}(k) + \underline{\underline{S}}_{m-1}\Delta\underline{u}(k+1)\ldots + \underline{\underline{S}}_{2}\Delta\underline{u}(k+m-2) + \underline{\underline{S}}_{1}\Delta\underline{u}(k+m-1) + \underline{d}_{k+1}$$

- A predição em *k*+1 para um instante genérico *m* e horizonte de controle *m*

$$\left[\underline{\hat{y}}^{C}_{k+m}\right]_{k+1} = \left[\underline{\hat{y}}^{C}_{k+m+1}\right]_{k} + \underline{\underline{S}}_{m+1}\Delta\underline{u}(k) + \underline{\underline{S}}_{m}\Delta\underline{u}(k+1)\ldots + \underline{\underline{S}}_{2}\Delta\underline{u}(k+m-1) + \underline{d}_{k+1}$$

M-1+1=M PREVISÃO

DMC por realinhamento

- A predição em *k*+1 para um instante genérico *j* e horizonte de controle *m*

$$\left[\underline{\hat{y}}^{C}_{k+m}\right]_{k+1} = \left[\underline{\hat{y}}^{C}_{k+m+1}\right]_{k} + \underline{\underline{S}}_{m+1}\Delta\underline{u}(k) + \underline{\underline{S}}_{m}\Delta\underline{u}(k+1)\ldots + \underline{\underline{S}}_{2}\Delta\underline{u}(k+m-1) + \underline{d}_{k+1}$$

REGRA PARA O ÍNDICE DE S:
(J+1)-(1)

$$\left[\underline{\hat{y}}^{C}_{k+j}\right]_{k+1} = \left[\underline{\hat{y}}^{C}_{k+j+1}\right]_{k} + \underline{\underline{S}}_{j+1}\Delta\underline{u}(k) + \underline{\underline{S}}_{j}\Delta\underline{u}(k+1)\ldots + \underline{\underline{S}}_{j-m+2}\Delta\underline{u}(k+m-1) + \underline{d}_{k+1}$$

SE J=M ÍNDICE DE S=2

DMC por realinhamento

- A predição em *k*+1 para um instante *j=np-1* e horizonte de controle *m*

$$\left[\underline{\hat{y}}^{C}_{k+j}\right]_{k+1} = \left[\underline{\hat{y}}^{C}_{k+j+1}\right]_{k} + \underline{\underline{S}}_{j+1}\Delta\underline{u}(k) + \underline{\underline{S}}_{j}\Delta\underline{u}(k+1)\ldots + \underline{\underline{S}}_{j-m+2}\Delta\underline{u}(k+m-1) + \underline{d}_{k+1}$$

$$\left[\underline{\hat{y}}^{C}_{k+np-1}\right]_{k+1} = \left[\underline{\hat{y}}^{C}_{k+np}\right]_{k} + \underline{\underline{S}}_{np}\Delta\underline{u}(k) + \underline{\underline{S}}_{np-1}\Delta\underline{u}(k+1)\ldots + \underline{\underline{S}}_{np-m+1}\Delta\underline{u}(k+m-1) + \underline{d}_{k+1}$$

DMC por realinhamento

- A predição em k+1 para um instante j=np e horizonte de controle m

$$\left[\underline{\hat{y}}^{C}_{k+j}\right]_{k+1} = \left[\underline{\hat{y}}^{C}_{k+j+1}\right]_{k} + \underline{\underline{S}}_{j+1}\Delta\underline{u}(k) + \underline{\underline{S}}_{j}\Delta\underline{u}(k+1)\ldots + \underline{\underline{S}}_{j-m+2}\Delta\underline{u}(k+m-1) + \underline{d}_{k+1}$$

$$\left[\underline{\hat{y}}^{C}_{k+np}\right]_{k+1} = \left[\underline{\hat{y}}^{C}_{k+np+1}\right]_{k} + \underline{\underline{S}}_{np+1}\Delta\underline{u}(k) + \underline{\underline{S}}_{np}\Delta\underline{u}(k+1)\ldots + \underline{\underline{S}}_{np-m+2}\Delta\underline{u}(k+m-1) + \underline{d}_{k+1}$$

ESSA PREDIÇÃO NÃO EXISTE NO INSTANTE K

$$\left[\underline{\hat{y}}^{C}_{k+np}\right]_{k+1} = \left[\underline{\hat{y}}^{C}_{k+np-1}\right]_{k+1} = \left[\underline{\hat{y}}^{C}_{k+np}\right]_{k}$$

DESLOCAMENTO DA PREDIÇÃO CORRIGIDA PELA LEITURA NO INSTANTE K

$$
\begin{bmatrix} \hat{\underline{y}}^{C}_{k+1} \\ \hat{\underline{y}}^{C}_{k+2} \\ \hat{\underline{y}}^{C}_{k+3} \\ \vdots \\ \hat{\underline{y}}^{C}_{k+np} \end{bmatrix}_{k+1}
=
\begin{bmatrix}
0_{ny\times ny} & I_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & I_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny}
\end{bmatrix}
\begin{bmatrix} \hat{\underline{y}}^{C}_{k+1} \\ \hat{\underline{y}}^{C}_{k+2} \\ \hat{\underline{y}}^{C}_{k+3} \\ \vdots \\ \hat{\underline{y}}^{C}_{k+np} \end{bmatrix}_{k}
+
\begin{bmatrix} \underline{d}_{k+1} \\ \underline{d}_{k+1} \\ \underline{d}_{k+1} \\ \vdots \\ \underline{d}_{k+1} \end{bmatrix}
+
$$

CORREÇÃO DA PREDIÇÃO NO INSTANTE K+1

$$
\begin{bmatrix}
0_{ny\times ny} & I_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & I_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \\
0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny}
\end{bmatrix}
\begin{bmatrix}
\underline{\underline{S}}_{1} & 0_{ny\times nu} & 0_{ny\times nu} & 0_{ny\times nu} & \cdots & 0_{ny\times nu} \\
\underline{\underline{S}}_{2} & \underline{\underline{S}}_{1} & 0_{ny\times nu} & 0_{ny\times nu} & \cdots & 0_{ny\times nu} \\
\underline{\underline{S}}_{3} & \underline{\underline{S}}_{2} & \underline{\underline{S}}_{1} & 0_{ny\times nu} & \cdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \ddots & 0_{ny\times nu} \\
\underline{\underline{S}}_{np-1} & \underline{\underline{S}}_{np-2} & \underline{\underline{S}}_{np-3} & \underline{\underline{S}}_{np-4} & \cdots & \underline{\underline{S}}_{np-m} \\
\underline{\underline{S}}_{np} & \underline{\underline{S}}_{np-1} & \underline{\underline{S}}_{np-2} & \underline{\underline{S}}_{np-3} & \cdots & \underline{\underline{S}}_{np-m+1}
\end{bmatrix}
\begin{bmatrix} \Delta\underline{u}(k) \\ \Delta\underline{u}(k+1) \\ \Delta\underline{u}(k+2) \\ \vdots \\ \Delta\underline{u}(k+m-2) \\ \Delta\underline{u}(k+m-1) \end{bmatrix}
$$

Notação Vetorial com correção da predição

$$
\begin{bmatrix} \hat{\underline{y}}^{C}_{k+1} \\ \hat{\underline{y}}^{C}_{k+2} \\ \hat{\underline{y}}^{C}_{k+3} \\ \vdots \\ \hat{\underline{y}}^{C}_{k+np} \end{bmatrix}_{k+1} = \begin{bmatrix} 0_{ny\times ny} & I_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\ 0_{ny\times ny} & 0_{ny\times ny} & I_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\ 0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \\ 0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \end{bmatrix} \begin{bmatrix} \hat{\underline{y}}^{C}_{k+1} \\ \hat{\underline{y}}^{C}_{k+2} \\ \hat{\underline{y}}^{C}_{k+3} \\ \vdots \\ \hat{\underline{y}}^{C}_{k+np} \end{bmatrix}_{k} + \begin{bmatrix} \underline{d}_{k+1} \\ \underline{d}_{k+1} \\ \underline{d}_{k+1} \\ \vdots \\ \underline{d}_{k+1} \end{bmatrix} +
$$

$$
\begin{bmatrix} 0_{ny\times ny} & I_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\ 0_{ny\times ny} & 0_{ny\times ny} & I_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\ 0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \\ 0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \end{bmatrix} \begin{bmatrix} \underline{\underline{S}}_{1} & 0_{ny\times nu} & 0_{ny\times nu} & 0_{ny\times nu} & \cdots & 0_{ny\times nu} \\ \underline{\underline{S}}_{2} & \underline{\underline{S}}_{1} & 0_{ny\times nu} & 0_{ny\times nu} & \cdots & 0_{ny\times nu} \\ \underline{\underline{S}}_{3} & \underline{\underline{S}}_{2} & \underline{\underline{S}}_{1} & 0_{ny\times nu} & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0_{ny\times nu} \\ \underline{\underline{S}}_{np-1} & \underline{\underline{S}}_{np-2} & \underline{\underline{S}}_{np-3} & \underline{\underline{S}}_{np-4} & \cdots & \underline{\underline{S}}_{np-m} \\ \underline{\underline{S}}_{np} & \underline{\underline{S}}_{np-1} & \underline{\underline{S}}_{np-2} & \underline{\underline{S}}_{np-3} & \cdots & \underline{\underline{S}}_{np-m+1} \end{bmatrix} \begin{bmatrix} \Delta\underline{u}(k) \\ \Delta\underline{u}(k+1) \\ \Delta\underline{u}(k+2) \\ \vdots \\ \Delta\underline{u}(k+m-2) \\ \Delta\underline{u}(k+m-1) \end{bmatrix}
$$

$$
\hat{\underline{y}}^{C}_{k+1} = \underline{\underline{M}}\,\hat{\underline{y}}^{C}_{k} + \underline{\underline{M}}\,\underline{\underline{D}}_{M}\Delta\underline{u}_{k} + \underline{\bar{d}}(k+1)
$$

DMC COM O USO DA MATRIZ AUXILIAR θ

DMC com uso de θ

**AO IDENTIFICAR MODELOS
DE SISTEMAS AUTO REGULADORES
AGUARDA-SE A ESTABILIZAÇÃO
ISSO FAZ COM QUE A MATRIZ DINÂMICA
TENHA UMA DIMENSÃO N**

DMC com uso de θ

$$\left[\underline{y}^{C}\right]_k = \underline{\underline{S}}\,\Delta\underline{u} + \overline{\underline{y}_k} + \overline{\underline{P}}$$

$$\overline{\underline{P}}_i = \sum_{n=1}^{i} \phi_n$$

$$\phi_n = \sum_{j=n+1}^{N} \underline{\underline{H}}_j \Delta\underline{u}(k+n-j)$$

$$\left[\underline{y}^{C}\right]_k = \underline{\underline{S}}\,\Delta\underline{u} + \left[\underline{y}^{P}\right]_k$$

DIMENSÃO DEPENDE DO PARÂMETRO SINTONIZÁVEL NP

DMC com uso de θ

$$\left[\underline{y}^{C} \right]_k = \underline{\underline{\theta}} \left[\underline{\underline{S}} \Delta \underline{u} + \left[y^{P} \right]_k \right]$$

$$\underline{\underline{\theta}} = \left[\underline{\underline{I}}_{np.ny} \quad \underline{\underline{0}}_{np.ny \times (N-np).ny} \right], \quad \left[\underline{\underline{\theta}} \right] = np.ny \times N.ny$$

$$\left[\underline{\underline{S}} \right] = N.ny \times m.nu$$

$$\left[y^{P} \right]_k = N.ny \times 1$$

$$\left[\underline{y}^{C} \right]_k = np.ny \times 1$$

$$\left[\Delta \underline{u} \right] = m.nu \times 1$$

Função erro do DMC com uso de θ

$$\left[\underline{y}^{C}\right]_k = \underline{\underline{\theta}}\left[\underline{\underline{S}}\,\Delta\underline{u} + \left[\underline{y}^{P}\right]_k\right]$$

$$\underline{e} = \underline{y}^{SP} - \left[\underline{y}^{C}\right]_k$$

$$\underline{e} = \underline{y}^{SP} - \underline{\underline{\theta}}\left[\underline{y}^{P}\right]_k - \underline{\underline{\theta}}\,\underline{\underline{S}}\,\Delta\underline{u}$$

Função erro do DMC com uso de θ

$$\underline{e} = \underline{y}^{SP} - \underline{\underline{\theta}}\left[\underline{y}^{P}\right]_k - \underline{\underline{\theta}}\,\underline{\underline{S}}\,\Delta\underline{u}$$

$$\left[\underline{e}^{'}\right] = np.ny \times 1$$

$$\underline{e} = \underline{e}^{'} - \underline{\underline{\theta}}\,\underline{\underline{S}}\,\Delta\underline{u}$$

Função objetivo do DMC com uso de θ

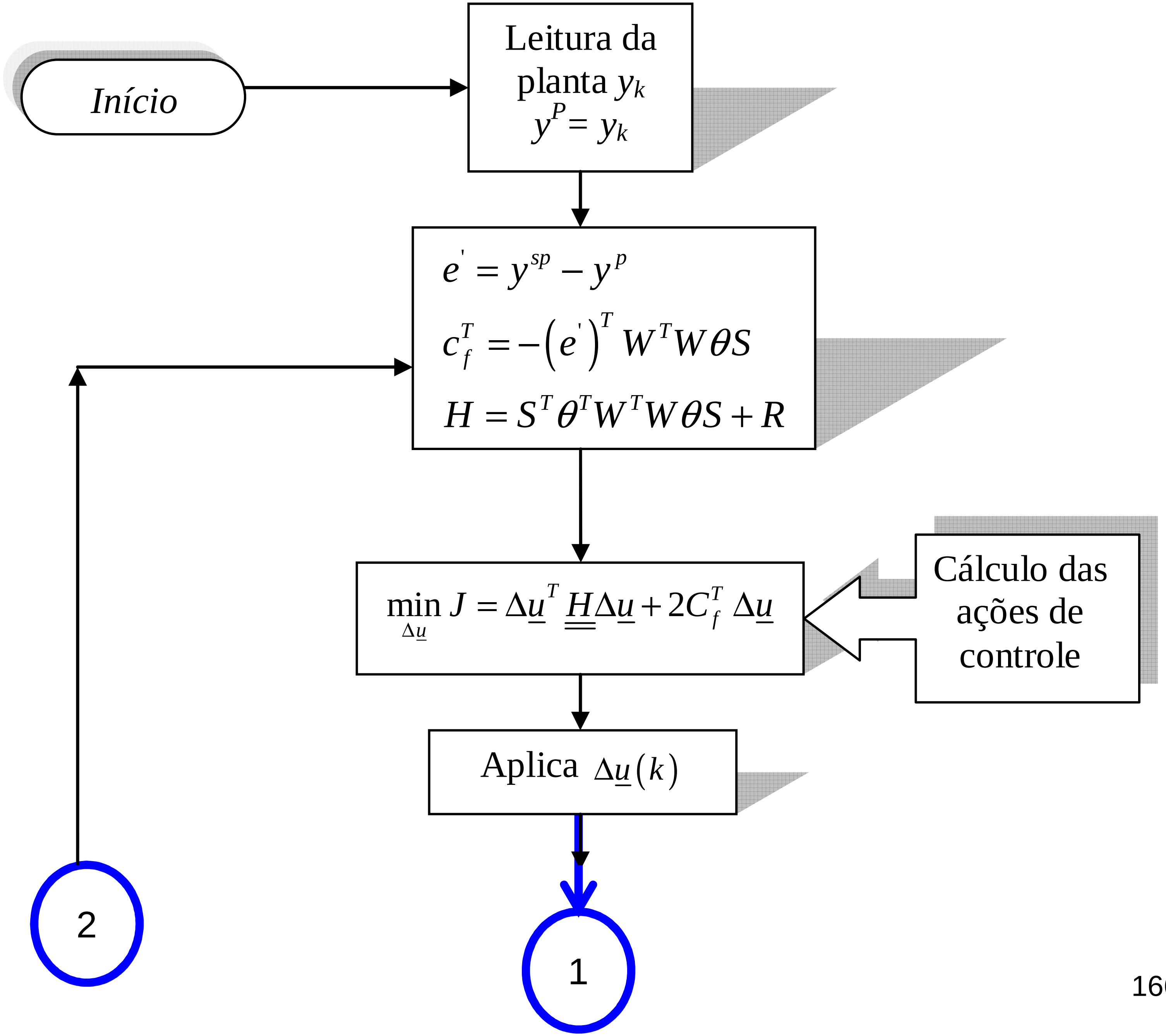
$$J = \underline{e}^T \underline{\underline{W}}^T \underline{\underline{W}}\, \underline{e} + \Delta \underline{u}^T \underline{\underline{R}}\, \Delta \underline{u}$$

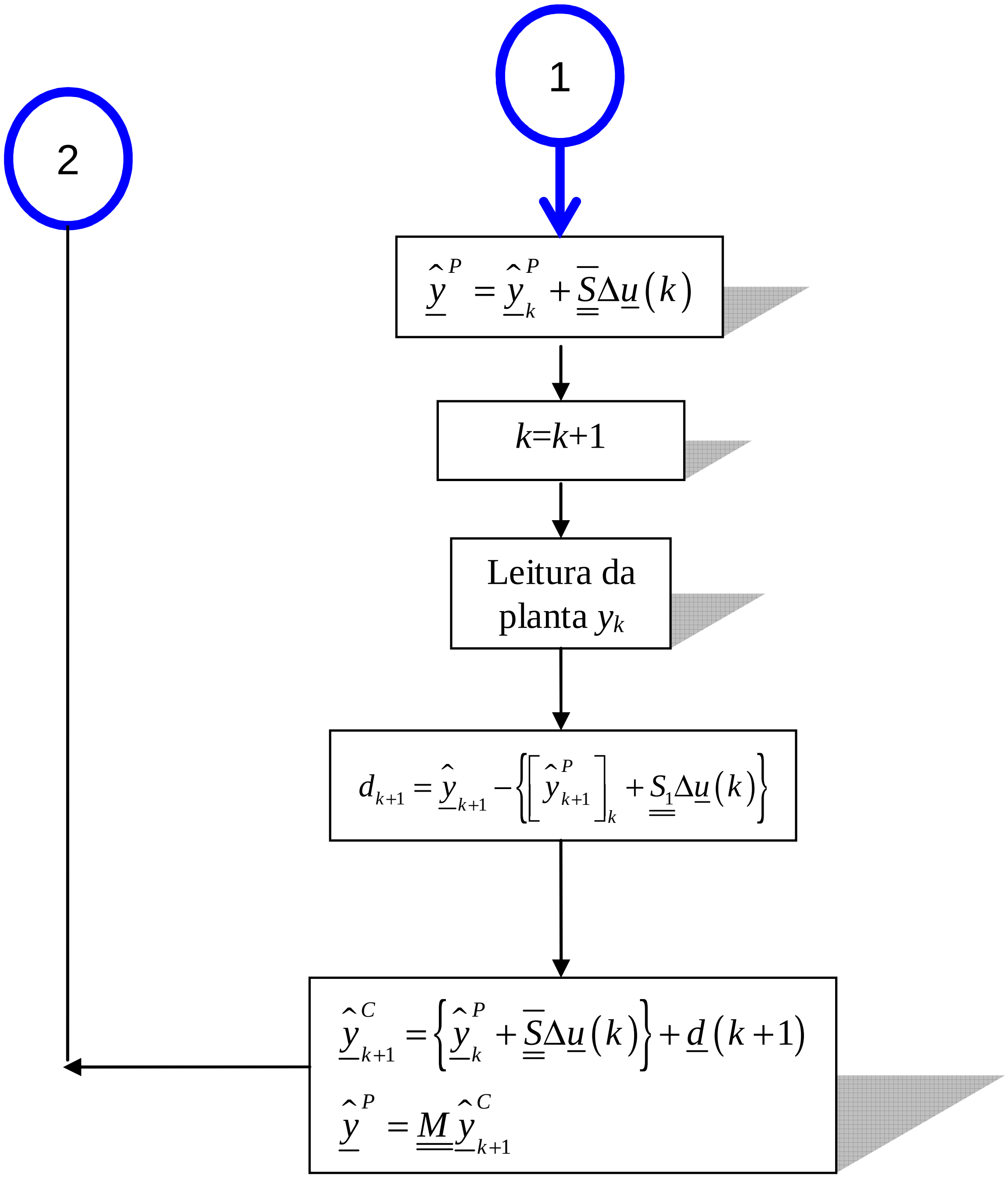
$$\underline{e} = \underline{e}' - \underline{\underline{\theta}}\,\underline{\underline{S}}\, \Delta \underline{u}$$

$$J = \Delta \underline{u}^T \left(\underline{\underline{S}}^T \underline{\underline{\theta}}^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{\underline{\theta}}\,\underline{\underline{S}} + \underline{\underline{R}} \right) \Delta \underline{u} - 2\underline{e}'^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{\underline{\theta}}\,\underline{\underline{S}}\, \Delta \underline{u} + \underline{e}'^T \underline{\underline{W}}^T \underline{\underline{W}}\, \underline{e}'$$

$$J = \Delta \underline{u}^T \underline{\underline{H}}\, \Delta \underline{u} + 2 C_f^T \Delta \underline{u}$$

CONSTANTE


Início
Leitura da planta y_k
$y^P = y_k$
$e' = y^{sp} - y^p$
$c_f^T = -(e')^T W^T W \theta S$
$H = S^T \theta^T W^T W \theta S + R$
$\min_{\Delta \underline{u}} J = \Delta \underline{u}^T \underline{\underline{H}} \Delta \underline{u} + 2 C_f^T \Delta \underline{u}$
Cálculo das ações de controle
Aplica $\Delta \underline{u}(k)$
2
1



1
2
$\underline{\hat{y}}^P = \underline{\hat{y}}^P_k + \underline{\underline{\bar{S}}}\Delta \underline{u}(k)$
$k=k+1$
Leitura da planta y_k
$d_{k+1} = \underline{\hat{y}}_{k+1} - \left\{\left[\underline{\hat{y}}^P_{k+1}\right]_k + \underline{\underline{S}}_1 \Delta \underline{u}(k)\right\}$
$\underline{\hat{y}}^C_{k+1} = \left\{\underline{\hat{y}}^P_k + \underline{\underline{\bar{S}}}\Delta \underline{u}(k)\right\} + \underline{d}(k+1)$
$\underline{\hat{y}}^P = \underline{\underline{M}}\,\underline{\hat{y}}^C_{k+1}$

LDMC

LDMC

- Função Objetivo de um QDMC

$$J = \Delta \underline{u}^T \left(\underline{\underline{S}}^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{\underline{S}} + \underline{\underline{R}} \right) \Delta \underline{u} - 2 \underline{\underline{S}}^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{e}' \Delta \underline{u} + \underline{e}'^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{e}'$$

$$\frac{\partial J}{\partial u} = 2 \left(\underline{\underline{S}}^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{\underline{S}} + \underline{\underline{R}} \right) \Delta \underline{u} - 2 \underline{\underline{S}}^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{e}'$$

PONTO ÓTIMO

$$\frac{\partial J}{\partial u} = 2 \left(\underline{\underline{S}}^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{\underline{S}} + \underline{\underline{R}} \right) \Delta \underline{u} - 2 \underline{\underline{S}}^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{e}' = 0$$

LDMC

- Se houver restrições, esse ponto ótimo não é alcançado.

SOLUÇÃO SUB-ÓTIMA

$$\frac{\partial J}{\partial u} = 2\left(\underline{\underline{S}}^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{\underline{S}} + \underline{\underline{R}}\right)\Delta \underline{u} - 2\underline{\underline{S}}^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{e}' \neq 0$$

$$\underline{\rho} = \left[\rho_1 \quad \rho_1 \quad \cdots \quad \rho_{nu.m}\right]^T$$

$$\left(\underline{\underline{S}}^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{\underline{S}} + \underline{\underline{R}}\right)\Delta \underline{u} - \underline{\underline{S}}^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{e}' = \underline{\rho}$$

LDMC

- Função Objetivo do LDMC

$$\min_{\Delta \underline{u}} \phi = \sum_{i=1}^{m.nu} \nu_i \left| \rho_i \right|$$

Não linear

PESOS ASSOCIADOS AS FOLGAS NORMALMENTE ASSUME-SE O VALOR DE 1 PARA TODAS FOLGAS

LDMC

- Para linearizar a Função Objetivo do LDMC

$$\min_{\Delta \underline{u}} \phi = \sum_{i=1}^{m.nu} |\rho_i| \qquad \rho_i = x_i - z_i, \quad x_i, z_i \geq 0$$

$$\rho_i \leq 0 \Rightarrow x_i = 0 \Rightarrow z_i = -\rho_i$$

$$\rho_i > 0 \Rightarrow z_i = 0 \Rightarrow x_i = \rho_i$$

Com essa definição $|\rho_i| = x_i + z_i$

LDMC

- Problema LDMC

$$\min_{\Delta \underline{u}} \phi = \sum_{i=1}^{m.nu} v_i \left(x_i + z_i \right)$$

sujeito a

RESTRIÇÃO DA DERIVADA

$$\underline{\rho} = x_i - z_i = \left(\underline{\underline{S}}^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{\underline{S}} + \underline{\underline{R}} \right) \Delta \underline{u} - \underline{\underline{S}}^T \underline{\underline{W}}^T \underline{\underline{W}}\, \underline{e}'$$

$$-\Delta \underline{u}_{\max} \le \Delta \underline{u} \le \Delta \underline{u}_{\max} \qquad \underline{u}_{\min} \le \underline{u} \le \underline{u}_{\max}$$

$$x_i \ge 0; z_i \ge 0;$$

RESTRIÇÕES E FUNÇÃO OBJETIVO LINEARES

LDMC

- Se não tivermos restrições ativas, os resultados obtidos no problema DMC (sem restrições), QDMC e LDMC são iguais.

$$\underline{\rho} = x_i - z_i = \left(\underline{\underline{S}}^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{\underline{S}} + \underline{\underline{R}}\right)\Delta\underline{u} - \underline{\underline{S}}^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{e}' = 0$$

$$\left(\underline{\underline{S}}^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{\underline{S}} + \underline{\underline{R}}\right)\Delta\underline{u} = \underline{\underline{S}}^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{e}'$$

$$\Delta\underline{u} = \left(\underline{\underline{S}}^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{\underline{S}} + \underline{\underline{R}}\right)^{-1} \underline{\underline{S}}^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{e}'$$

$$\Delta\underline{u} = \underline{\underline{K}}_{DMC}\,\underline{e}'$$

$$\underline{e}' = \underline{e}_k - \overline{\underline{P}}$$

$$\overline{\underline{P}}_i = \sum_{n=1}^{i} \phi_n$$

$$\phi_n = \sum_{j=n+1}^{N} \underline{\underline{H}}_j \Delta\underline{u}(k+n-j)$$

LDMC

$$\left[\underline{e}'\right] = N.ny \times 1$$

$$\Delta \underline{u} = \underline{\underline{K}}_{DMC}\,\underline{e}'$$

Erro atual e resposta do processo conforme modelo e ações aplicados no passado

$$\begin{bmatrix} \Delta \underline{u}(k) \\ \Delta \underline{u}(k+1) \\ \vdots \\ \Delta \underline{u}(k+m-1) \end{bmatrix} = \begin{bmatrix} \underline{\underline{K}}_{11} & \underline{\underline{K}}_{12} & \cdots & \underline{\underline{K}}_{1N} \\ \underline{\underline{K}}_{21} & \underline{\underline{K}}_{22} & \cdots & \underline{\underline{K}}_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \underline{\underline{K}}_{m1} & \underline{\underline{K}}_{m2} & \cdots & \underline{\underline{K}}_{mN} \end{bmatrix} \begin{bmatrix} \underline{e}'(k+1) \\ \underline{e}'(k+2) \\ \vdots \\ \underline{e}'(k+N) \end{bmatrix}$$

$$\Delta \underline{u}(k) = \left[\underline{\underline{K}}_{11} \quad \underline{\underline{K}}_{12} \quad \cdots \quad \underline{\underline{K}}_{1N} \right] \underline{e}'(k+1)$$

$$\left[\underline{\underline{K}}_{DMC}\right] = m.nu \times N.ny$$

Primeira ação aplicada

LDMC

$$\Delta \underline{u} = \underline{\underline{K_{DMC}}}\, \underline{e}'$$

Se o erro persistir, a ação do controlador funciona como uma ação integradora, removendo offset

MPC POR MATRIZ DE REALINHAMENTO ANÁLISE DA ESTABILIDADE EM MALHA FECHADA

MPC em Malha Fechada

- Equação de Predição do DMC com matriz de realinhamento

$$\underline{\hat{y}}^{C}_{k+1} = \underline{\underline{M}}\,\underline{\hat{y}}^{C}_{k} + \underline{\underline{M}}\,\underline{\underline{\bar{S}}}\Delta \underline{u}(k) + \underline{\bar{d}}(k+1)$$

$$\begin{cases} \underline{x}(k+1) = \underline{\underline{A}}\,\underline{x}(k) + \underline{\underline{B}}\,\underline{u}(k) \\ \underline{y}(k) = \underline{\underline{C}}\,\underline{x}(k) \end{cases}$$

CONSIDERANDO QUE O ESTADO POSSA SER A PREDIÇÃO DAS VARIÁVEIS

$$C = \begin{bmatrix} I_{ny} & 0_{ny} & \dots & 0_{ny} \end{bmatrix}$$

Recordando que...

$$\underline{\hat{y}}^{C}_{k+1} = \underline{\underline{M}}\,\underline{\hat{y}}^{C}_{k} + \underline{\underline{M}}\,\underline{\underline{\bar{S}}}\Delta\underline{u}(k) + \underline{\bar{d}}(k+1)$$

$$\underline{\underline{M}} = \begin{bmatrix} 0_{ny\times ny} & I_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\ 0_{ny\times ny} & 0_{ny\times ny} & I_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\ 0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \\ 0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdot & & \end{bmatrix}$$

$$\underline{\hat{y}}^{C}_{k} = \begin{bmatrix} \underline{\hat{y}}^{C}_{k+1} \\ \underline{\hat{y}}^{C}_{k+2} \\ \underline{\hat{y}}^{C}_{k+3} \\ \vdots \\ \underline{\hat{y}}^{C}_{k+np} \end{bmatrix} \qquad \underline{\underline{\bar{S}}} = \begin{bmatrix} \underline{\underline{S}}_{1} \\ \underline{\underline{S}}_{2} \\ \underline{\underline{S}}_{3} \\ \vdots \\ \underline{\underline{S}}_{np} \end{bmatrix} \qquad \underline{\bar{d}}(k+1) = \begin{bmatrix} \underline{d}_{k+1} \\ \underline{d}_{k+1} \\ \underline{d}_{k+1} \\ \vdots \\ \underline{d}_{k+1} \end{bmatrix}$$

$$d_{k+1} = \underline{\hat{y}}_{k+1} - \left\{ \left[\underline{\hat{y}}^{P}_{k+1} \right]_{k} + \underline{\underline{S}}_{1}\Delta\underline{u}(k) \right\}$$

MPC em Malha Fechada

Valor lido da planta com as ações de controle

MODELO REAL

$$\overline{\underline{y}}_{k+1} = \underline{\underline{C}}\left(\overline{\underline{y}}_k + \overline{\underline{\underline{S}}}_P \Delta \underline{u}(k)\right)$$

Valor predito da planta com as ações de controle pelo controlador com o modelo configurado no mesmo

$$\hat{\underline{y}}^{P}_{k+1} = \underline{\underline{C}}\left(\hat{\underline{y}}^{P}_k + \overline{\underline{\underline{S}}} \Delta \underline{u}(k)\right)$$

MODELO ESPERADO

MPC em Malha Fechada

$$\overline{\underline{y}}_{k+1} = \underline{\underline{C}}\left(\overline{\underline{y}}_k + \overline{\underline{\underline{S}}}_P \Delta \underline{u}(k)\right)$$

$$\overline{\underline{y}}_k = \begin{bmatrix} \underline{y}_{k+1} & \underline{y}_{k+2} & \cdots & \underline{y}_{k+N} \end{bmatrix}_k^T$$

$$\overline{\underline{\underline{S}}}_P = \begin{bmatrix} \underline{\underline{S}}_{P1} & \underline{\underline{S}}_{P2} & \cdots & \underline{\underline{S}}_{PN} \end{bmatrix}^T$$

MPC em Malha Fechada

$$\underline{d}(k+1) = \underline{y}_{k+1} - \hat{\underline{y}}^{C}_{k+1} = \underline{\underline{C}}\left(\bar{\underline{y}}_{k} + \bar{\underline{\underline{S}}}_{P}\Delta\underline{u}(k)\right) - \underline{\underline{C}}\left(\hat{\underline{y}}^{C}_{k} + \bar{\underline{\underline{S}}}\Delta\underline{u}(k)\right)$$

VALOR REAL DA PLANTA

VALOR PREDITO PARA O INSTANTE SEGUINTE PELO MODELO REAL

VALOR PREDITO PELO CONTROLADOR COM O MODELO AJUSTADO

MPC em Malha Fechada

$$\underline{\hat{y}}^{C}_{k+1} = \underline{\underline{M}}\,\underline{\hat{y}}^{C}_{k} + \underline{\underline{M}}\,\underline{\underline{\bar{S}}}\Delta\underline{u}(k) + \underline{d}(k+1)$$

$$\underline{d}(k+1) = \underline{\underline{C}}\left(\underline{\bar{y}}_{k} + \underline{\underline{\bar{S}}}_{P}\Delta\underline{u}(k)\right) - \underline{\underline{C}}\left(\underline{\hat{y}}^{C}_{k} + \underline{\underline{\bar{S}}}\Delta\underline{u}(k)\right)$$

$$\underline{\hat{y}}^{C}_{k+1} = \underline{\underline{M}}\,\underline{\hat{y}}^{C}_{k} + \underline{\underline{M}}\,\underline{\underline{\bar{S}}}\Delta\underline{u}(k) + K_{F}\left\{\underline{\underline{C}}\left(\underline{\bar{y}}_{k} + \underline{\underline{\bar{S}}}_{P}\Delta\underline{u}(k)\right) - \underline{\underline{C}}\left(\underline{\hat{y}}^{C}_{k} + \underline{\underline{\bar{S}}}\Delta\underline{u}(k)\right)\right\}$$

Observador de estados

MPC em Malha Fechada

PARA O ESTADO EM QUESTÃO QUE SÃO AS PRÓPRIAS SAÍDAS PREDITAS

$$K_F = \begin{bmatrix} \underline{\underline{I}}_{ny} & \underline{\underline{I}}_{ny} & \cdots & \underline{\underline{I}}_{ny} \end{bmatrix}^T$$

$$\underline{\hat{y}}_k^C = \begin{bmatrix} \underline{\hat{y}}_{k+1}^C & \underline{\hat{y}}_{k+2}^C & \cdots & \underline{\hat{y}}_{k+N}^C \end{bmatrix}_k^T$$

MPC em Malha Fechada

$$\hat{\underline{y}}^{C}_{k+1} = \underline{\underline{M}}\,\hat{\underline{y}}^{C}_{k} + \underline{\underline{M}}\,\underline{\underline{\bar{S}}}\,\Delta\underline{u}(k) + K_F\left\{\underline{\underline{C}}\left(\underline{\bar{y}}_{k} + \underline{\underline{\bar{S}}}_{P}\,\Delta\underline{u}(k)\right) - \underline{\underline{C}}\left(\hat{\underline{y}}^{C}_{k} + \underline{\underline{\bar{S}}}\,\Delta\underline{u}(k)\right)\right\}$$

$$\hat{\underline{y}}^{C}_{k+1} = \underline{\underline{M}}\,\hat{\underline{y}}^{C}_{k} + \underline{\underline{M}}\,\underline{\underline{\bar{S}}}\,\Delta\underline{u}(k) + K_F\,\underline{\underline{C}}\,\underline{\bar{y}}_{k} + K_F\,\underline{\underline{C}}\,\underline{\underline{\bar{S}}}_{P}\,\Delta\underline{u}(k) - K_F\,\underline{\underline{C}}\,\hat{\underline{y}}^{C}_{k} - K_F\,\underline{\underline{C}}\,\underline{\underline{\bar{S}}}\,\Delta\underline{u}(k)$$

$$\hat{\underline{y}}^{C}_{k+1} = \left(\underline{\underline{M}} - K_F\,\underline{\underline{C}}\right)\hat{\underline{y}}^{C}_{k} + \left(\underline{\underline{M}}\,\underline{\underline{\bar{S}}} + K_F\,\underline{\underline{C}}\,\underline{\underline{\bar{S}}}_{P} - K_F\,\underline{\underline{C}}\,\underline{\underline{\bar{S}}}\right)\Delta\underline{u}(k) + K_F\,\underline{\underline{C}}\,\underline{\bar{y}}_{k}$$

$$\hat{\underline{y}}^{C}_{k+1} = \left(\underline{\underline{M}} - K_F\,\underline{\underline{C}}\right)\hat{\underline{y}}^{C}_{k} + \left(\underline{\underline{M}}\,\underline{\underline{\bar{S}}} - K_F\,\underline{\underline{C}}\left[\underline{\underline{\bar{S}}} - \underline{\underline{\bar{S}}}_{P}\right]\right)\Delta\underline{u}(k) + K_F\,\underline{\underline{C}}\,\underline{\bar{y}}_{k}$$

MPC em Malha Fechada

- Predição para a planta

$$\overline{\underline{y}}_{k+1} = \underline{\underline{M}}\,\overline{\underline{y}}_k + \underline{\underline{M}}\,\overline{\underline{\underline{S}}}_P \Delta \underline{u}(k) + \underline{d}(k+1)$$

zero

$$\overline{\underline{y}}_{k+1} = \underline{\underline{M}}\,\overline{\underline{y}}_k + \underline{\underline{M}}\,\overline{\underline{\underline{S}}}_P \Delta \underline{u}(k)$$

MPC em Malha Fechada

CONSIDERANDO UM ESTADO COMPOSTO DE DUAS COMPONENTES A PREDIÇÃO E A PLANTA

$$\hat{\underline{y}}_{k+1}^{C} = \left(\underline{\underline{M}} - K_F\,\underline{\underline{C}}\right)\hat{\underline{y}}_{k}^{C} + \left(\underline{\underline{M}}\,\overline{\underline{\underline{S}}} - K_F\,\underline{\underline{C}}\left[\overline{\underline{\underline{S}}} - \overline{\underline{\underline{S}}}_P\right]\right)\Delta\underline{u}(k) + K_F\,\underline{\underline{C}}\,\overline{\underline{y}}_k$$

$$\overline{\underline{y}}_{k+1} = \underline{\underline{M}}\,\overline{\underline{y}}_k + \underline{\underline{M}}\,\overline{\underline{\underline{S}}}_P\,\Delta\underline{u}(k)$$

MPC em Malha Fechada

- Sistema Controlador e Processo

$$\begin{bmatrix} \hat{\underline{y}}^C \\ \\ \underline{\bar{y}} \end{bmatrix}_{k+1} = \begin{bmatrix} \left(\underline{\underline{M}} - K_F \underline{\underline{C}}\right) & K_F \underline{\underline{C}} \\ 0 & \underline{\underline{M}} \end{bmatrix} \begin{bmatrix} \hat{\underline{y}}^C \\ \\ \underline{\bar{y}} \end{bmatrix}_k + \begin{bmatrix} \underline{\underline{M}}\,\underline{\underline{\bar{S}}} - K_F \underline{\underline{C}}\left[\underline{\underline{\bar{S}}} - \underline{\underline{\bar{S}}}_P\right] \\ \underline{\underline{M}}\,\underline{\underline{\bar{S}}}_P \end{bmatrix} \Delta \underline{u}(k)$$

⇕

$$\begin{cases} \underline{x}(k+1) = \underline{\underline{A}}\,\underline{x}(k) + \underline{\underline{B}}\Delta\underline{u}(k) \\ \underline{y}(k) = \underline{\underline{C}}\,\underline{x}(k) \end{cases}$$

MPC em Malha Fechada

- Subtraindo do set point: $\begin{bmatrix} \underline{y}^{SP} & \underline{y}^{SP} \end{bmatrix}^T$

$$\begin{bmatrix} \underline{y}^{SP} \\ \underline{y}^{SP} \end{bmatrix} - \begin{bmatrix} \hat{\underline{y}}^{C} \\ \bar{\underline{y}} \end{bmatrix}_{k+1} = \begin{bmatrix} \underline{y}^{SP} \\ \underline{y}^{SP} \end{bmatrix} - \begin{bmatrix} \left(\underline{\underline{M}} - K_F \underline{\underline{C}}\right) & K_F \underline{\underline{C}} \\ 0 & \underline{\underline{M}} \end{bmatrix} \begin{bmatrix} \hat{\underline{y}}^{C} \\ \bar{\underline{y}} \end{bmatrix}_{k} - \begin{bmatrix} \underline{\underline{M}}\,\bar{\underline{\underline{S}}} - K_F \underline{\underline{C}}\left[\bar{\underline{\underline{S}}} - \bar{\underline{\underline{S}}}_P\right] \\ \underline{\underline{M}}\,\bar{\underline{\underline{S}}}_P \end{bmatrix} \Delta \underline{u}(k)$$

MAS

$$\begin{bmatrix} \underline{y}^{SP} \\ \underline{y}^{SP} \end{bmatrix} = \begin{bmatrix} \left(\underline{\underline{M}} - K_F \underline{\underline{C}}\right) & K_F \underline{\underline{C}} \\ 0 & \underline{\underline{M}} \end{bmatrix} \begin{bmatrix} \underline{y}^{SP} \\ \underline{y}^{SP} \end{bmatrix} = \begin{bmatrix} \left(\underline{\underline{M}} - K_F \underline{\underline{C}}\right) \underline{y}^{SP} + K_F \underline{\underline{C}}\,\underline{y}^{SP} \\ \underline{\underline{M}}\,\underline{y}^{SP} \end{bmatrix} = \begin{bmatrix} \underline{\underline{M}}\,\underline{y}^{SP} \\ \underline{\underline{M}}\,\underline{y}^{SP} \end{bmatrix} = \begin{bmatrix} \underline{y}^{SP} \\ \underline{y}^{SP} \end{bmatrix}$$

A matriz M é uma matriz identidade de deslocamento

MPC em Malha Fechada

$$\begin{bmatrix} \underline{y}^{SP} \\ \underline{y}^{SP} \end{bmatrix} - \begin{bmatrix} \hat{\underline{y}}^{C} \\ \overline{\underline{y}} \end{bmatrix}_{k+1} = \begin{bmatrix} \left(\underline{\underline{M}} - K_F \underline{\underline{C}}\right) & K_F \underline{\underline{C}} \\ 0 & \underline{\underline{M}} \end{bmatrix} \begin{bmatrix} \underline{y}^{SP} \\ \underline{y}^{SP} \end{bmatrix} - \begin{bmatrix} \left(\underline{\underline{M}} - K_F \underline{\underline{C}}\right) & K_F \underline{\underline{C}} \\ 0 & \underline{\underline{M}} \end{bmatrix} \begin{bmatrix} \hat{\underline{y}}^{C} \\ \overline{\underline{y}} \end{bmatrix}_{k} - \begin{bmatrix} \underline{\underline{M}}\,\overline{\underline{\underline{S}}} - K_F \underline{\underline{C}} \left[\overline{\underline{\underline{S}}} - \overline{\underline{\underline{S}}}_P \right] \\ \underline{\underline{M}}\,\overline{\underline{\underline{S}}}_P \end{bmatrix} \Delta \underline{u}(k)$$

$$\begin{bmatrix} \underline{y}^{SP} - \hat{\underline{y}}^{C} \\ \underline{y}^{SP} - \overline{\underline{y}} \end{bmatrix} = \begin{bmatrix} \left(\underline{\underline{M}} - K_F \underline{\underline{C}}\right) & K_F \underline{\underline{C}} \\ 0 & \underline{\underline{M}} \end{bmatrix} \begin{bmatrix} \underline{y}^{SP} - \hat{\underline{y}}^{C} \\ \underline{y}^{SP} - \overline{\underline{y}} \end{bmatrix} - \begin{bmatrix} \underline{\underline{M}}\,\overline{\underline{\underline{S}}} - K_F \underline{\underline{C}} \left[\overline{\underline{\underline{S}}} - \overline{\underline{\underline{S}}}_P \right] \\ \underline{\underline{M}}\,\overline{\underline{\underline{S}}}_P \end{bmatrix} \Delta \underline{u}(k)$$

$$\begin{bmatrix} \underline{e}' \\ \overline{\underline{e}}' \end{bmatrix}_{k+1} = \begin{bmatrix} \left(\underline{\underline{M}} - K_F \underline{\underline{C}}\right) & K_F \underline{\underline{C}} \\ 0 & \underline{\underline{M}} \end{bmatrix} \begin{bmatrix} \underline{e}' \\ \overline{\underline{e}}' \end{bmatrix}_{k} - \begin{bmatrix} \underline{\underline{M}}\,\overline{\underline{\underline{S}}} - K_F \underline{\underline{C}} \left[\overline{\underline{\underline{S}}} - \overline{\underline{\underline{S}}}_P \right] \\ \underline{\underline{M}}\,\overline{\underline{\underline{S}}}_P \end{bmatrix} \Delta \underline{u}(k)$$

MPC em Malha Fechada

$$\begin{bmatrix} \underline{e}' \\ \underline{e}' \end{bmatrix}_{k+1} = \begin{bmatrix} \left(\underline{\underline{M}} - K_F \underline{\underline{C}}\right) & K_F \underline{\underline{C}} \\ 0 & \underline{\underline{M}} \end{bmatrix} \begin{bmatrix} \underline{e}' \\ \underline{e}' \end{bmatrix}_k - \begin{bmatrix} \underline{\underline{M}}\,\underline{\underline{\bar{S}}} - K_F \underline{\underline{C}}\left[\underline{\underline{\bar{S}}} - \underline{\underline{\bar{S}}}_P\right] \\ \underline{\underline{M}}\,\underline{\underline{\bar{S}}}_P \end{bmatrix} \Delta \underline{u}(k)$$

SUPONDO A LEI DE CONTROLE

$$\Delta \underline{u} = \underline{\underline{K}}_{DMC}\, \underline{e}'$$

$$\begin{bmatrix} \underline{e}' \\ \underline{e}' \end{bmatrix}_{k+1} = \begin{bmatrix} \left(\underline{\underline{M}} - K_F \underline{\underline{C}}\right) & K_F \underline{\underline{C}} \\ 0 & \underline{\underline{M}} \end{bmatrix} \begin{bmatrix} \underline{e}' \\ \underline{e}' \end{bmatrix}_k - \begin{bmatrix} \underline{\underline{M}}\,\underline{\underline{\bar{S}}} - K_F \underline{\underline{C}}\left[\underline{\underline{\bar{S}}} - \underline{\underline{\bar{S}}}_P\right] \\ \underline{\underline{M}}\,\underline{\underline{\bar{S}}}_P \end{bmatrix} \begin{bmatrix} \underline{\underline{K}}_{DMC}\, \underline{e}' \\ \underline{\underline{K}}_{DMC}\, \underline{e}' \end{bmatrix}_k$$

MPC em Malha Fechada

$$\begin{bmatrix} \underline{e}' \\ \underline{\bar{e}}' \end{bmatrix}_{k+1} = \begin{bmatrix} \left(\underline{\underline{M}} - K_F \underline{\underline{C}}\right) & K_F \underline{\underline{C}} \\ 0 & \underline{\underline{M}} \end{bmatrix} \begin{bmatrix} \underline{e}' \\ \underline{\bar{e}}' \end{bmatrix}_k - \begin{bmatrix} \underline{\underline{M}}\,\underline{\underline{\bar{S}}} - K_F \underline{\underline{C}}\left[\underline{\underline{\bar{S}}} - \underline{\underline{\bar{S}}}_P\right] \\ \underline{\underline{M}}\,\underline{\underline{\bar{S}}}_P \end{bmatrix} \begin{bmatrix} \underline{\underline{K}}_{DMC}\, \underline{e}' \\ \underline{\underline{K}}_{DMC}\, \underline{\bar{e}}' \end{bmatrix}_k$$

$$\begin{bmatrix} \underline{e}' \\ \underline{\bar{e}}' \end{bmatrix}_{k+1} = \begin{bmatrix} \left(\underline{\underline{M}} - K_F \underline{\underline{C}}\right) & K_F \underline{\underline{C}} \\ 0 & \underline{\underline{M}} \end{bmatrix} \begin{bmatrix} \underline{e}' \\ \underline{\bar{e}}' \end{bmatrix}_k - \begin{bmatrix} \left\{\underline{\underline{M}}\,\underline{\underline{\bar{S}}} - K_F \underline{\underline{C}}\left[\underline{\underline{\bar{S}}} - \underline{\underline{\bar{S}}}_P\right]\right\} \underline{\underline{K}}_{DMC} & \left\{\underline{\underline{M}}\,\underline{\underline{\bar{S}}} - K_F \underline{\underline{C}}\left[\underline{\underline{\bar{S}}} - \underline{\underline{\bar{S}}}_P\right]\right\} \underline{\underline{K}}_{DMC} \\ \underline{\underline{M}}\,\underline{\underline{\bar{S}}}_P \underline{\underline{K}}_{DMC} & \underline{\underline{M}}\,\underline{\underline{\bar{S}}}_P \underline{\underline{K}}_{DMC} \end{bmatrix} \begin{bmatrix} \underline{e}' \\ \underline{\bar{e}}' \end{bmatrix}_k$$

$$\begin{bmatrix} \underline{e}' \\ \underline{\bar{e}}' \end{bmatrix}_{k+1} = \begin{bmatrix} \left(\underline{\underline{M}} - K_F \underline{\underline{C}}\right) - \left\{\underline{\underline{M}}\,\underline{\underline{\bar{S}}} - K_F \underline{\underline{C}}\left[\underline{\underline{\bar{S}}} - \underline{\underline{\bar{S}}}_P\right]\right\} \underline{\underline{K}}_{DMC} & K_F \underline{\underline{C}} - \left\{\underline{\underline{M}}\,\underline{\underline{\bar{S}}} - K_F \underline{\underline{C}}\left[\underline{\underline{\bar{S}}} - \underline{\underline{\bar{S}}}_P\right]\right\} \underline{\underline{K}}_{DMC} \\ -\underline{\underline{M}}\,\underline{\underline{\bar{S}}}_P \underline{\underline{K}}_{DMC} & \underline{\underline{M}} - \underline{\underline{M}}\,\underline{\underline{\bar{S}}}_P \underline{\underline{K}}_{DMC} \end{bmatrix} \begin{bmatrix} \underline{e}' \\ \underline{\bar{e}}' \end{bmatrix}_k$$

$x(k+1)$ $\qquad$ A $\qquad$ $x(k)$

MPC em Malha Fechada

- Autovalores de A definem a estabilidade da malha fechada

$$\begin{bmatrix} e' \\ \bar{e}' \end{bmatrix}_{k+1} = \begin{bmatrix} \left(\underline{\underline{M}} - K_F \underline{\underline{C}}\right) - \left\{\underline{\underline{M}}\,\underline{\underline{\bar{S}}} - K_F \underline{\underline{C}}\left[\underline{\underline{\bar{S}}} - \underline{\underline{\bar{S}}}_P\right]\right\} \underline{\underline{K}}_{DMC} & K_F \underline{\underline{C}} - \left\{\underline{\underline{M}}\,\underline{\underline{\bar{S}}} - K_F \underline{\underline{C}}\left[\underline{\underline{\bar{S}}} - \underline{\underline{\bar{S}}}_P\right]\right\} \underline{\underline{K}}_{DMC} \\ -\underline{\underline{M}}\,\underline{\underline{\bar{S}}}_P \underline{\underline{K}}_{DMC} & \underline{\underline{M}} - \underline{\underline{M}}\,\underline{\underline{\bar{S}}}_P \underline{\underline{K}}_{DMC} \end{bmatrix} \begin{bmatrix} e' \\ \bar{e}' \end{bmatrix}_{k}$$

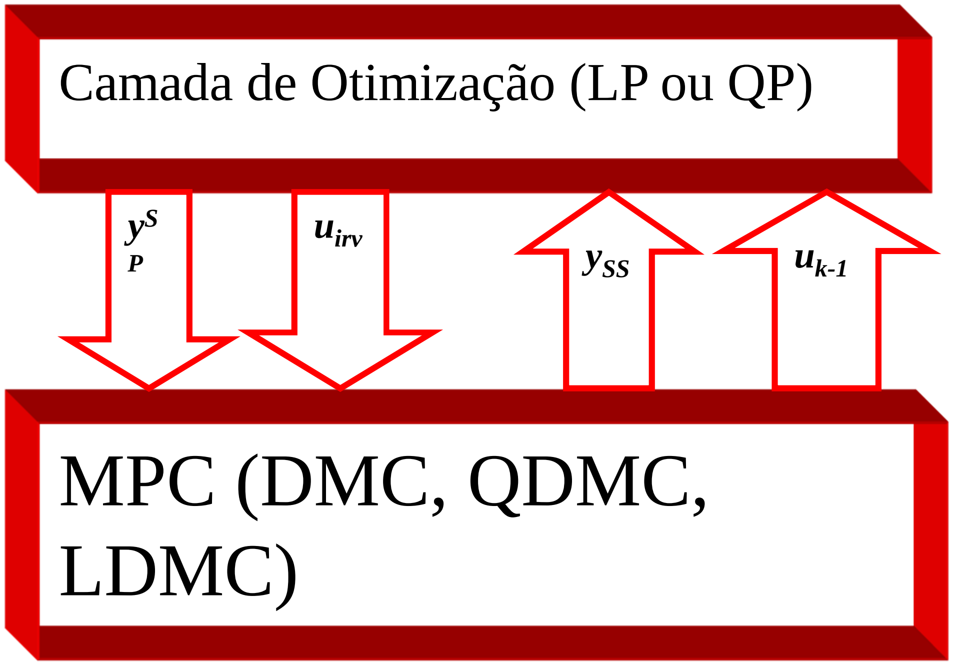
ESTABILIDADE DO MPC EM MALHA FECHADA

MPC EM DUAS CAMADAS

MPC em duas camadas

- Camada estática e camada dinâmica

Uirv = ideal rest value para as manipuladas



MPC em duas camadas

- Predição do MPC

$$\begin{bmatrix} \underline{\hat{y}}^{C}_{k+1} \\ \underline{\hat{y}}^{C}_{k+2} \\ \underline{\hat{y}}^{C}_{k+3} \\ \vdots \\ \underline{\hat{y}}^{C}_{k+np} \end{bmatrix} = \begin{bmatrix} \underline{\underline{S}}_1 & 0_{ny\times nu} & 0_{ny\times nu} & \cdots & 0_{ny\times nu} \\ \underline{\underline{S}}_2 & \underline{\underline{S}}_1 & 0_{ny\times nu} & \cdots & 0_{ny\times nu} \\ \underline{\underline{S}}_3 & \underline{\underline{S}}_2 & \underline{\underline{S}}_1 & \cdots & 0_{ny\times nu} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \underline{\underline{S}}_{np} & \underline{\underline{S}}_{np-1} & \underline{\underline{S}}_{np-2} & \cdots & \underline{\underline{S}}_{np-m+1} \end{bmatrix} \begin{bmatrix} \Delta\underline{u}(k) \\ \Delta\underline{u}(k+1) \\ \Delta\underline{u}(k+2) \\ \vdots \\ \Delta\underline{u}(k+m-1) \end{bmatrix} + \begin{bmatrix} \underline{\hat{y}}_k + \underline{P}_1 \\ \underline{\hat{y}}_k + \underline{P}_2 \\ \underline{\hat{y}}_k + \underline{P}_3 \\ \vdots \\ \underline{\hat{y}}_k + \underline{P}_{np} \end{bmatrix}$$

$$\underline{\hat{y}}^{C}_{k+np} = \begin{bmatrix} \underline{\underline{S}}_{np} & \underline{\underline{S}}_{np-1} & \underline{\underline{S}}_{np-2} & \cdots & \underline{\underline{S}}_{np-m+1} \end{bmatrix} \begin{bmatrix} \Delta\underline{u}(k) \\ \Delta\underline{u}(k+1) \\ \Delta\underline{u}(k+2) \\ \vdots \\ \Delta\underline{u}(k+m-1) \end{bmatrix} + \underline{\hat{y}}_k + \underline{P}_{np}$$

PREDIÇÃO COM O VALOR ATUAL MAIS AÇÕES DO PASSADO

MPC em duas camadas

VALOR DAS VARIÁVEIS NO TEMPO DE ESTABILIZAÇÃO CONSIDERANDO APENAS AS AÇÕES PASSADAS

$\underline{y}^{P}_{k+N}$

np→∞

$$\underline{\hat{y}}^{C}_{k+\infty} = \begin{bmatrix} \underline{\underline{S}}_{\infty} & \underline{\underline{S}}_{\infty-1} & \underline{\underline{S}}_{\infty-2} & \cdots & \underline{\underline{S}}_{\infty-m+1} \end{bmatrix} \begin{bmatrix} \Delta\underline{u}(k) \\ \Delta\underline{u}(k+1) \\ \Delta\underline{u}(k+2) \\ \vdots \\ \Delta\underline{u}(k+m-1) \end{bmatrix} + \underline{\hat{y}}_{k} + \underline{P}_{\infty}$$

$\underline{\underline{S}}_{N}$

AÇÕES PASSADAS ANTES DAS M FUTURAS AÇÕES E QUE FAZEM COM QUE AS VARIÁVEIS TENDEM PARA

$\underline{\underline{S}}_{N}\Delta u$

MPC em duas camadas

$$\underline{\hat{y}}^{C}_{k+\infty} = \begin{bmatrix} \underline{\underline{S}}_N & \underline{\underline{S}}_N & \underline{\underline{S}}_N & \cdots & \underline{\underline{S}}_N \end{bmatrix} \begin{bmatrix} \Delta \underline{u}(k) \\ \Delta \underline{u}(k+1) \\ \Delta \underline{u}(k+2) \\ \vdots \\ \Delta \underline{u}(k+m-1) \end{bmatrix} + \underline{y}^{P}_{k+N}$$

$$\underline{\hat{y}}^{C}_{k+\infty} = \underline{\underline{S}}_N \sum_{i=0}^{m-1} \Delta \underline{u}(k+i) + \underline{y}^{P}_{k+N}$$

MPC em duas camadas

$$\underline{\hat{y}}^{C}_{k+\infty} = \underline{\underline{S}}_N \sum_{i=0}^{m-1} \Delta \underline{u}(k+i) + \underline{y}^{P}_{k+N}$$

$$\sum_{i=0}^{m-1} \Delta \underline{u}(k+i) = \Delta \underline{u}(k) + \Delta \underline{u}(k+1) + \ldots \Delta \underline{u}(k+m-1) = \underline{u}(k+m-1) - \underline{u}(k-1)$$

$$\underline{u}(k) - \underline{u}(k-1)$$

$$\underline{u}(k+m-1) - \underline{u}(k-1)$$

MPC em duas camadas

$$\underline{\hat{y}}^{C}_{k+\infty} = \underline{\underline{S}}_N \sum_{i=0}^{m-1} \Delta \underline{u}(k+i) + \underline{y}^{P}_{k+N}$$

$$\underline{u}(k+m-1) - \underline{u}(k-1)$$

$$\underline{\hat{y}}^{C}_{k+\infty} = \underline{\underline{S}}_N \left[\underline{u}(k+m-1) - \underline{u}(k-1) \right] + \underline{y}^{P}_{k+N}$$

MPC em duas camadas

$$\hat{\underline{y}}^{C}_{k+\infty} - \underline{y}^{P}_{k+N} = \underline{\underline{S}}_{N}\left[\underline{u}(k+m-1) - \underline{u}(k-1)\right]$$

Com as ações de controle

Sem as futuras ações de controle

Ganho estático do processo

$$\Delta \underline{y} = \underline{\underline{S}}_{N} \Delta \underline{u}$$

MPC em duas camadas

- A camada interna do MPC está preocupada com a dinâmica para se atingir o *setpoint* definido pela camada externa, tendo como índice o somatório quadrático dos erros durante o horizonte de predição do controlador.
- A camada externa de otimização está preocupada em definir esse *setpoint*, também chamado de *target*, observando, normalmente, aspectos econômicos. Dentro desta camada de otimização tem-se apenas a informação do ganho estático, visto que a dinâmica não é relevante para esta camada, enquanto que na camada do MPC a informação completa do processo, dinâmica e estática, é importante.

MPC em duas camadas

$$\hat{\underline{y}}^{C}_{k+\infty} - \underline{y}^{P}_{k+N} = \underline{\underline{S}}_N \left[\underline{u}(k+m-1) - \underline{u}(k-1) \right]$$

$$\hat{\underline{y}}^{C}_{SS} - \underline{y}^{P}_{k+N} = \underline{\underline{S}}_N \left[\underline{u}_{SS} - \underline{u}_{k-1} \right]$$

uirv

MPC em duas camadas

$\underline{\hat{y}}_{SS}^{C}$: predição para o novo estado estacionário obtido de uma função objetivo econômica

$\underline{y}_{k+N}^{P}$: situação no futuro *N* caso nenhuma ação de controle seja tomada

$\underline{\underline{S}}_{N}$: ganho estático do processo

$\underline{u}_{k-1}$: valor atual das manipuladas

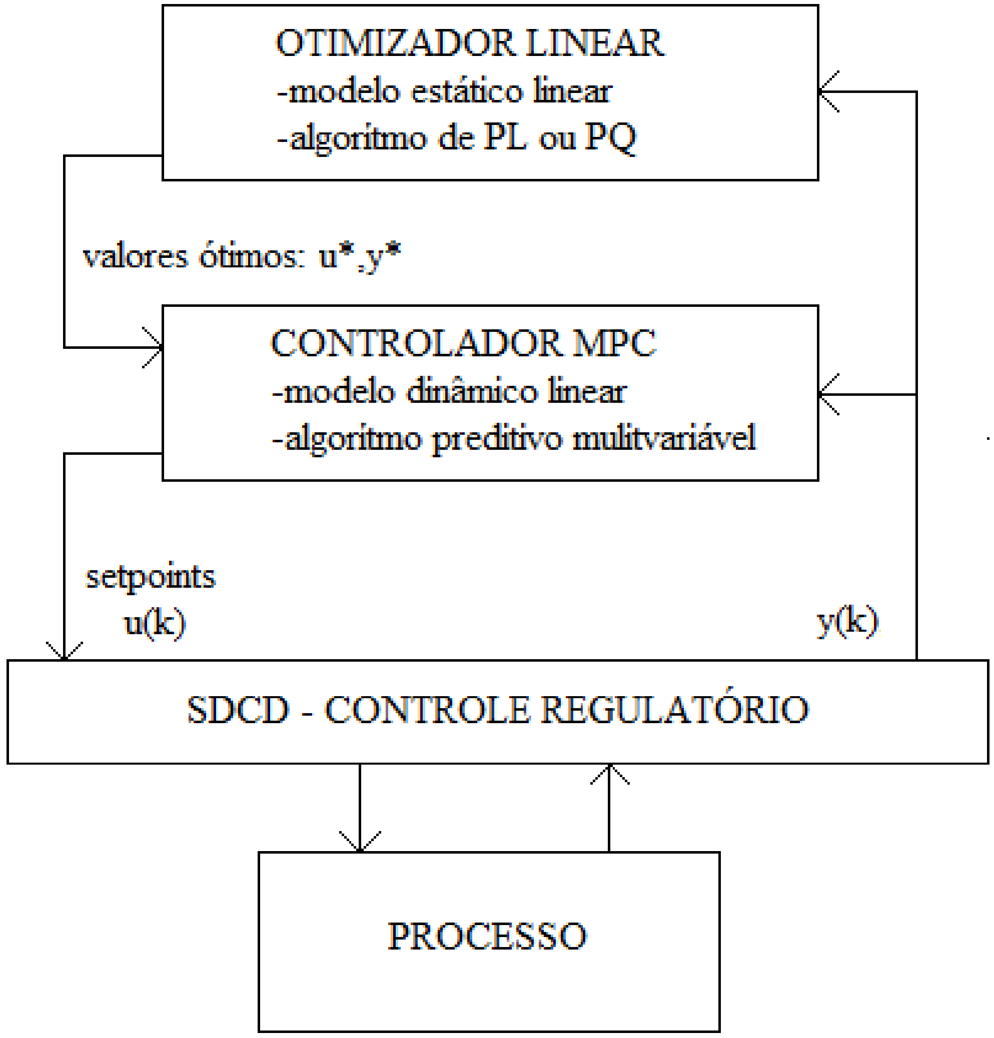
$\underline{u}_{SS}$: valor desejado para as variáveis manipuladas, $\underline{u}_{irv}$

Alteração da função objetivo MPC em duas camadas

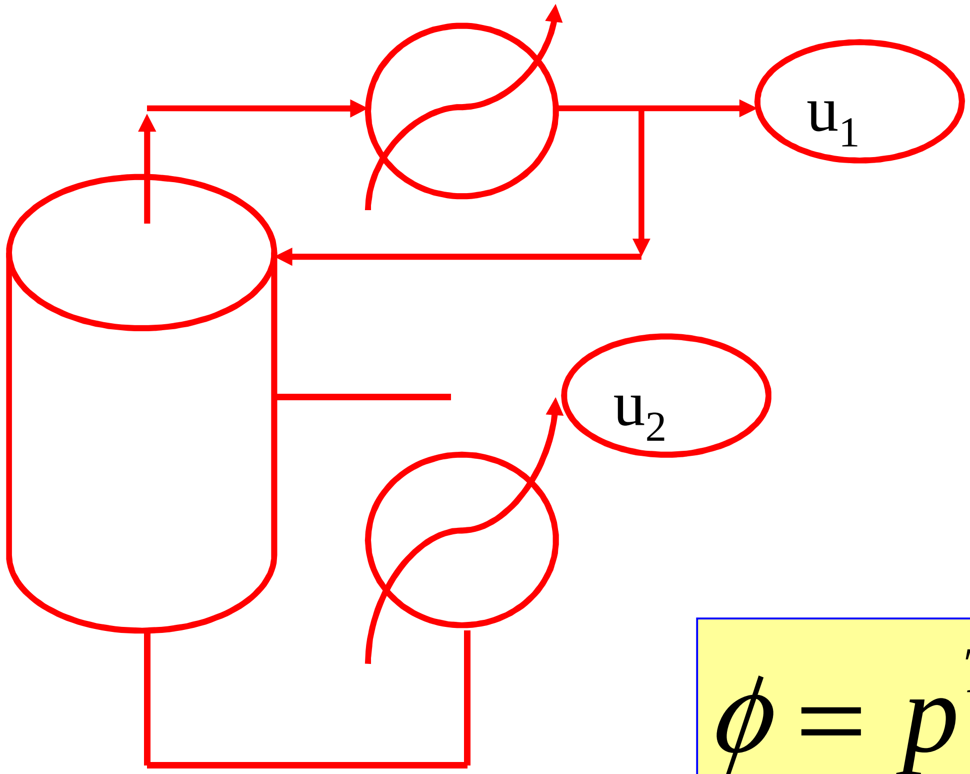
$$J = \underline{e}^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{e} + \left(\underline{u}_{k+m-1} - \underline{u}_{irv}\right)^T \underline{\underline{R}}_u \left(\underline{u}_{k+m-1} - \underline{u}_{irv}\right) + \Delta\underline{u}^T \underline{\underline{R}}\,\Delta\underline{u}$$

GRAU DE IMPORTÂNCIA DA MANIPULADA ATINGIR O VALOR IDEAL

MPC em duas camadas



MPC em duas camadas



$$\phi = \underline{p}_y^T \underline{y}_{SS} + \underline{p}_u^T \underline{u}_{SS}$$

MPC em duas camadas

- Programação Linear na camada **superior**

$$\min_{\underline{y}_{SS}^{C},\,\underline{u}_{SS}} \phi = \underline{p}_y^T \underline{y}_{SS}^C + \underline{p}_u^T \underline{u}_{SS}$$

s. a.

$$\hat{\underline{y}}_{SS}^{C} - \underline{y}_{k+N}^{P} = \underline{\underline{S}}_N \left[\underline{u}_{SS} - \underline{u}_{k-1}\right]$$

$$\underline{u}_{\min} \le \underline{u}_{SS} \le \underline{u}_{\max}$$

$$\underline{y}_{\min}^{C} \le \underline{y}_{SS}^{C} \le \underline{y}_{\max}^{C}$$

$$\underline{y}_{SS}^{C} \rightarrow \underline{y}^{SP}$$

$$\underline{u}_{SS} \rightarrow \underline{u}_{irv}$$

MPC em duas camadas

- Se o modelo fosse exatamente o real, ao definir a controlada, fica definida a manipulada e vice-versa. Como isso não ocorre, não conseguimos satisfazer o par controlada-manipuladas desejado.

$$\underline{\hat{y}}^{C}_{SS} - \underline{y}^{P}_{k+N} = \underline{\underline{S}}_{N}\left[\underline{u}_{SS} - \underline{u}_{k-1}\right]$$

- Ex: Se o ganho estático do modelo for maior que o real, tem-se, para um dado yss, um valor menor de u1 que poderia ser retirado:

$$\underline{u}_{SS} = \frac{\underline{\hat{y}}^{C}_{SS} - \underline{y}^{P}_{k+N}}{\underline{\underline{S}}_{N}} + \underline{u}_{k-1}$$

MPC em duas camadas

- Para resolver este problema, a função objetivo do MPC pode ser alterada:

$$J = \underline{e}^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{e} + \left(\underline{u}_{k+m-1} - \underline{u}_{irv}\right)^T \underline{\underline{R}}_u \left(\underline{u}_{k+m-1} - \underline{u}_{irv}\right) + \Delta\underline{u}^T \underline{\underline{R}}\,\Delta\underline{u}$$

Grau de importância para a manipulada atingir uirv

Objetivo: Ao final do horizonte m, u(k+m=1)~uirv

$$u_{k+m-1} = \begin{bmatrix} \underline{\underline{I}}_{nu} & \underline{\underline{I}}_{nu} & \cdots & \underline{\underline{I}}_{nu} \end{bmatrix} \begin{bmatrix} \Delta\underline{u}(k) \\ \Delta\underline{u}(k+1) \\ \vdots \\ \Delta\underline{u}(k+m-1) \end{bmatrix} + \underline{u}_{k-1} = \underline{\underline{\bar{I}}}\,\Delta\underline{u} + \underline{u}_{k-1}$$

QDMC em duas camadas

- Função objetivo do QDMC em duas camadas

$$J = \underline{e}^T \underline{\underline{W}}^T \underline{\underline{W}}\, \underline{e} + \left(\underline{u}_{k+m-1} - \underline{u}_{irv}\right)^T \underline{\underline{R}}_u \left(\underline{u}_{k+m-1} - \underline{u}_{irv}\right) + \Delta \underline{u}^T \underline{\underline{R}}\, \Delta \underline{u}$$

$$\left(\underline{\underline{\bar{I}}}\Delta \underline{u} + \underline{u}_{k-1} - \underline{u}_{irv}\right)^T \underline{\underline{R}}_u \left(\underline{\underline{\bar{I}}}\Delta \underline{u} + \underline{u}_{k-1} - \underline{u}_{irv}\right)$$

QDMC em duas camadas

$$\left(\underline{\underline{\bar{I}}}\Delta\underline{u}+\underline{u}_{k-1}-\underline{u}_{irv}\right)^T\underline{\underline{R}}_u\left(\underline{\underline{\bar{I}}}\Delta\underline{u}+\underline{u}_{k-1}-\underline{u}_{irv}\right)$$

$$\left(\Delta\underline{u}^T\underline{\underline{\bar{I}}}^T+\left[\underline{u}^T_{k-1}-\underline{u}^T_{irv}\right]\right)\underline{\underline{R}}_u\left(\underline{\underline{\bar{I}}}\Delta\underline{u}+\left[\underline{u}_{k-1}-\underline{u}_{irv}\right]\right)$$

$$\Delta\underline{u}^T\underline{\underline{\bar{I}}}^T\underline{\underline{R}}_u\underline{\underline{\bar{I}}}\Delta\underline{u}+\left(\underline{u}^T_{k-1}-\underline{u}^T_{irv}\right)\underline{\underline{R}}_u\underline{\underline{\bar{I}}}\Delta\underline{u}+\Delta\underline{u}^T\underline{\underline{\bar{I}}}^T\underline{\underline{R}}_u\left(\underline{u}_{k-1}-\underline{u}_{irv}\right)$$
$$+\left(\underline{u}^T_{k-1}-\underline{u}^T_{irv}\right)\underline{\underline{R}}_u\left(\underline{u}_{k-1}-\underline{u}_{irv}\right)$$

$$\Delta\underline{u}^T\underline{\underline{\bar{I}}}^T\underline{\underline{R}}_u\underline{\underline{\bar{I}}}\Delta\underline{u}+2\left(\underline{u}_{k-1}-\underline{u}_{irv}\right)^T\underline{\underline{R}}_u\underline{\underline{\bar{I}}}\Delta\underline{u}+\left(\underline{u}_{k-1}-\underline{u}_{irv}\right)^T\underline{\underline{R}}_u\left(\underline{u}_{k-1}-\underline{u}_{irv}\right)$$

QDMC em duas camadas

- Função objetivo do QDMC com a inclusão de uirv

$$J = \Delta \underline{u}^T \underline{\underline{H}} \Delta \underline{u} + 2 C_f^T \Delta \underline{u}$$

$$\left(\underline{\underline{\bar{I}}} \Delta \underline{u} + \underline{u}_{k-1} - \underline{u}_{irv}\right)^T \underline{\underline{R}}_u \left(\underline{\underline{\bar{I}}} \Delta \underline{u} + \underline{u}_{k-1} - \underline{u}_{irv}\right) = \Delta \underline{u}^T \underline{\underline{\bar{I}}}^T \underline{\underline{R}}_u \underline{\underline{\bar{I}}} \Delta \underline{u} + 2\left(\underline{u}_{k-1} - \underline{u}_{irv}\right)^T \underline{\underline{R}}_u \underline{\underline{\bar{I}}} \Delta \underline{u} + \\ + \left(\underline{u}_{k-1} - \underline{u}_{irv}\right)^T \underline{\underline{R}}_u \left(\underline{u}_{k-1} - \underline{u}_{irv}\right)$$

$$J = \Delta \underline{u}^T \left(\underline{\underline{H}} + \underline{\underline{\bar{I}}}^T \underline{\underline{R}}_u \underline{\underline{\bar{I}}}\right) \Delta \underline{u} + 2\left(C_f^T + \left(\underline{u}_{k-1} - \underline{u}_{irv}\right)^T \underline{\underline{R}}_u \underline{\underline{\bar{I}}}\right) \Delta \underline{u}$$

$$\underline{\underline{H}} = \underline{\underline{S}}^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{\underline{S}} + \underline{\underline{R}}$$

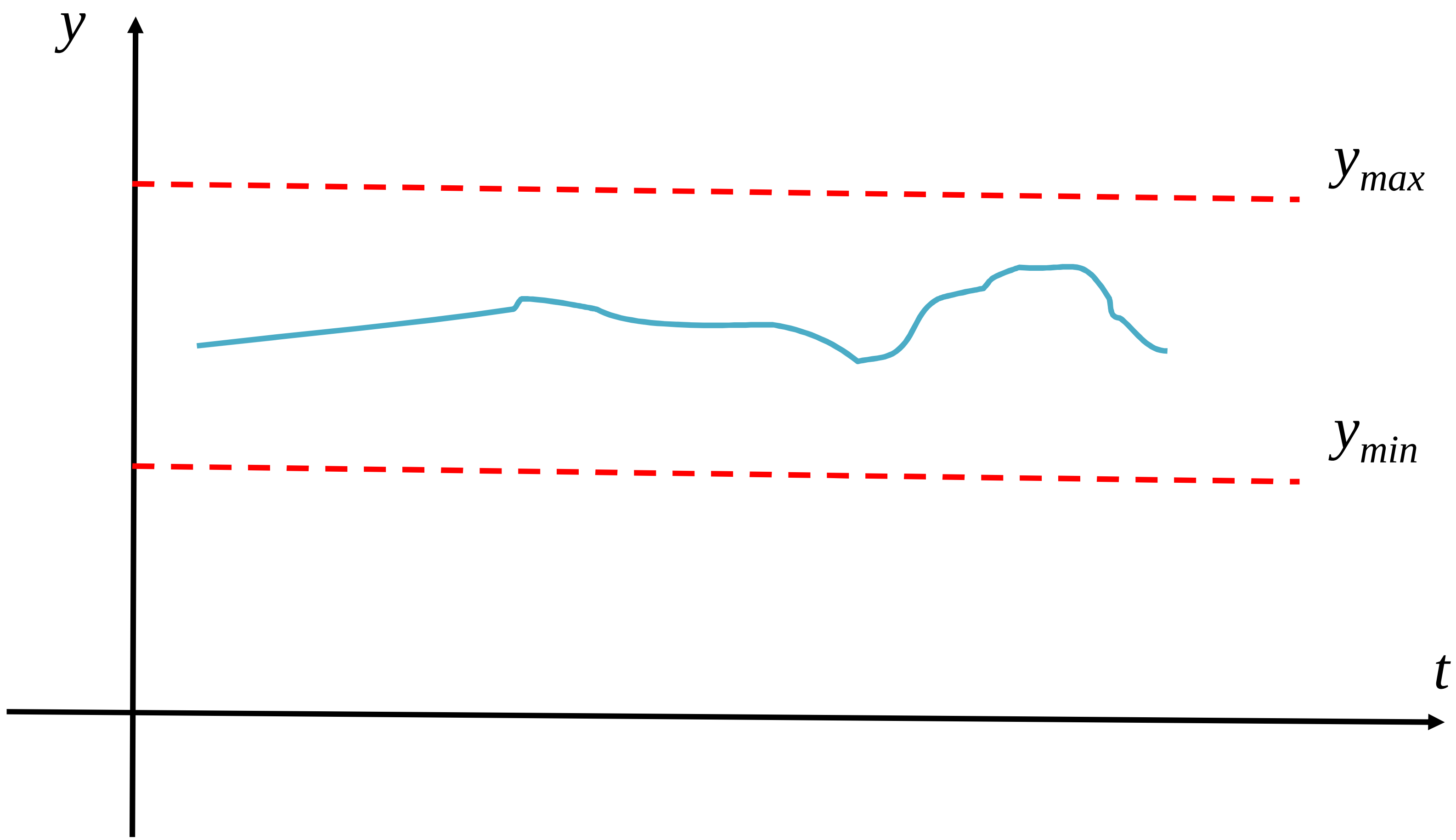
$$C_f^T = -\underline{e}'^T \underline{\underline{W}}^T \underline{\underline{W}}\,\underline{\underline{S}}$$

MPC COM CONTROLE DAS SAÍDAS POR FAIXA

Controle de saídas por faixa

- Nos processos a serem controlados, a maioria das saídas não tem um *setpoint* bem definido e sim uma faixa onde a saída, variável controlada, tem que ser mantida.
- Essa faixa é conhecida como restrição leve ou, do inglês, "*soft constraints*".
- Desta forma, permite-se um grau de liberdade para as controladas, o que "relaxa" o problema de otimização. Estas variáveis só passam a ser efetivamente controladas pelas manipuladas disponíveis quando uma das restrições for atingida (*ymax*, *ymin*).

Controle de saídas por faixa



Controle de saídas por faixa

- Função objetivo do MPC por faixa

$$J = \sum_{j=1}^{np} \left[\underline{y}^{C}_{k+j} - \underline{y}^{b}_{k+j}\right]^T \underline{\underline{W}}^T \underline{\underline{W}} \left[\underline{y}^{C}_{k+j} - \underline{y}^{b}_{k+j}\right] + \left(\underline{u}_{k+m-1} - \underline{u}_{irv}\right)^T \underline{\underline{R}}_u \left(\underline{u}_{k+m-1} - \underline{u}_{irv}\right) + \sum_{j=0}^{m-1} \Delta\underline{u}^T_{k+j} \underline{\underline{R}} \Delta\underline{u}_{k+j}$$

$$\underline{y}^{\min} \le \underline{y}^{P}_{k+j} \le \underline{y}^{\max} \Rightarrow W_{i,k+j} = 0$$

$$y^{P}_{i,k+j} > y^{\max}_i \Rightarrow y^{b}_{i,k+j} = y^{\max}_i$$

$$y^{P}_{i,k+j} < y^{\min}_i \Rightarrow y^{b}_{i,k+j} = y^{\min}_i$$

FUNÇÃO OBJETIVO ECONÔMICA COMO SAÍDA CONTROLADA DO MPC

MPC sem camada de otimização

- Função objetivo econômica

$$\phi = \underline{p}_y^T \underline{y}_{SS} + \underline{p}_u^T \underline{u}_{SS}$$

DEFINIÇÃO DA NOVA VARIÁVEL DE SAÍDA

$$\phi = y_{ny+1} = \underline{p}_y^T \underline{y}^C + \underline{p}_u^T \underline{u}$$

Qual o valor desejado para essa nova variável?

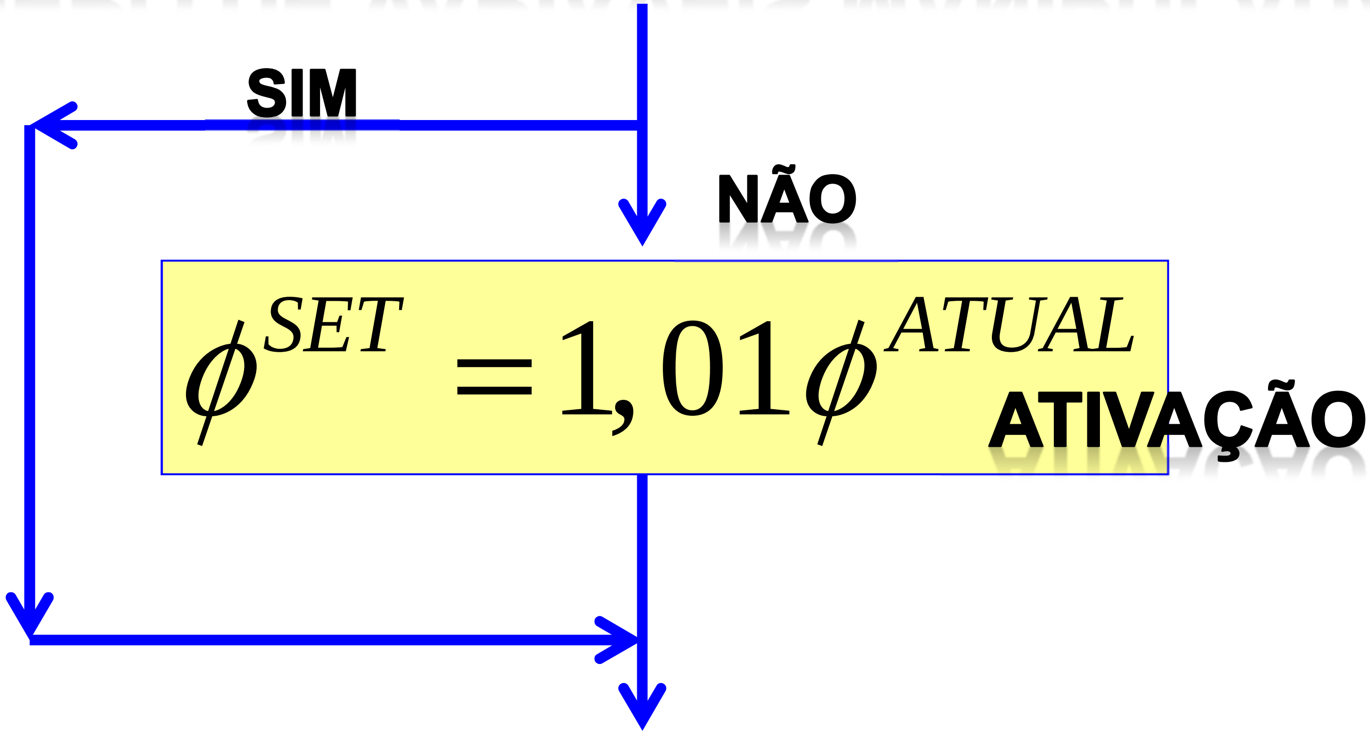
- Função objetivo deve ser maximizada

$$\phi^{SET} = 1,01\phi^{ATUAL}$$

O MPC AUMENTARÁ A FUNÇÃO OBJETIVO ATÉ ATINGIR UMA RESTRIÇÃO

Quando ativar essa variável?

NÚMERO DE SAÍDAS ATIVAS > NÚMERO DE VARIÁVEIS MANIPULADAS



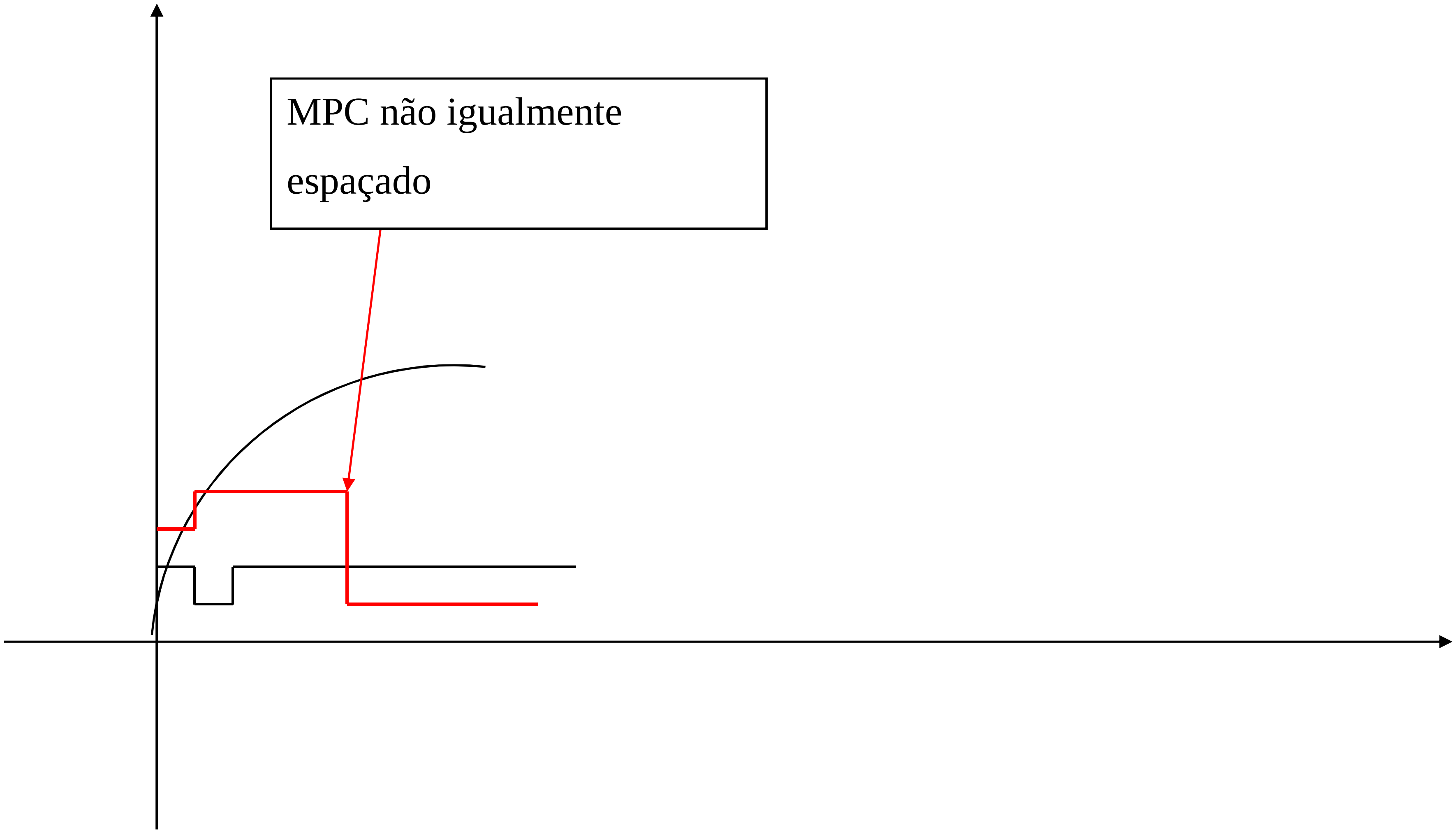
$$\phi^{SET} = 1,01\phi^{ATUAL}$$

MPC COM AÇÕES DE CONTROLE NÃO IGUALMENTE ESPAÇADAS

MPC com ações não igualmente espaçadas

- ✓ Para processos com período de estabilização *N* muito grande, o horizonte de controle *m* tende ao valor unitário, pois tudo se passa como, apesar de ter ocorrido várias ações de controle, esta ação fosse única.
- ✓ Se a ação de controle fosse executada em tempos maiores, mais espaçados, teríamos uma ação mais eficaz do controlador.

MPC com ações não igualmente espaçadas



MPC com ações não igualmente espaçadas

- Equação de predição para m=3.

$$\Delta u(k), \Delta u(k+n_1), \Delta u(k+n_2)$$

$$n_1 < n_2$$

n1 intervalos

n2-n1 intervalos

np –n2 intervalos

$$
\begin{bmatrix}
\underline{\hat{y}}^{C}_{k+1} \\
\underline{\hat{y}}^{C}_{k+2} \\
\vdots \\
\underline{\hat{y}}^{C}_{k+n_1+1} \\
\underline{\hat{y}}^{C}_{k+n_1+2} \\
\vdots \\
\underline{\hat{y}}^{C}_{k+n_2+1} \\
\underline{\hat{y}}^{C}_{k+n_2+2} \\
\vdots \\
\underline{\hat{y}}^{C}_{k+np}
\end{bmatrix}
=
\begin{bmatrix}
\underline{\underline{S}}_{1} & 0_{ny\times nu} & 0_{ny\times nu} \\
\underline{\underline{S}}_{2} & 0_{ny\times nu} & 0_{ny\times nu} \\
\vdots & \vdots & \vdots \\
\underline{\underline{S}}_{n_1+1} & \underline{\underline{S}}_{1} & 0_{ny\times nu} \\
\underline{\underline{S}}_{n_1+2} & \underline{\underline{S}}_{2} & 0_{ny\times nu} \\
\vdots & \vdots & \vdots \\
\underline{\underline{S}}_{n_2+1} & \underline{\underline{S}}_{n_2-n_1} & \underline{\underline{S}}_{1} \\
\underline{\underline{S}}_{n_2+2} & \underline{\underline{S}}_{n_2-n_1+1} & \underline{\underline{S}}_{2} \\
\vdots & \vdots & \vdots \\
\underline{\underline{S}}_{np} & \underline{\underline{S}}_{np-n_1} & \underline{\underline{S}}_{np-n_2}
\end{bmatrix}
\begin{bmatrix}
\Delta \underline{u}(k) \\
\Delta \underline{u}(k+n_1) \\
\Delta \underline{u}(k+n_2)
\end{bmatrix}
+
\begin{bmatrix}
\underline{\hat{y}}_{k} + \underline{P}_{1} \\
\underline{\hat{y}}_{k} + \underline{P}_{2} \\
\underline{\hat{y}}_{k} + \underline{P}_{3} \\
\vdots \\
\underline{\hat{y}}_{k} + \underline{P}_{np}
\end{bmatrix}
$$

MPC com ações não igualmente espaçadas

- Exemplo para m=3. $\Delta u(k), \Delta u(k+n_1), \Delta u(k+n_2)$

$$\begin{bmatrix} \hat{\underline{y}}^C_{k+1} \\ \hat{\underline{y}}^C_{k+2} \\ \vdots \\ \hat{\underline{y}}^C_{k+n_1+1} \\ \hat{\underline{y}}^C_{k+n_1+2} \\ \vdots \\ \hat{\underline{y}}^C_{k+n_2+1} \\ \hat{\underline{y}}^C_{k+n_2+2} \\ \vdots \\ \hat{\underline{y}}^C_{k+np} \end{bmatrix} = \begin{bmatrix} \underline{\underline{S}}_1 & 0_{ny\times nu} & 0_{ny\times nu} \\ \underline{\underline{S}}_2 & 0_{ny\times nu} & 0_{ny\times nu} \\ \vdots & \vdots & \vdots \\ \underline{\underline{S}}_{n_1+1} & \underline{\underline{S}}_1 & 0_{ny\times nu} \\ \underline{\underline{S}}_{n_1+2} & \underline{\underline{S}}_2 & 0_{ny\times nu} \\ \vdots & \vdots & \vdots \\ \underline{\underline{S}}_{n_2+1} & \underline{\underline{S}}_{n_2-n_1} & \underline{\underline{S}}_1 \\ \underline{\underline{S}}_{n_2+2} & \underline{\underline{S}}_{n_2-n_1+1} & \underline{\underline{S}}_2 \\ \vdots & \vdots & \vdots \\ \underline{\underline{S}}_{np} & \underline{\underline{S}}_{np-n_1} & \underline{\underline{S}}_{np-n_2} \end{bmatrix} \begin{bmatrix} \Delta\underline{u}(k) \\ \Delta\underline{u}(k+n_1) \\ \Delta\underline{u}(k+n_2) \end{bmatrix} + \begin{bmatrix} \hat{\underline{y}}_k + \underline{P}_1 \\ \hat{\underline{y}}_k + \underline{P}_2 \\ \hat{\underline{y}}_k + \underline{P}_3 \\ \vdots \\ \hat{\underline{y}}_k + \underline{P}_{np} \end{bmatrix}$$

$$n_2 = 2n_1$$

Sugestão

SISTEMA INTEGRADOR

Sistema Integrador

- ✓ São sistema que apresentam polo na origem em uma função de transferência em Laplace ou no círculo unitário na função de transferência em Z.

$$G_P(s) = \frac{N(s)}{s}$$

$$HG_P(z) = \frac{N(z)}{1 - z^{-1}}$$

$$\begin{bmatrix} \hat{\underline{y}}^P_{k+1} \\ \hat{\underline{y}}^P_{k+2} \\ \vdots \\ \hat{\underline{y}}^P_{k+np-1} \\ \hat{\underline{y}}^P_{k+np} \end{bmatrix}_{k+1} = \underbrace{\begin{bmatrix} 0_{ny\times ny} & I & 0 & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\ 0_{ny\times ny} & & & & 0 & 0 \\ 0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \\ 0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \end{bmatrix}}_{\underline{\underline{M}}} \begin{bmatrix} \hat{\underline{y}}^P_{k+1} \\ \hat{\underline{y}}^P_{k+2} \\ \hat{\underline{y}}^P_{k+3} \\ \vdots \\ \hat{\underline{y}}^P_{k+np} \end{bmatrix}_k + \underbrace{\begin{bmatrix} \vdots \\ \underline{\underline{S}}_{np-1} \\ \underline{\underline{S}}_{np} \end{bmatrix}}_{\underline{\underline{S}}} \Delta \underline{u}(k)$$

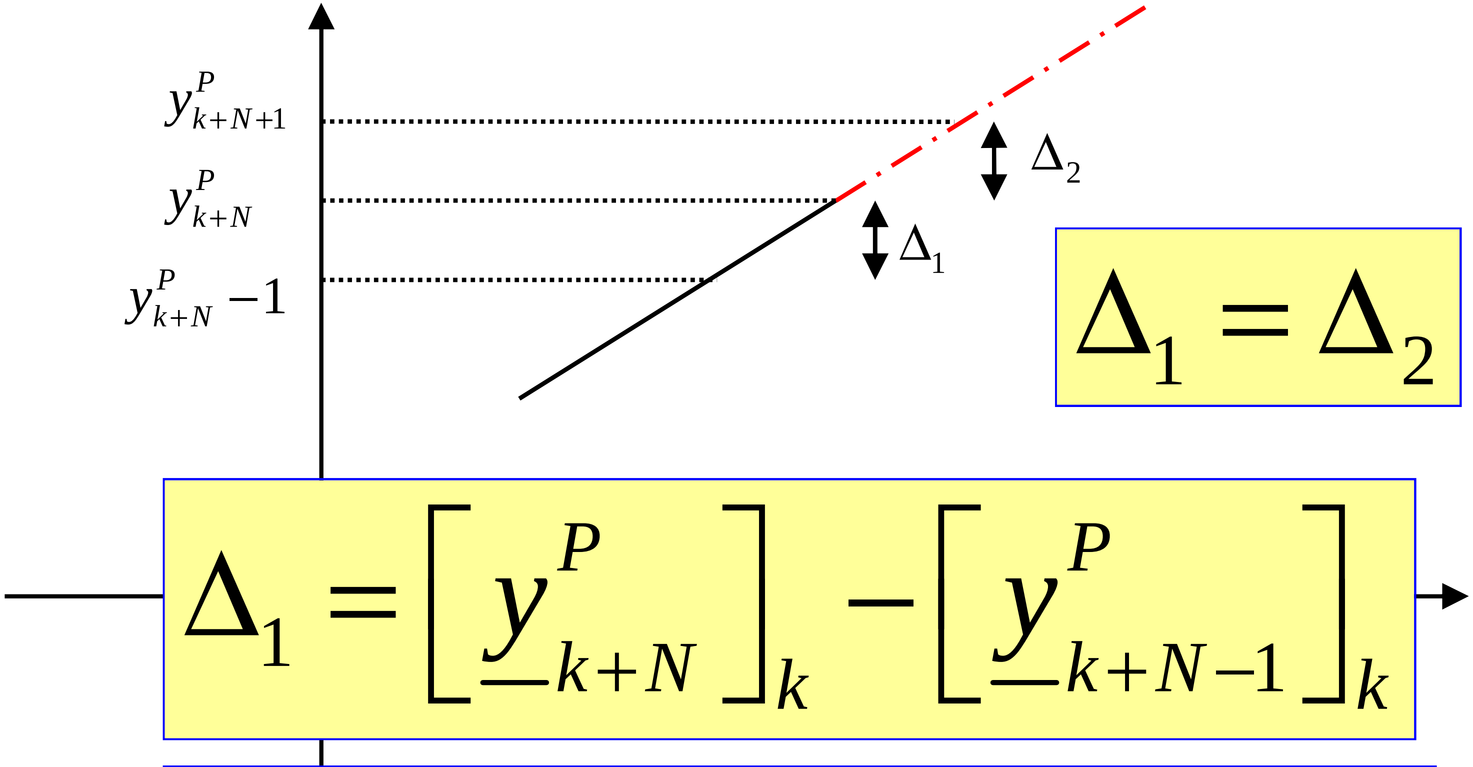
$$\underline{\underline{S}}_{np} \approx \underline{\underline{S}}_{np-1}$$

VÁLIDO PARA SISTEMAS ESTÁVEIS

$$\left[\hat{y}^P_{k+np}\right]_{k+1} = \left[\hat{y}^P_{k+np-1}\right]_{k+1} = \left[\hat{y}^P_{k+np}\right]_k$$

Nesse caso não é possível a hipótese da predição em np ser igual à predição em np-1.

Extrapolação de Saída do Modelo



$$\Delta_1 = \Delta_2$$

$$\Delta_1 = \left[\underline{y}^P_{k+N} \right]_k - \left[\underline{y}^P_{k+N-1} \right]_k$$

$$\Delta_2 = \left[\underline{y}^P_{k+N+1} \right]_k - \left[\underline{y}^P_{k+N} \right]_k$$

Extrapolação de Saída do Modelo

$$\Delta_1 = \Delta_2$$

$$\left[\underline{y}^P_{k+N}\right]_k - \left[\underline{y}^P_{k+N-1}\right]_k = \left[\underline{y}^P_{k+N+1}\right]_k - \left[\underline{y}^P_{k+N}\right]_k$$

$$\left[\underline{y}^P_{k+N+1}\right]_k = \left[\underline{y}^P_{k+N}\right]_k - \left[\underline{y}^P_{k+N-1}\right]_k + \left[\underline{y}^P_{k+N}\right]_k$$

$$\left[\underline{y}^P_{k+N+1}\right]_k = 2\left[\underline{y}^P_{k+N}\right]_k - \left[\underline{y}^P_{k+N-1}\right]_k$$

Extrapolação de Saída do Modelo

$$\left[\underline{y}^{P}_{k+N+1}\right]_k = \left[\underline{y}^{P}_{k+N}\right]_k - \left[\underline{y}^{P}_{k+N-1}\right]_k + \left[\underline{y}^{P}_{k+N}\right]_k$$

$$\left[\underline{y}^{P}_{k+N+1}\right]_k = 2\left[\underline{y}^{P}_{k+N}\right]_k - \left[\underline{y}^{P}_{k+N-1}\right]_k$$

$$\underline{\underline{M}} = \begin{bmatrix} 0_{ny\times ny} & I_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\ 0_{ny\times ny} & 0_{ny\times ny} & I_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\ 0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \\ 0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & -I_{ny\times ny} & 2I_{ny\times ny} \end{bmatrix}$$

ANÁLISE DA ESTABILIDADE DA MATRIZ DE DESLOCAMENTO PARA UM SISTEMA GERAL

Análise de Estabilidade de um Sistema Geral

PREDIÇÃO CORRIGIDA

$$\underline{\hat{y}}^{C}_{k+1} = \underline{\underline{M}}\,\underline{\hat{y}}^{C}_{k} + \underline{\underline{M}}\,\underline{\underline{\bar{S}}}\Delta \underline{u}(k) + \underline{d}(k+1)$$

$$\underline{d}(k+1) = \underline{\underline{K}}_F \left\{ \underline{y}_{k+1} - \underline{\underline{C}}\left(\underline{\hat{y}}^{C}_{k} + \underline{\underline{\bar{S}}}\Delta \underline{u}(k) \right) \right\}$$

$$\underline{\hat{y}}^{C}_{k+1} = \underline{\underline{M}}\,\underline{\hat{y}}^{C}_{k} + \underline{\underline{M}}\,\underline{\underline{\bar{S}}}\Delta \underline{u}(k) + \underline{\underline{K}}_F \left[\underline{y}_{k+1} - \underline{\underline{C}}\left(\underline{\hat{y}}^{C}_{k} + \underline{\underline{\bar{S}}}\Delta \underline{u}(k) \right) \right]$$

OBSERVADOR DE ESTADOS

$$\underline{\underline{K}}_F = \left[\underline{\underline{I}}_{ny} \quad \underline{\underline{I}}_{ny} \quad \cdots \quad \underline{\underline{I}}_{ny} \right]^T$$

ESTADO MEDIDO

Análise de Estabilidade de um Sistema Geral

$$\underline{\hat{y}}^{C}_{k+1} = \underline{\underline{M}}\,\underline{\hat{y}}^{C}_{k} + \underline{\underline{M}}\,\underline{\underline{\bar{S}}}\Delta\underline{u}(k) + \underline{\underline{K_F}}\left\{\underline{y}_{k+1} - \underline{\underline{C}}\left(\underline{\hat{y}}^{C}_{k} + \underline{\underline{\bar{S}}}\Delta\underline{u}(k)\right)\right\}$$

↓

$$\underline{\hat{y}}^{C}_{k+1} = \underline{\underline{M}}\,\underline{\hat{y}}^{C}_{k} + \underline{\underline{M}}\,\underline{\underline{\bar{S}}}\Delta\underline{u}(k) + \underline{\underline{K_F}}\,\underline{y}_{k+1} - \underline{\underline{K_F}}\,\underline{\underline{C}}\,\underline{\hat{y}}^{C}_{k} - \underline{\underline{K_F}}\,\underline{\underline{C}}\,\underline{\underline{\bar{S}}}\Delta\underline{u}(k)$$

↓

$$\underline{\hat{y}}^{C}_{k+1} = \left[\underline{\underline{M}} - \underline{\underline{K_F}}\,\underline{\underline{C}}\right]\underline{\hat{y}}^{C}_{k} + \left[\underline{\underline{M}} - \underline{\underline{K_F}}\,\underline{\underline{C}}\right]\underline{\underline{\bar{S}}}\Delta\underline{u}(k) + \underline{\underline{K_F}}\,\underline{y}_{k+1}$$

Análise de Estabilidade de um Sistema Geral

$$\underline{\hat{y}}^{C}_{k+1} = \left[\underline{\underline{M}} - \underline{\underline{K}}_F \underline{\underline{C}}\right] \underline{\hat{y}}^{C}_{k} + \left[\underline{\underline{M}} - \underline{\underline{K}}_F \underline{\underline{C}}\right] \underline{\underline{\bar{S}}} \Delta \underline{u}(k) + \underline{\underline{K}}_F \underline{y}_{k+1}$$

$x(k+1)$ $\quad A \quad$ $x(k)$ $\quad B \quad$ Δu

AUTOVALORES DA MATRIZ A

ANÁLISE DA ESTABILIDADE DA MATRIZ DE DESLOCAMENTO PARA UM SISTEMA AUTORREGULADOR

Análise de Estabilidade de um Sistema Autorregulador

$$Det\left\{\left[\underline{\underline{M}} - \underline{\underline{K}}_F \underline{\underline{C}}\right] - \underline{\lambda}\underline{\underline{I}}\right\} = 0$$

$$\underline{\underline{M}} - \underline{\underline{K}}_F \underline{\underline{C}} = \begin{bmatrix} 0_{ny\times ny} & I_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\ 0_{ny\times ny} & 0_{ny\times ny} & I_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\ 0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \\ 0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \end{bmatrix} - \begin{bmatrix} I_{ny\times ny} \\ I_{ny\times ny} \\ I_{ny\times ny} \\ \vdots \\ I_{ny\times ny} \\ I_{ny\times ny} \end{bmatrix} \begin{bmatrix} I_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \end{bmatrix}$$

TODOS AUTOVALORES EM ZERO

SISTEMA ESTÁVEL

ANÁLISE DA ESTABILIDADE DA MATRIZ DE DESLOCAMENTO PARA SISTEMAS INTEGRADORES

Análise de Estabilidade de um Sistema Integrador

$$Det\left\{\left[\underline{\underline{M}} - \underline{\underline{K}}_F \underline{\underline{C}}\right] - \underline{\lambda}\underline{\underline{I}}\right\} = 0$$

$$\underline{\underline{M}} - \underline{\underline{K}}_F \underline{\underline{C}} = \begin{bmatrix} 0_{ny\times ny} & I_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\ 0_{ny\times ny} & 0_{ny\times ny} & I_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\ 0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \\ 0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & -I_{ny\times ny} & 2I_{ny\times ny} \end{bmatrix} - \begin{bmatrix} I_{ny\times ny} \\ I_{ny\times ny} \\ I_{ny\times ny} \\ \vdots \\ I_{ny\times ny} \\ I_{ny\times ny} \end{bmatrix} \begin{bmatrix} I_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \end{bmatrix}$$

NY AUTOVALORES EM UM
DEMAIS AUTOVALORES EM ZERO
SISTEMA INSTÁVEL

Análise de Estabilidade de um Sistema Integrador

$$Det\left\{\left[\underline{\underline{M}} - \underline{\underline{K}}_F \underline{\underline{C}}\right] - \underline{\lambda}\underline{\underline{I}}\right\} = 0$$

$$\underline{\underline{M}} - \underline{\underline{K}}_F \underline{\underline{C}} = \begin{bmatrix} 0_{ny\times ny} & I_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\ 0_{ny\times ny} & 0_{ny\times ny} & I_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\ 0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \\ 0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & -I_{ny\times ny} & 2I_{ny\times ny} \end{bmatrix} - \begin{bmatrix} I_{ny\times ny} \\ I_{ny\times ny} \\ I_{ny\times ny} \\ \vdots \\ I_{ny\times ny} \\ 1,1I_{ny\times ny} \end{bmatrix} \begin{bmatrix} I_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \end{bmatrix}$$

MODIFICAÇÃO NO OBSERVADOR DE ESTADOS
SISTEMA ESTÁVEL

REPRESENTAÇÃO DE ESTADOS E MODELO DE REALINHAMENTO PARA UM SISTEMA INTEGRADOR PURO

Sistema Integrador com Espaço de Estados

$$y(k+1) = y(k) + \left[y(k) - y(k-1) \right] + \underline{\underline{S}}_1 \Delta \underline{u}(k)$$

QUANTO Y AUMENTOU NO PERÍODO ATUAL E ANTERIOR

$$y(k+1) = 2y(k) - y(k-1) + \underline{\underline{S}}_1 \Delta \underline{u}(k)$$

Sistema Integrador com Espaço de Estados

$$\underline{y}(k+1) = 2\underline{y}(k) - \underline{y}(k-1) + \underline{\underline{S}}_1 \Delta \underline{u}(k)$$

$$\underline{x}(k) = \begin{bmatrix} \underline{y}(k) \\ \underline{y}(k-1) \\ \underline{y}(k-2) \end{bmatrix}$$

$$\underline{y}(k) = \begin{bmatrix} I_{ny} & 0_{ny} & 0_{ny} \end{bmatrix} \underline{x}(k)$$

C

DEFINIÇÃO DO ESTADO

Sistema Integrador com Espaço de Estados

$$y(k+1) = 2y(k) - y(k-1) + \underline{\underline{S}}_1 \Delta \underline{u}(k)$$

NOTAÇÃO ESPAÇO DE ESTADO

$$\begin{bmatrix} \underline{y}(k+1) \\ \underline{y}(k) \\ \underline{y}(k-1) \end{bmatrix}_{k+1} = \begin{bmatrix} 2I_{ny} & -I_{ny} & 0 \\ I_{ny} & 0 & 0 \\ 0 & I_{ny} & 0 \end{bmatrix} \begin{bmatrix} \underline{y}(k) \\ \underline{y}(k-1) \\ \underline{y}(k-2) \end{bmatrix}_{k} + \begin{bmatrix} \underline{\underline{S}}_1 \\ 0 \\ 0 \end{bmatrix} \Delta \underline{u}(k)$$

X(K+1)　　A　　X(K)　　B

Sistema Integrador com Espaço de Estados

- Colocando a correção da leitura da planta

$$\underline{x}(k+1) = \underline{\underline{A}}\,\underline{x}(k) + \underline{\underline{B}}\Delta\underline{u}(k) + \underline{\underline{K}}_F\left(\underline{y}_{k+1} - \underline{\underline{C}}\left(\underline{\underline{A}}\,\underline{x}(k) + \underline{\underline{B}}\Delta\underline{u}(k)\right)\right)$$

LEITURA DA PLANTA NO INSTANTE K+1

PREDIÇÃO DE Y PARA K+1

$$K_F = \begin{bmatrix} I_{ny} & 0 & 0 \end{bmatrix}^T$$

APENAS Y(K+1) SERÁ CORRIGIDO. OS DEMAIS COMPONENTES DO ESTADO SERÃO OBTIDOS POR DESLOCAMENTO

$$\underline{x}(k+1) = \left[\underline{\underline{A}} - \underline{\underline{K}}_F C \underline{\underline{A}}\right]\underline{x}(k) + \left[\underline{\underline{B}} - \underline{\underline{K}}_F C \underline{\underline{B}}\right]\Delta\underline{u}(k) + \underline{\underline{K}}_F\,\overline{y}_{k+1}\Delta\underline{u}(k)$$

Estabilidade no Sistema Integrador com Espaço de Estados

$$\underline{x}(k+1)=\left[\underline{\underline{A}}-\underline{\underline{K}}_F C\underline{\underline{A}}\right]\underline{x}(k)+\left[\underline{\underline{B}}-\underline{\underline{K}}_F C\underline{\underline{B}}\right]\Delta\underline{u}(k)+\underline{\underline{K}}_F\,\overline{y}_{k+1}\Delta\underline{u}(k)$$

$$\underline{\underline{A}}=\begin{bmatrix}2 & -1 & 0\\ 1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}$$

$$K_F=\begin{bmatrix}I_{ny} & 0 & 0\end{bmatrix}^T$$

$$C=\begin{bmatrix}1 & 0 & 0\end{bmatrix}$$

AUTOVALORES EM ZERO
SISTEMA ESTÁVEL

EQUAÇÃO DE PREDIÇÃO PARA UM SISTEMA EM ESPAÇO DE ESTADO

MPC com Modelo em Espaço de Estados

- Equação de predição para o instante 1

$$\underline{x}(k+1)=\underline{\underline{A}}\,\underline{x}(k)+\underline{\underline{B}}\Delta\underline{u}(k)$$

$$\underline{y}(k)=\underline{\underline{C}}\,\underline{x}(k)$$

- Equação de predição para o instante 2

$$\underline{x}(k+2)=\underline{\underline{A}}\,\underline{x}(k+1)+\underline{\underline{B}}\Delta\underline{u}(k+1)$$

$$\underline{x}(k+1)=\underline{\underline{A}}\,\underline{x}(k)+\underline{\underline{B}}\Delta\underline{u}(k)$$

$$\underline{x}(k+2)=\underline{\underline{A}}\left[\underline{\underline{A}}\,\underline{x}(k)+\underline{\underline{B}}\Delta\underline{u}(k)\right]+\underline{\underline{B}}\Delta\underline{u}(k+1)$$

MPC com Modelo em Espaço de Estados

$$\underline{x}(k+2) = \underline{\underline{A}}\left[\underline{\underline{A}}\,\underline{x}(k) + \underline{\underline{B}}\Delta\underline{u}(k)\right] + \underline{\underline{B}}\Delta\underline{u}(k+1)$$

$$\underline{x}(k+2) = \underline{\underline{A}}^2\,\underline{x}(k) + \underline{\underline{A}}\,\underline{\underline{B}}\Delta\underline{u}(k) + \underline{\underline{B}}\Delta\underline{u}(k+1)$$

MPC com Modelo em Espaço de Estados

$$\underline{y}(k+2) = \underline{\underline{C}}\,\underline{x}(k+2)$$

$$\underline{x}(k+2) = \underline{\underline{A}}^2\underline{x}(k) + \underline{\underline{A}}\,\underline{\underline{B}}\Delta\underline{u}(k) + \underline{\underline{B}}\Delta\underline{u}(k+1)$$

$$\underline{y}(k+2) = \underline{\underline{C}}\left[\underline{\underline{A}}^2\underline{x}(k) + \underline{\underline{A}}\,\underline{\underline{B}}\Delta\underline{u}(k) + \underline{\underline{B}}\Delta\underline{u}(k+1)\right]$$

$$\underline{y}(k+2) = \underline{\underline{C}}\,\underline{\underline{A}}^2\underline{x}(k) + \underline{\underline{C}}\,\underline{\underline{A}}\,\underline{\underline{B}}\Delta\underline{u}(k) + \underline{\underline{C}}\,\underline{\underline{B}}\Delta\underline{u}(k+1)$$

MPC com Modelo em Espaço de Estados

- Equação de predição para o instante 3

$$\underline{x}(k+3) = \underline{\underline{A}}\,\underline{x}(k+2) + \underline{\underline{B}}\Delta\underline{u}(k+2)$$

$$\underline{x}(k+2) = \underline{\underline{A}}^2\underline{x}(k) + \underline{\underline{A}}\,\underline{\underline{B}}\Delta\underline{u}(k) + \underline{\underline{B}}\Delta\underline{u}(k+1)$$

$$\underline{x}(k+3) = \underline{\underline{A}}\left[\underline{\underline{A}}^2\underline{x}(k) + \underline{\underline{A}}\,\underline{\underline{B}}\Delta\underline{u}(k) + \underline{\underline{B}}\Delta\underline{u}(k+1)\right] + \underline{\underline{B}}\Delta\underline{u}(k+2)$$

$$\underline{x}(k+3) = \underline{\underline{A}}^3\underline{x}(k) + \underline{\underline{A}}^2\underline{\underline{B}}\Delta\underline{u}(k) + \underline{\underline{A}}\,\underline{\underline{B}}\Delta\underline{u}(k+1) + \underline{\underline{B}}\Delta\underline{u}(k+2)$$

MPC com Modelo em Espaço de Estados

$$\underline{y}(k+3)=\underline{\underline{C}}\,\underline{x}(k+3)$$

$$\underline{x}(k+3)=\underline{\underline{A}}^3\underline{x}(k)+\underline{\underline{A}}^2\underline{\underline{B}}\Delta\underline{u}(k)+\underline{\underline{A}}\,\underline{\underline{B}}\Delta\underline{u}(k+1)+\underline{\underline{B}}\Delta\underline{u}(k+2)$$

$$\underline{y}(k+2)=\underline{\underline{C}}\left[\underline{\underline{A}}^3\underline{x}(k)+\underline{\underline{A}}^2\underline{\underline{B}}\Delta\underline{u}(k)+\underline{\underline{A}}\,\underline{\underline{B}}\Delta\underline{u}(k+1)+\underline{\underline{B}}\Delta\underline{u}(k+2)\right]$$

$$\underline{y}(k+2)=\underline{\underline{C}}\,\underline{\underline{A}}^3\underline{x}(k)+\underline{\underline{C}}\,\underline{\underline{A}}^2\underline{\underline{B}}\Delta\underline{u}(k)+\underline{\underline{C}}\,\underline{\underline{A}}\,\underline{\underline{B}}\Delta\underline{u}(k+1)+\underline{\underline{C}}\,\underline{\underline{B}}\Delta\underline{u}(k+2)$$

MPC com Modelo em Espaço de Estados

- Equação de predição para o instante *np*

$$\underline{x}(k+3) = \underline{\underline{A}}^{3}\underline{x}(k) + \underline{\underline{A}}^{2}\underline{\underline{B}}\Delta\underline{u}(k) + \underline{\underline{A}}\,\underline{\underline{B}}\Delta\underline{u}(k+1) + \underline{\underline{B}}\Delta\underline{u}(k+2)$$

$$\underline{x}(k+np) = \underline{\underline{A}}^{np}\underline{x}(k) + \underline{\underline{A}}^{np-1}\underline{\underline{B}}\Delta\underline{u}(k) + \underline{\underline{A}}^{np-2}\underline{\underline{B}}\Delta\underline{u}(k+1) + \cdots + \underline{\underline{A}}^{np-m}\underline{\underline{B}}\Delta\underline{u}(k+m-1)$$

MPC com Modelo em Espaço de Estados

$$\underline{y}(k+np)=\underline{\underline{C}}\,\underline{x}(k+np)$$

$$\underline{x}(k+np)=\underline{\underline{A}}^{np}\underline{x}(k)+\underline{\underline{A}}^{np-1}\underline{\underline{B}}\Delta\underline{u}(k)+\underline{\underline{A}}^{np-2}\underline{\underline{B}}\Delta\underline{u}(k+1)+\cdots\underline{\underline{\underline{\underline{A}}^{np-m}\underline{\underline{B}}}}\Delta\underline{u}(k+m-1)$$

$$\underline{y}(k+np)=\underline{\underline{C}}\left[\underline{\underline{A}}^{np}\underline{x}(k)+\underline{\underline{A}}^{np-1}\underline{\underline{B}}\Delta\underline{u}(k)+\underline{\underline{A}}^{np-2}\underline{\underline{B}}\Delta\underline{u}(k+1)+\cdots\underline{\underline{\underline{\underline{A}}^{np-m}\underline{\underline{B}}}}\Delta\underline{u}(k+m-1)\right]$$

$$\underline{y}(k+np)=\underline{\underline{C}}\,\underline{\underline{A}}^{np}\underline{x}(k)+\underline{\underline{C}}\,\underline{\underline{A}}^{np-1}\underline{\underline{B}}\Delta\underline{u}(k)+\underline{\underline{C}}\,\underline{\underline{A}}^{np-2}\underline{\underline{B}}\Delta\underline{u}(k+1)+\cdots\underline{\underline{C}}\,\underline{\underline{\underline{\underline{A}}^{np-m}\underline{\underline{B}}}}\Delta\underline{u}(k+m-1)$$

MPC com Modelo em Espaço de Estados

- Equação de predição para um instante *j* genérico

$$\underline{y}(k+np)=\underline{\underline{C}}\,\underline{\underline{A}}^{np}\,\underline{x}(k)+\underline{\underline{C}}\,\underline{\underline{A}}^{np-1}\underline{\underline{B}}\Delta\underline{u}(k)+\underline{\underline{C}}\,\underline{\underline{A}}^{np-2}\underline{\underline{B}}\Delta\underline{u}(k+1)+\underline{\underline{C}}\,\underline{\underline{A}}^{np-3}\underline{\underline{B}}\Delta\underline{u}(k+2)+$$
$$+\ldots+\underline{\underline{C}}\,\underline{\underline{A}}^{np-m}\underline{\underline{B}}\Delta\underline{u}(k+m-1)$$

$$\underline{y}(k+j)=\underline{\underline{C}}\,\underline{\underline{A}}^{j}\,\underline{x}(k)+\underline{\underline{C}}\,\underline{\underline{A}}^{j-1}\underline{\underline{B}}\Delta\underline{u}(k)+\underline{\underline{C}}\,\underline{\underline{A}}^{j-2}\underline{\underline{B}}\Delta\underline{u}(k+1)+\underline{\underline{C}}\,\underline{\underline{A}}^{j-3}\underline{\underline{B}}\Delta\underline{u}(k+2)+$$
$$+\ldots+\underline{\underline{C}}\,\underline{\underline{A}}^{j-m}\underline{\underline{B}}\Delta\underline{u}(k+m-1)$$

SE J-M<0, MATRIZ DE ZEROS COM NY LINHAS E NU COLUNAS

MPC com Modelo em Espaço de Estados

- Equação de predição forma matricial

$$\underline{y}(k+j) = \underline{\underline{C}}\,\underline{\underline{A}}^{j}\,\underline{x}(k) + \underline{\underline{C}}\,\underline{\underline{A}}^{j-1}\,\underline{\underline{B}}\Delta\underline{u}(k) + \underline{\underline{C}}\,\underline{\underline{A}}^{j-2}\,\underline{\underline{B}}\Delta\underline{u}(k+1) + \underline{\underline{C}}\,\underline{\underline{A}}^{j-3}\,\underline{\underline{B}}\Delta\underline{u}(k+2) + \\ + \ldots + \underline{\underline{C}}\,\underline{\underline{A}}^{j-m}\,\underline{\underline{B}}\Delta\underline{u}(k+m-1)$$

$$\Downarrow$$

$$\begin{bmatrix} \underline{y}(k+1) \\ \underline{y}(k+2) \\ \underline{y}(k+3) \\ \vdots \\ \underline{y}(k+np) \end{bmatrix} = \begin{bmatrix} \underline{\underline{C}}\,\underline{\underline{A}} \\ \underline{\underline{C}}\,\underline{\underline{A}}^{2} \\ \underline{\underline{C}}\,\underline{\underline{A}}^{3} \\ \vdots \\ \underline{\underline{C}}\,\underline{\underline{A}}^{np} \end{bmatrix} \underline{x}(k) + \begin{bmatrix} \underline{\underline{C}}\,\underline{\underline{B}} & 0_{ny\times nu} & 0_{ny\times nu} & \cdots & 0_{ny\times nu} \\ \underline{\underline{C}}\,\underline{\underline{A}}\,\underline{\underline{B}} & \underline{\underline{C}}\,\underline{\underline{B}} & 0_{ny\times nu} & \cdots & 0_{ny\times nu} \\ \underline{\underline{C}}\,\underline{\underline{A}}^{2}\,\underline{\underline{B}} & \underline{\underline{C}}\,\underline{\underline{A}}\,\underline{\underline{B}} & \underline{\underline{C}}\,\underline{\underline{B}} & \cdots & 0_{ny\times nu} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \underline{\underline{C}}\,\underline{\underline{A}}^{np-1}\,\underline{\underline{B}} & \underline{\underline{C}}\,\underline{\underline{A}}^{np-2}\,\underline{\underline{B}} & \underline{\underline{C}}\,\underline{\underline{A}}^{np-3}\,\underline{\underline{B}} & \cdots & \underline{\underline{C}}\,\underline{\underline{A}}^{np-m}\,\underline{\underline{B}} \end{bmatrix} \begin{bmatrix} \Delta\underline{u}(k) \\ \Delta\underline{u}(k+1) \\ \Delta\underline{u}(k+2) \\ \vdots \\ \Delta\underline{u}(k+m-1) \end{bmatrix}$$

$$\overline{\underline{x}}_{k+1} = \Psi\,\overline{\underline{x}}_{k} + \Theta\Delta\overline{\underline{u}}_{k}$$

SISTEMA INTEGRADOR: PREDIÇÃO COM REALINHAMENTO E ESPAÇO DE ESTADO

INTEGRADOR EM ESPAÇO DE ESTADO COM REALINHAMENTO

- Equação de predição do DMC Clássico

$$\underline{\hat{y}}^{C}_{k+1} = \underline{\underline{M}}\,\underline{\hat{y}}^{C}_{k} + \underline{\underline{M}}\,\overline{\underline{\underline{S}}}\Delta\underline{u}(k) + \overline{\underline{d}}(k+1)$$

$$\underline{\underline{M}} = \begin{bmatrix} 0_{ny\times ny} & I_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\ 0_{ny\times ny} & 0_{ny\times ny} & I_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\ 0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & 0_{ny\times ny} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & 0_{ny\times ny} & I_{ny\times ny} \\ 0_{ny\times ny} & 0_{ny\times ny} & 0_{ny\times ny} & \cdots & -I_{ny\times ny} & 2I_{ny\times ny} \end{bmatrix}$$

INTEGRADOR EM ESPAÇO DE ESTADO COM REALINHAMENTO

- Considerando a predição como uma equação em espaço de estado:

$$\underbrace{\underline{\hat{y}}^{C}_{k+1}}_{x(k+1)} = \underbrace{\left(\underline{\underline{M}} - K_F \underline{\underline{C}}\right)}_{A} \underbrace{\underline{\hat{y}}^{C}_{k}}_{x(k)} + \underbrace{\left(\underline{\underline{M}} - K_F \underline{\underline{C}}\right)\underline{\underline{\bar{S}}}}_{B} \Delta \underline{u}(k) + K_F \underline{\bar{y}}_{k+1}$$

NY AUTOVALORES EM +1 PARA ESTABILIZAR:

$$\underline{\underline{K_F}} = \begin{bmatrix} \underline{\underline{I}}_{ny} & \underline{\underline{I}}_{ny} & \dots & 1{,}1\underline{\underline{I}}_{ny} \end{bmatrix}^T$$

INTEGRADOR EM ESPAÇO DE ESTADO COM REALINHAMENTO

- Uma abordagem utilizada pelo SICON:

$$\underline{y}_{k+1}^{P} = \underline{y}_{k} + \Delta \underline{y} = \underline{y}_{k} + \underline{y}_{k} - \underline{y}_{k-1} = 2\underline{y}_{k} - \underline{y}_{k-1}$$

ESSA É A MESMA IDEIA UTILIZADA NA ÚLTIMA LINHA DA MATRIZ DE DESLOCAMENTO. SÓ QUE AGORA SERÁ UTILIZADA AO LONGO DE TODO HORIZONTE DE PREDIÇÃO

INTEGRADOR EM ESPAÇO DE ESTADO COM REALINHAMENTO

- Uma abordagem utilizada pelo SICON:

$$\begin{bmatrix} \underline{y}(k+1) \\ \underline{y}(k) \\ \underline{y}(k-1) \end{bmatrix}_{k+1} = \begin{bmatrix} 2_{ny} & -I_{ny} & 0_{ny} \\ I_{ny} & 0_{ny} & 0_{ny} \\ 0_{ny} & I_{ny} & 0_{ny} \end{bmatrix} \begin{bmatrix} \underline{y}(k) \\ \underline{y}(k-1) \\ \underline{y}(k-2) \end{bmatrix}_{k} + \begin{bmatrix} \underline{\underline{S}}_1 \\ 0 \\ 0 \end{bmatrix} \Delta \underline{u}(k)$$

X(K+1) | **A** | **X(K)** | **B**

INTEGRADOR EM ESPAÇO DE ESTADO COM REALINHAMENTO

- Equação de Predição com correção da estimativa

$$\hat{\underline{x}}^{C}_{k+1} = \underline{\underline{A}}\,\hat{\underline{x}}^{C}_{k} + \underline{\underline{B}}\,\Delta\underline{u}(k) + \underline{\underline{K}}_{F}\left\{\underline{\underline{C}}\left(\overline{\underline{y}}_{k} + \overline{\underline{\underline{S}}}_{P}\,\Delta\underline{u}(k)\right) - \underline{\underline{C}}\left(\hat{\underline{y}}^{C}_{k} + \overline{\underline{\underline{S}}}\,\Delta\underline{u}(k)\right)\right\}$$

REALINHAMENTO DA PREDIÇÃO CORRIGIDA

AÇÕES FUTURAS DE CONTROLE

PREDIÇÃO COM O MODELO DA PLANTA

PREDIÇÃO COM O MODELO DO CONTROLADOR

INTEGRADOR EM ESPAÇO DE ESTADO COM REALINHAMENTO

$$\hat{\underline{x}}_{k+1}^{C} = \underline{\underline{A}}\,\hat{\underline{x}}_{k}^{C} + \underline{\underline{B}}\Delta\underline{u}(k) + \underline{\underline{K}}_{F}\left\{\underline{\underline{C}}\left(\overline{\underline{y}}_{k} + \overline{\underline{\underline{S}}}_{P}\Delta\underline{u}(k)\right) - \underline{\underline{C}}\left(\hat{\underline{y}}_{k}^{C} + \overline{\underline{\underline{S}}}\Delta\underline{u}(k)\right)\right\}$$

$$\hat{\underline{x}}_{k+1}^{C} = \underline{\underline{A}}\,\hat{\underline{x}}_{k}^{C} + \underline{\underline{B}}\Delta\underline{u}(k) + \underline{\underline{K}}_{F}\left\{\underline{\underline{C}}\,\overline{\underline{y}}_{k} + \underline{\underline{C}}\,\overline{\underline{\underline{S}}}_{P}\Delta\underline{u}(k) - \underline{\underline{C}}\,\hat{\underline{y}}_{k}^{C} - \underline{\underline{C}}\,\overline{\underline{\underline{S}}}\Delta\underline{u}(k)\right\}$$

$$\underline{\underline{C}} = \begin{bmatrix} \underline{\underline{I}}_{ny} & \underline{\underline{0}}_{ny} & \cdots & \underline{\underline{0}}_{ny} \end{bmatrix}$$

$$\overline{\underline{y}}_{k} = \begin{bmatrix} \underline{y}_{k} & \underline{y}_{k} & \cdots & \underline{y}_{k} \end{bmatrix}_{k}^{T}$$

$$\underline{\underline{C}}\,\overline{\underline{y}}_{k} = \begin{bmatrix} \underline{\underline{I}}_{ny} & \underline{\underline{0}}_{ny} & \cdots & \underline{\underline{0}}_{ny} \end{bmatrix} \begin{bmatrix} \underline{y}_{k} \\ \underline{y}_{k} \\ \vdots \\ \underline{y}_{k} \end{bmatrix} = \underline{y}_{k}$$

INTEGRADOR EM ESPAÇO DE ESTADO COM REALINHAMENTO

$$\underline{\hat{x}}_{k+1}^{C} = \underline{\underline{A}}\,\underline{\hat{x}}_{k}^{C} + \underline{\underline{B}}\Delta\underline{u}(k) + \underline{\underline{K}}_{F}\left\{\underline{\underline{C}}\left(\underline{\bar{y}}_{k} + \underline{\underline{\bar{S}}}_{P}\Delta\underline{u}(k)\right) - \underline{\underline{C}}\left(\underline{\hat{y}}_{k}^{C} + \underline{\underline{\bar{S}}}\Delta\underline{u}(k)\right)\right\}$$

$$\underline{\hat{x}}_{k+1}^{C} = \underline{\underline{A}}\,\underline{\hat{x}}_{k}^{C} + \underline{\underline{B}}\Delta\underline{u}(k) + \underline{\underline{K}}_{F}\left\{\underline{\underline{C}}\,\underline{\bar{y}}_{k} + \underline{\underline{C}}\,\underline{\underline{\bar{S}}}_{P}\Delta\underline{u}(k) - \underline{\underline{C}}\,\underline{\hat{y}}_{k}^{C} - \underline{\underline{C}}\,\underline{\underline{\bar{S}}}\Delta\underline{u}(k)\right\}$$

$$\underline{\underline{C}} = \begin{bmatrix} \underline{\underline{I}}_{ny} & \underline{\underline{0}}_{ny} & \cdots & \underline{\underline{0}}_{ny} \end{bmatrix}$$

$$\underline{\underline{\bar{S}}}_{P} = \begin{bmatrix} \underline{\underline{S}}_{P1} & \underline{\underline{S}}_{P2} & \cdots & \underline{\underline{S}}_{PN} \end{bmatrix}^{T}$$

$$\underline{\underline{C}}\,\underline{\underline{\bar{S}}}_{P} = \begin{bmatrix} \underline{\underline{I}}_{ny} & \underline{\underline{0}}_{ny} & \cdots & \underline{\underline{0}}_{ny} \end{bmatrix} \begin{bmatrix} \underline{\underline{S}}_{P1} \\ \underline{\underline{S}}_{P2} \\ \vdots \\ \underline{\underline{S}}_{PN} \end{bmatrix} = \underline{\underline{S}}_{P1}$$

INTEGRADOR EM ESPAÇO DE ESTADO COM REALINHAMENTO

$$\hat{\underline{x}}^C_{k+1} = \underline{\underline{A}}\,\hat{\underline{x}}^C_k + \underline{\underline{B}}\Delta\underline{u}(k) + \underline{\underline{K}}_F\left\{\underline{\underline{C}}\left(\bar{\underline{y}}_k + \bar{\underline{\underline{S}}}_P\Delta\underline{u}(k)\right) - \underline{\underline{C}}\left(\hat{\underline{y}}^C_k + \bar{\underline{\underline{S}}}\Delta\underline{u}(k)\right)\right\}$$

$$\hat{\underline{x}}^C_{k+1} = \underline{\underline{A}}\,\hat{\underline{x}}^C_k + \underline{\underline{B}}\Delta\underline{u}(k) + \underline{\underline{K}}_F\left\{\underline{\underline{C}}\,\bar{\underline{y}}_k + \underline{\underline{C}}\,\bar{\underline{\underline{S}}}_P\Delta\underline{u}(k) - \underline{\underline{C}}\,\hat{\underline{y}}^C_k - \underline{\underline{C}}\,\bar{\underline{\underline{S}}}\Delta\underline{u}(k)\right\}$$

$$\underline{\underline{C}} = \begin{bmatrix} \underline{\underline{I}}_{ny} & \underline{\underline{0}}_{ny} & \cdots & \underline{\underline{0}}_{ny} \end{bmatrix}$$

$$\hat{\underline{y}}^C_k = \begin{bmatrix} \hat{\underline{y}}^C_k & \hat{\underline{y}}^C_{k+1} & \cdots & \hat{\underline{y}}^C_{k+np-1} \end{bmatrix}^T$$

$$\underline{\underline{C}}\,\hat{\underline{y}}^C_k = \begin{bmatrix} \underline{\underline{I}}_{ny} & \underline{\underline{0}}_{ny} & \cdots & \underline{\underline{0}}_{ny} \end{bmatrix} \begin{bmatrix} \hat{\underline{y}}^C_k \\ \hat{\underline{y}}^C_{k+1} \\ \vdots \\ \hat{\underline{y}}^C_{k+np-1} \end{bmatrix} = \hat{\underline{y}}^C_k$$

INTEGRADOR EM ESPAÇO DE ESTADO COM REALINHAMENTO

$$\underline{\hat{y}}^{C}_{k+1} = \underline{\hat{x}}^{C}_{k+1} = \underline{\underline{A}}\,\underline{\hat{x}}^{C}_{k} + \underline{\underline{B}}\Delta\underline{u}(k) + \underline{\underline{K}}_{F}\left\{\underline{\underline{C}}\left(\underline{\bar{y}}_{k} + \underline{\underline{\bar{S}}}_{P}\Delta\underline{u}(k)\right) - \underline{\underline{C}}\left(\underline{\hat{y}}^{C}_{k} + \underline{\underline{\bar{S}}}\Delta\underline{u}(k)\right)\right\}$$

$$\underline{\hat{x}}^{C}_{k+1} = \underline{\underline{A}}\,\underline{\hat{x}}^{C}_{k} + \underline{\underline{B}}\Delta\underline{u}(k) + \underline{\underline{K}}_{F}\left\{\underline{\underline{C}}\,\underline{\bar{y}}_{k} + \underline{\underline{C}}\,\underline{\underline{\bar{S}}}_{P}\Delta\underline{u}(k) - \underline{\underline{C}}\,\underline{\hat{y}}^{C}_{k} - \underline{\underline{C}}\,\underline{\underline{\bar{S}}}\Delta\underline{u}(k)\right\}$$

$$\underline{\underline{C}} = \begin{bmatrix} \underline{\underline{I}}_{ny} & \underline{\underline{0}}_{ny} & \cdots & \underline{\underline{0}}_{ny} \end{bmatrix}$$

$$\underline{\underline{\bar{S}}} = \begin{bmatrix} \underline{\underline{S}}_{1} & \underline{\underline{S}}_{2} & \cdots & \underline{\underline{S}}_{np} \end{bmatrix}^{T}$$

$$\underline{\underline{C}}\,\underline{\underline{\bar{S}}} = \begin{bmatrix} \underline{\underline{I}}_{ny} & \underline{\underline{0}}_{ny} & \cdots & \underline{\underline{0}}_{ny} \end{bmatrix} \begin{bmatrix} \underline{\underline{S}}_{1} \\ \underline{\underline{S}}_{2} \\ \vdots \\ \underline{\underline{S}}_{np} \end{bmatrix} = \underline{\underline{S}}_{1}$$

INTEGRADOR EM ESPAÇO DE ESTADO COM REALINHAMENTO

$$\hat{x}^{C}_{k+1} = A\hat{x}^{C}_{k} + B\Delta u(k) + K_F\left\{C\bar{y}_k + C\bar{S}_P\Delta u(k) - C\hat{y}^{C}_{k} + C\bar{S}\Delta u(k)\right\}$$

$$\hat{x}^{C}_{k+1} = A\hat{x}^{C}_{k} + B\Delta u(k) + K_F\left\{y_k + S_{P1}\Delta u(k) - \hat{y}^{C}_{k} - S_1\Delta u(k)\right\}$$

$$\hat{x}^{C}_{k+1} = A\hat{x}^{C}_{k} + B\Delta u(k) + K_F\left\{y_{k+1} - \hat{y}^{C}_{k} - S_1\Delta u(k)\right\}$$

VALOR REAL DA PLANTA

INTEGRADOR EM ESPAÇO DE ESTADO COM REALINHAMENTO

$$\underline{\hat{x}}^{C}_{k+1} = \underline{\underline{A}}\,\underline{\hat{x}}^{C}_{k} + \underline{\underline{B}}\Delta \underline{u}(k) + \underline{\underline{K}}_{F}\left\{\underline{y}_{k+1} - \underline{\hat{y}}^{C}_{k} - \underline{\underline{S}}_{1}\Delta \underline{u}(k)\right\}$$

PREDIÇÃO PARA O INSTANTE K+1

$$\underline{y}(k+1) = \underline{\underline{C}}\left[\underline{\underline{A}}\,\underline{x}(k) + \underline{\underline{B}}\Delta \underline{u}(k)\right]$$

$$\underline{\hat{x}}^{C}_{k+1} = \underline{\underline{A}}\,\underline{\hat{x}}^{C}_{k} + \underline{\underline{B}}\Delta \underline{u}(k) + \underline{\underline{K}}_{F}\left\{\underline{y}_{k+1} - \underline{\underline{C}}\left[\underline{\underline{A}}\,\underline{x}(k) + \underline{\underline{B}}\Delta \underline{u}(k)\right]\right\}$$

VOLTANDO PARA A NOTAÇÃO ESPAÇO DE ESTADO

INTEGRADOR EM ESPAÇO DE ESTADO COM REALINHAMENTO

$$\hat{\underline{x}}^{C}_{k+1} = \underline{\underline{A}}\,\hat{\underline{x}}^{C}_{k} + \underline{\underline{B}}\Delta\underline{u}(k) + \underline{\underline{K}}_F\left\{\underline{y}_{k+1} - \underline{\underline{C}}\left[\underline{\underline{A}}\,\underline{x}(k) + \underline{\underline{B}}\Delta\underline{u}(k)\right]\right\}$$

POLOS DENTRO DO CÍRCULO UNITÁRIO (ZERO)

$$\underline{x}(k+1) = \left[\underline{\underline{A}} - \underline{\underline{K}}_F\underline{\underline{C}}\,\underline{\underline{A}}\right]\underline{x}(k) + \left[\underline{\underline{B}} - \underline{\underline{K}}_F\underline{\underline{C}}\,\underline{\underline{B}}\right]\Delta\underline{u}(k) + \underline{\underline{K}}_F\,\underline{y}_{k+1}$$

$$\begin{bmatrix} \underline{y}(k+1) \\ \underline{y}(k) \\ \underline{y}(k-1) \end{bmatrix} \qquad \begin{bmatrix} 2_{ny} & -1_{ny} & 0_{ny} \\ 1_{ny} & 0_{ny} & 0_{ny} \\ 0_{ny} & 1_{ny} & 0_{ny} \end{bmatrix}$$

$$\underline{\underline{K}}_F = \begin{bmatrix} I_{ny} & 0_{ny} & 0_{ny} \end{bmatrix}^T$$

$$\underline{\underline{C}} = \begin{bmatrix} I_{ny} & 0_{ny} & 0_{ny} \end{bmatrix}$$

$$\underline{\underline{B}} = \begin{bmatrix} \underline{\underline{S}}_1 \\ 0_{ny} \\ 0_{ny} \end{bmatrix}$$

$$\hat{\underline{x}}^{C}_{k+1} = \underline{\underline{A}}\,\hat{\underline{x}}^{C}_{k} + \underline{\underline{B}}\Delta\underline{u}(k) + \underline{\underline{K}}_F\left\{\underline{y}_{k+1} - \underline{\underline{C}}\left[\underline{\underline{A}}\,\underline{x}(k) + \underline{\underline{B}}\Delta\underline{u}(k)\right]\right\}$$

$$\begin{bmatrix} \underline{y}(k+1) \\ \underline{y}(k) \\ \underline{y}(k-1) \end{bmatrix}$$

$$\begin{bmatrix} 2_{ny} & -1_{ny} & 0_{ny} \\ 1_{ny} & 0_{ny} & 0_{ny} \\ 0_{ny} & 1_{ny} & 0_{ny} \end{bmatrix}$$

$$\underline{\underline{K}}_F = \begin{bmatrix} I_{ny} & 0_{ny} & 0_{ny} \end{bmatrix}^T$$

$$\underline{\underline{C}} = \begin{bmatrix} I_{ny} & 0_{ny} & 0_{ny} \end{bmatrix}$$

$$\underline{\underline{B}} = \begin{bmatrix} \underline{\underline{S}}_1 \\ 0_{ny} \\ 0_{ny} \end{bmatrix}$$

$$\begin{bmatrix} \underline{y}(k+1) \\ \underline{y}(k) \\ \underline{y}(k-1) \end{bmatrix}_{k+1} = \begin{bmatrix} 2_{ny} & -1_{ny} & 0_{ny} \\ 1_{ny} & 0_{ny} & 0_{ny} \\ 0_{ny} & 1_{ny} & 0_{ny} \end{bmatrix} \begin{bmatrix} \underline{y}(k) \\ \underline{y}(k-1) \\ \underline{y}(k-2) \end{bmatrix}_{k} + \begin{bmatrix} \underline{\underline{S}}_1 \\ 0_{ny} \\ 0_{ny} \end{bmatrix} \Delta\underline{u}(k) +$$

$$+ \begin{bmatrix} I_{ny} \\ 0_{ny} \\ 0_{ny} \end{bmatrix} \left\{ \underline{y}_{k+1} - \begin{bmatrix} I_{ny} & 0_{ny} & 0_{ny} \end{bmatrix} \left[\begin{bmatrix} 2_{ny} & -1_{ny} & 0_{ny} \\ 1_{ny} & 0_{ny} & 0_{ny} \\ 0_{ny} & 1_{ny} & 0_{ny} \end{bmatrix} \begin{bmatrix} \underline{y}(k) \\ \underline{y}(k-1) \\ \underline{y}(k-2) \end{bmatrix}_{k} + \begin{bmatrix} \underline{\underline{S}}_1 \\ 0_{ny} \\ 0_{ny} \end{bmatrix} \Delta\underline{u}(k) \right] \right\}$$

INTEGRADOR EM ESPAÇO DE ESTADO COM REALINHAMENTO

$$\begin{bmatrix} \underline{y}(k+1) \\ \underline{y}(k) \\ \underline{y}(k-1) \end{bmatrix}_{k+1} = \begin{bmatrix} 2_{ny} & -1_{ny} & 0_{ny} \\ 1_{ny} & 0_{ny} & 0_{ny} \\ 0_{ny} & 1_{ny} & 0_{ny} \end{bmatrix} \begin{bmatrix} \underline{y}(k) \\ \underline{y}(k-1) \\ \underline{y}(k-2) \end{bmatrix}_{k} + \begin{bmatrix} \underline{\underline{S}}_1 \\ 0_{ny} \\ 0_{ny} \end{bmatrix} \Delta\underline{u}(k) +$$

$$+ \begin{bmatrix} I_{ny} \\ 0_{ny} \\ 0_{ny} \end{bmatrix} \left\{ \underline{y}_{k+1} - \begin{bmatrix} I_{ny} & 0_{ny} & 0_{ny} \end{bmatrix} \left[\begin{bmatrix} 2_{ny} & -1_{ny} & 0_{ny} \\ 1_{ny} & 0_{ny} & 0_{ny} \\ 0_{ny} & 1_{ny} & 0_{ny} \end{bmatrix} \begin{bmatrix} \underline{y}(k) \\ \underline{y}(k-1) \\ \underline{y}(k-2) \end{bmatrix}_{k} + \begin{bmatrix} \underline{\underline{S}}_1 \\ 0_{ny} \\ 0_{ny} \end{bmatrix} \Delta\underline{u}(k) \right] \right\}$$

$$\begin{bmatrix} \underline{y}(k+1) \\ \underline{y}(k) \\ \underline{y}(k-1) \end{bmatrix}_{k+1} = \begin{bmatrix} 2_{ny} & -1_{ny} & 0_{ny} \\ 1_{ny} & 0_{ny} & 0_{ny} \\ 0_{ny} & 1_{ny} & 0_{ny} \end{bmatrix} \begin{bmatrix} \underline{y}(k) \\ \underline{y}(k-1) \\ \underline{y}(k-2) \end{bmatrix}_{k} + \begin{bmatrix} \underline{\underline{S}}_1 \Delta\underline{u}(k) \\ 0_{ny} \\ 0_{ny} \end{bmatrix} + \begin{bmatrix} \underline{y}_{k+1} - 2_{ny}\underline{y}(k) + \underline{y}(k-1) - \underline{\underline{S}}_1 \Delta\underline{u}(k) \\ 0_{ny} \\ 0_{ny} \end{bmatrix}$$

INTEGRADOR EM ESPAÇO DE ESTADO COM REALINHAMENTO

$$\begin{bmatrix} \underline{y}(k+1) \\ \underline{y}(k) \\ \underline{y}(k-1) \end{bmatrix}_{k+1} = \begin{bmatrix} 2_{ny} & -1_{ny} & 0_{ny} \\ 1_{ny} & 0_{ny} & 0_{ny} \\ 0_{ny} & 1_{ny} & 0_{ny} \end{bmatrix} \begin{bmatrix} \underline{y}(k) \\ \underline{y}(k-1) \\ \underline{y}(k-2) \end{bmatrix}_{k} + \begin{bmatrix} \underline{\underline{S}}_1 \Delta \underline{u}(k) \\ 0_{ny} \\ 0_{ny} \end{bmatrix} + \begin{bmatrix} \underline{y}_{k+1} - 2_{ny}\,\underline{y}(k) + \underline{y}(k-1) - \underline{\underline{S}}_1 \Delta \underline{u}(k) \\ 0_{ny} \\ 0_{ny} \end{bmatrix}$$

$$\begin{bmatrix} \underline{y}(k+1) \\ \underline{y}(k) \\ \underline{y}(k-1) \end{bmatrix}_{k+1} = \begin{bmatrix} 2_{ny}\,\underline{y}(k) - \underline{y}(k-1) \\ \underline{y}(k) \\ \underline{y}(k-1) \end{bmatrix} + \begin{bmatrix} \underline{y}_{k+1} - 2_{ny}\,\underline{y}(k) + \underline{y}(k-1) \\ 0_{ny} \\ 0_{ny} \end{bmatrix}$$

$$\begin{bmatrix} \underline{y}(k+1) \\ \underline{y}(k) \\ \underline{y}(k-1) \end{bmatrix}_{k+1} = \begin{bmatrix} \underline{y}_{k+1} \\ \underline{y}(k) \\ \underline{y}(k-1) \end{bmatrix}$$

Com o observador escolhido e o vetor de deslocamento, o vetor de estados é atualizado com os dados da planta, equivalente ao modelo de realinhamento

MODELO DE REALINHAMENTO SISTEMA DE ORDEM GENÉRICA NA E NB

MPC com Modelo em Espaço de Estados

- Um modelo pode ser escrito no domínio discreto z

$$\frac{Y(z)}{U(z)} = \frac{\sum_{i=1}^{i=nb} b_i z^{-i}}{\sum_{i=0}^{i=na} a_i z^{-i}}$$

$$Y(z)\left(1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_{na} z^{-na}\right) = \left(b_1 z^{-1} + b_2 z^{-2} + \ldots b_{nb} z^{-nb}\right) U(z)$$

$$Y(z) + a_1 z^{-1} Y(z) + a_2 z^{-2} Y(z) \ldots + a_{na} z^{-na} Y(z) = b_1 z^{-1} U(z) + b_2 z^{-2} U(z) + \ldots b_{nb} z^{-nb} U(z)$$

MPC com Modelo em Espaço de Estados

- Transformando em equações de diferenças

$$Y(z)+a_1 z^{-1}Y(z)+a_2 z^{-2}Y(z)\ldots+a_{na}z^{-na}Y(z)=b_1 z^{-1}U(z)+b_2 z^{-2}U(z)+\ldots b_{nb}z^{-nb}U(z)$$

$$y(k)+a_1 y(k-1)+a_2 y(k-2)\ldots+a_{na}y(k-na)=b_1 u(k-1)+b_2 u(k-2)+\ldots b_{nb}u(k-nb)$$

$$\underline{y}(k)+\sum_{i=1}^{na}\underline{\underline{A}}_i\,\underline{y}(k-i)=\sum_{i=1}^{nb}\underline{\underline{B}}_i\,\underline{u}(k-i)$$

MPC com Modelo em Espaço de Estados

- Definindo como estado não mínimo:

$$\underline{x}(k) = \left[\underline{y}(k) \quad \underline{y}(k-1) \quad \cdots \quad \underline{y}(k-na+1) \quad \underline{u}(k-1) \quad \cdots \quad \underline{u}(k-nb+1) \right]^T$$

- Predição para um instante genérico *k*

$$\underline{y}(k) = -\underline{\underline{A}}_1 \underline{y}(k-1) - \underline{\underline{A}}_2 y(k-2) \ldots - \underline{\underline{A}}_{na} \underline{y}(k-na) + \underline{\underline{B}}_1 \underline{u}(k-1) + \underline{\underline{B}}_2 \underline{u}(k-2) + \ldots \underline{\underline{B}}_{nb} \underline{u}(k-nb)$$

MPC com Modelo em Espaço de Estados

- As leituras do passado podem ser obtidas a partir das informações disponíveis da planta

$$
\begin{bmatrix} \underline{y}(k) \\ \underline{y}(k-1) \\ \underline{y}(k-2) \\ \vdots \\ \underline{y}(k-na+1) \\ \underline{u}(k-1) \\ \underline{u}(k-2) \\ \vdots \\ \underline{u}(k-nb+1) \end{bmatrix}_k =
\begin{bmatrix}
-\underline{\underline{A}}_1 & -\underline{\underline{A}}_2 & -\underline{\underline{A}}_3 & \cdots & -\underline{\underline{A}}_{na-1} & -\underline{\underline{A}}_{na} & \underline{\underline{B}}_2 & \cdots & \underline{\underline{B}}_{nb-2} & \underline{\underline{B}}_{nb-1} & \underline{\underline{B}}_{nb} \\
\underline{\underline{I}} & \underline{\underline{0}} & \underline{\underline{0}} & \cdots & \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} & \vdots & \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} \\
\underline{\underline{0}} & \underline{\underline{I}} & \underline{\underline{0}} & \cdots & \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} & \vdots & \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
\underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} & \cdots & \underline{\underline{I}} & \underline{\underline{0}} & \underline{\underline{0}} & \cdots & \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} \\
\underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} & \cdots & \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} & \cdots & \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} \\
\underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} & \cdots & \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{I}} & \cdots & \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
\underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} & \cdots & \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} & \cdots & \underline{\underline{0}} & \underline{\underline{I}} & \underline{\underline{0}}
\end{bmatrix}
\begin{bmatrix} \underline{y}(k-1) \\ \underline{y}(k-2) \\ \underline{y}(k-3) \\ \vdots \\ \underline{y}(k-na) \\ \underline{u}(k-2) \\ \underline{u}(k-3) \\ \vdots \\ \underline{u}(k-nb) \end{bmatrix}_{k-1} +
\begin{bmatrix} \underline{\underline{B}}_1 \\ \underline{\underline{0}} \\ \underline{\underline{0}} \\ \vdots \\ \underline{\underline{0}} \\ \underline{\underline{I}} \\ \underline{\underline{0}} \\ \vdots \\ \underline{\underline{0}} \end{bmatrix} \underline{u}(k-1)
$$

MPC com Modelo em Espaço de Estados

- As leituras do passado podem ser obtidas a partir das informações disponíveis da planta

$$
\begin{bmatrix} \underline{y}(k) \\ \underline{y}(k-1) \\ \underline{y}(k-2) \\ \vdots \\ \underline{y}(k-na+1) \\ \underline{u}(k-1) \\ \underline{u}(k-2) \\ \vdots \\ \underline{u}(k-nb+1) \end{bmatrix}_k
=
\begin{bmatrix}
-\underline{\underline{A}}_1 & -\underline{\underline{A}}_2 & -\underline{\underline{A}}_3 & \cdots & -\underline{\underline{A}}_{na-1} & -\underline{\underline{A}}_{na} & \underline{\underline{B}}_2 & \cdots & \underline{\underline{B}}_{nb-2} & \underline{\underline{B}}_{nb-1} & \underline{\underline{B}}_{nb} \\
\underline{\underline{I}} & \underline{\underline{0}} & \underline{\underline{0}} & \cdots & \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} & \vdots & \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} \\
\underline{\underline{0}} & \underline{\underline{I}} & \underline{\underline{0}} & \cdots & \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} & \vdots & \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
\underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} & \cdots & \underline{\underline{I}} & \underline{\underline{0}} & \underline{\underline{0}} & \cdots & \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} \\
\underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} & \cdots & \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} & \cdots & \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} \\
\underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} & \cdots & \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{I}} & \cdots & \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
\underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} & \cdots & \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} & \cdots & \underline{\underline{0}} & \underline{\underline{I}} & \underline{\underline{0}}
\end{bmatrix}
\begin{bmatrix} \underline{y}(k-1) \\ \underline{y}(k-2) \\ \underline{y}(k-3) \\ \vdots \\ \underline{y}(k-na) \\ \underline{u}(k-2) \\ \underline{u}(k-3) \\ \vdots \\ \underline{u}(k-nb) \end{bmatrix}_{k-1}
+
\begin{bmatrix} \underline{\underline{B}}_1 \\ \underline{\underline{0}} \\ \underline{\underline{0}} \\ \vdots \\ \underline{\underline{0}} \\ \underline{\underline{I}} \\ \underline{\underline{0}} \\ \vdots \\ \underline{\underline{0}} \end{bmatrix}
\underline{u}(k-1)
$$

MPC com Modelo em Espaço de Estados

- As leituras do passado podem ser obtidas a partir das informações disponíveis da planta

$$
\begin{bmatrix} \underline{y}(k) \\ \underline{y}(k-1) \\ \underline{y}(k-2) \\ \vdots \\ \underline{y}(k-na+1) \\ \underline{u}(k-1) \\ \underline{u}(k-2) \\ \vdots \\ \underline{u}(k-nb+1) \end{bmatrix}_k =
\begin{bmatrix}
-\underline{\underline{A}}_1 & -\underline{\underline{A}}_2 & -\underline{\underline{A}}_3 & \cdots & -\underline{\underline{A}}_{na-1} & -\underline{\underline{A}}_{na} & \underline{\underline{B}}_2 & \cdots & \underline{\underline{B}}_{nb-2} & \underline{\underline{B}}_{nb-1} & \underline{\underline{B}}_{nb} \\
\underline{\underline{I}} & \underline{\underline{0}} & \underline{\underline{0}} & \cdots & \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} & \vdots & \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} \\
\underline{\underline{0}} & \underline{\underline{I}} & \underline{\underline{0}} & \cdots & \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} & \vdots & \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
\underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} & \cdots & \underline{\underline{I}} & \underline{\underline{0}} & \underline{\underline{0}} & \cdots & \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} \\
\underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} & \cdots & \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} & \cdots & \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} \\
\underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} & \cdots & \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{I}} & \cdots & \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
\underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} & \cdots & \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{0}} & \cdots & \underline{\underline{0}} & \underline{\underline{I}} & \underline{\underline{0}}
\end{bmatrix}
\begin{bmatrix} \underline{y}(k-1) \\ \underline{y}(k-2) \\ \underline{y}(k-3) \\ \vdots \\ \underline{y}(k-na) \\ \underline{u}(k-2) \\ \underline{u}(k-3) \\ \vdots \\ \underline{u}(k-nb) \end{bmatrix}_{k-1} +
\begin{bmatrix} \underline{\underline{B}}_1 \\ \underline{\underline{0}} \\ \underline{\underline{0}} \\ \vdots \\ \underline{\underline{0}} \\ \underline{\underline{I}} \\ \underline{\underline{0}} \\ \vdots \\ \underline{\underline{0}} \end{bmatrix} \underline{u}(k-1)
$$

MODELO POSICIONAL

$$\underline{x}(k) = \underline{\underline{A}}\,\underline{x}(k-1) + \underline{\underline{B}}\,\underline{u}(k-1)$$

MODELO INCREMENTAL

MPC com Modelo em Espaço de Estados na forma incremental

$$\begin{cases} \underline{x}(k+1) = \underline{\underline{A}}\,\underline{x}(k) + \underline{\underline{B}}\,\underline{u}(k) \\ \underline{y}(k) = \underline{\underline{C}}\,\underline{x}(k) \end{cases}$$

$$\begin{cases} \underline{\tilde{x}}(k+1) = \underline{\underline{\tilde{A}}}\,\underline{\tilde{x}}(k) + \underline{\underline{\tilde{B}}}\,\Delta\underline{u}(k) \\ \underline{y}(k) = \underline{\underline{\tilde{C}}}\,\underline{\tilde{x}}(k) \end{cases}$$

FORMAS DE OBTENÇÃO DE UM MODELO INCREMENTAL A PARTIR DE UM POSICIONAL

Primeira forma

$$\underline{x}(k+1) = \underline{\underline{A}}\,\underline{x}(k) + \underline{\underline{B}}\,\underline{u}(k)$$

$$\underline{x}(k) = \underline{\underline{A}}\,\underline{x}(k-1) + \underline{\underline{B}}\,\underline{u}(k-1)$$

$\downarrow \Delta$

$$\underline{x}(k+1) = \underline{\underline{B}}\left[\Delta \underline{u}(k)\right] + \left[\underline{\underline{I}} + \underline{\underline{A}}\right]\underline{x}(k) - \underline{\underline{A}}\,\underline{x}(k-1)$$

$$\tilde{\underline{x}}_1(k) = \underline{x}(k) \qquad \tilde{\underline{x}}_2(k) = \left[\underline{\underline{I}} + \underline{\underline{A}}\right]\underline{x}(k) - \underline{\underline{A}}\,\underline{x}(k-1)$$

Primeira forma

$$\underline{x}(k+1) = \underline{\underline{B}}\left[\Delta \underline{u}(k)\right] + \left[\underline{\underline{I}} + \underline{\underline{A}}\right]\underline{x}(k) - \underline{\underline{A}}\,\underline{x}(k-1)$$

$$\tilde{\underline{x}}_1(k) = \underline{x}(k)$$

$$\tilde{\underline{x}}_2(k) = \left[\underline{\underline{I}} + \underline{\underline{A}}\right]\underline{x}(k) - \underline{\underline{A}}\,\underline{x}(k-1)$$

$$\tilde{\underline{x}}_1(k+1) = \underline{\underline{B}}\left[\Delta \underline{u}(k)\right] + \tilde{x}_2(k)$$

$$\tilde{\underline{x}}_2(k+1) = \left[\underline{\underline{I}} + \underline{\underline{A}}\right]\underline{x}(k+1) - \underline{\underline{A}}\,\underline{x}(k)$$

$$\tilde{\underline{x}}_2(k+1) = \left[\underline{\underline{I}} + \underline{\underline{A}}\right]\left\{\underline{\underline{B}}\left[\Delta \underline{u}(k)\right] + \tilde{x}_2(k)\right\} - \underline{\underline{A}}\,\tilde{\underline{x}}_1(k)$$

Primeira forma

$$\underline{\tilde{x}}_1(k+1) = \underline{\underline{B}}\left[\Delta \underline{u}(k)\right] + \tilde{x}_2(k)$$

$$\underline{\tilde{x}}_1(k) = \underline{x}(k)$$

$$\underline{\tilde{x}}_2(k+1) = \left[\underline{\underline{I}} + \underline{\underline{A}}\right]\left\{\underline{\underline{B}}\left[\Delta \underline{u}(k)\right] + \tilde{x}_2(k)\right\} - \underline{\underline{A}}\,\underline{\tilde{x}}_1(k)$$

NOTAÇÃO MATRICIAL

$$\begin{bmatrix} \tilde{x}_1(k+1) \\ \tilde{x}_2(k+1) \end{bmatrix} = \begin{bmatrix} \underline{\underline{0}} & \underline{\underline{I}} \\ -\underline{\underline{A}} & \underline{\underline{I}} + \underline{\underline{A}} \end{bmatrix} \begin{bmatrix} \tilde{x}_1(k) \\ \tilde{x}_2(k) \end{bmatrix} + \begin{bmatrix} \underline{\underline{B}} \\ \left(\underline{\underline{I}} + \underline{\underline{A}}\right)\underline{\underline{B}} \end{bmatrix} \Delta \underline{u}(k)$$

$$\underline{y} = \underline{\underline{C}}\,\underline{x(k) = C}\left[\underline{\underline{I}} \quad \underline{\underline{0}}\right] \begin{bmatrix} \underline{\tilde{x}}_1(k) \\ \underline{\tilde{x}}_2(k) \end{bmatrix} = \underline{\underline{\tilde{C}}}\,\underline{\tilde{x}}(k)$$

Primeira forma

$$\begin{bmatrix} \tilde{x}_1(k+1) \\ \tilde{x}_2(k+1) \end{bmatrix} = \begin{bmatrix} \underline{\underline{0}} & \underline{\underline{I}} \\ -\underline{\underline{A}} & \underline{\underline{I}} + \underline{\underline{A}} \end{bmatrix} \begin{bmatrix} \tilde{x}_1(k) \\ \tilde{x}_2(k) \end{bmatrix} + \begin{bmatrix} \underline{\underline{B}} \\ (\underline{\underline{I}} + \underline{\underline{A}})\underline{\underline{B}} \end{bmatrix} \Delta \underline{u}(k)$$

MESMOS AUTOVALORES DE A E MAIS NX AUTOVALORES EM 1

Segunda forma

$$\Delta \underline{u}(k) = \underline{u}(k) - \underline{u}(k-1) \Rightarrow \underline{u}(k) = \underline{u}(k-1) + \Delta \underline{u}(k)$$

$$\underline{x}(k+1) = \underline{\underline{A}}\,\underline{x}(k) + \underline{\underline{B}}\left[\underline{u}(k-1) + \Delta \underline{u}(k)\right]$$

$$\begin{bmatrix} \underline{x}(k+1) \\ \underline{u}(k) \end{bmatrix} = \begin{bmatrix} \underline{\underline{A}} & \underline{\underline{B}} \\ \underline{\underline{0}} & \underline{\underline{I}} \end{bmatrix} \begin{bmatrix} \underline{x}(k) \\ \underline{u}(k-1) \end{bmatrix} + \begin{bmatrix} \underline{\underline{B}} \\ \underline{\underline{I}} \end{bmatrix} \Delta \underline{u}(k)$$

$$\underline{\tilde{x}}(k+1) = \underline{\underline{\tilde{A}}}\,\underline{\tilde{x}}(k) + \underline{\underline{\tilde{B}}}\Delta \underline{u}(k)$$

Segunda forma

$$\underline{y} = \underline{\underline{C}} \begin{bmatrix} \underline{\underline{I}} & \underline{\underline{0}} \end{bmatrix} \begin{bmatrix} \underline{x}(k) \\ \underline{u}(k) \end{bmatrix} = \begin{bmatrix} \underline{\underline{C}} & \underline{\underline{0}} \end{bmatrix} \begin{bmatrix} \underline{x}(k) \\ \underline{u}(k) \end{bmatrix}$$

Segunda forma

$$\begin{bmatrix} \underline{x}(k+1) \\ \underline{u}(k) \end{bmatrix} = \begin{bmatrix} \underline{\underline{A}} & \underline{\underline{B}} \\ \underline{\underline{0}} & \underline{\underline{I}} \end{bmatrix} \begin{bmatrix} \underline{x}(k) \\ \underline{u}(k-1) \end{bmatrix} + \begin{bmatrix} \underline{\underline{B}} \\ \underline{\underline{I}} \end{bmatrix} \Delta \underline{u}(k)$$

MESMOS AUTOVALORES DE A E MAIS NX+NU AUTOVALORES EM 1

APLICAÇÃO DA SEGUNDA FORMA

MPC com Modelo em Espaço de Estados na forma incremental

$$y(k+1)+a_1 y(k)+a_2 y(k-1)+a_3 y(k-2)+\ldots+a_{na} y(k-na+1)=$$
$$b_1 u(k)+b_2 u(k-1)+b_3 u(k-2)+\ldots+b_{nb} u(k-nb+1)$$

$$y(k)+a_1 y(k-1)+a_2 y(k-2)+a_3 y(k-3)+\ldots+a_{na} y(k-na)=$$
$$b_1 u(k-1)+b_2 u(k-2)+b_3 u(k-3)+\ldots+b_{nb} u(k-nb)$$

$$y(k+1)=y(k)\left(I_{ny}-a_1\right)+y(k-1)\left(-a_2+a_1\right)+y(k-2)\left(-a_3+a_2\right)+\ldots+$$
$$+y(k-na+1)\left(-a_{na}+a_{na-1}\right)+a_{na} y(k-na)+$$
$$+b_1 \Delta u(k)+b_2 \Delta u(k-1)+b_3 \Delta u(k-2)+\ldots+b_{nb} \Delta u(k-nb+1)$$

MPC com Modelo em Espaço de Estados

- As leituras do passado podem ser obtidas a partir das informações disponíveis da planta

$$
\begin{bmatrix} y(k+1) \\ y(k) \\ y(k-1) \\ \vdots \\ y(k-na+1) \\ \Delta u(k) \\ \vdots \\ \Delta u(k-nb+2) \end{bmatrix}
=
\begin{bmatrix}
-a_1+I_{ny} & -a_2+a_1 & -a_3+a_2 & -a_4+a_3 & \cdots & -a_{na}+a_{na-1} & a_{na} & b_2 & b_3 & b_4 & \cdots & b_{nb} \\
I_{ny} & 0_{ny} & 0_{ny} & 0_{ny} & \cdots & 0_{ny} & 0_{ny} & 0_{ny\times nu} & 0_{ny\times nu} & 0_{ny\times nu} & \cdots & 0_{ny\times nu} \\
0_{ny} & I_{ny} & 0_{ny} & 0_{ny} & \cdots & 0_{ny} & 0_{ny} & 0_{ny\times nu} & 0_{ny\times nu} & 0_{ny\times nu} & \cdots & 0_{ny\times nu} \\
0_{ny} & 0_{ny} & I_{ny} & 0_{ny} & \cdots & 0_{ny} & 0_{ny} & 0_{ny\times nu} & 0_{ny\times nu} & 0_{ny\times nu} & \cdots & 0_{ny\times nu} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0_{ny} & 0_{ny} & 0_{ny} & 0_{ny} & \cdots & I_{ny} & 0_{ny} & 0_{ny\times nu} & 0_{ny\times nu} & 0_{ny\times nu} & 0_{ny\times nu} & 0_{ny\times nu} \\
0_{nu\times ny} & 0_{nu\times ny} & 0_{nu\times ny} & 0_{nu\times ny} & \cdots & 0_{nu\times ny} & 0_{nu\times ny} & 0_{nu} & 0_{nu} & 0_{nu} & 0_{nu} & 0_{nu} \\
0_{nu\times ny} & 0_{nu\times ny} & 0_{nu\times ny} & 0_{nu\times ny} & \cdots & 0_{nu\times ny} & 0_{nu\times ny} & I_{nu} & 0_{nu} & 0_{nu} & 0_{nu} & 0_{nu} \\
0_{nu\times ny} & 0_{nu\times ny} & 0_{nu\times ny} & 0_{nu\times ny} & \cdots & 0_{nu\times ny} & 0_{nu\times ny} & 0_{nu} & I_{nu} & 0_{nu} & 0_{nu} & 0_{nu} \\
0_{nu\times ny} & 0_{nu\times ny} & 0_{nu\times ny} & 0_{nu\times ny} & \cdots & 0_{nu\times ny} & 0_{nu\times ny} & 0_{nu} & 0_{nu} & I_{nu} & 0_{nu} & 0_{nu} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0_{nu\times ny} & 0_{nu\times ny} & 0_{nu\times ny} & 0_{nu\times ny} & \cdots & 0_{nu\times ny} & 0_{nu\times ny} & 0_{nu} & 0_{nu} & 0_{nu} & I_{nu} & 0_{nu}
\end{bmatrix}
\begin{bmatrix} y(k) \\ y(k-1) \\ \vdots \\ y(k-na+1) \\ y(k-na) \\ \Delta u(k-1) \\ \vdots \\ \Delta u(k-nb+2) \\ \Delta u(k-nb+1) \end{bmatrix}
+
\begin{bmatrix} b_1 \\ 0_{ny\times nu} \\ 0_{ny\times nu} \\ 0_{ny\times nu} \\ \cdots \\ 0_{ny\times nu} \\ I_{nu} \\ 0_{nu} \\ 0_{nu} \\ 0_{nu} \\ \cdots \\ 0_{nu} \end{bmatrix}
\Delta u(k)
$$

MPC com Modelo em Espaço de Estados

- As leituras do passado podem ser obtidas a partir das informações disponíveis da planta

$$
\begin{bmatrix} y(k+1) \\ y(k) \\ y(k-1) \\ \vdots \\ y(k-na+1) \\ \Delta u(k) \\ \vdots \\ \Delta u(k-nb+2) \end{bmatrix}
=
\begin{bmatrix}
-a_1+I_{ny} & -a_2+a_1 & -a_3+a_2 & -a_4+a_3 & & -a_{na}+a_{na-1} & a_{na} & b_2 & b_3 & b_4 & & b_{nb} \\
I_{ny} & 0_{ny} & 0_{ny} & 0_{ny} & \cdots & 0_{ny} & 0_{ny} & 0_{ny\times nu} & 0_{ny\times nu} & 0_{ny\times nu} & \cdots & 0_{ny\times nu} \\
0_{ny} & I_{ny} & 0_{ny} & 0_{ny} & \cdots & 0_{ny} & 0_{ny} & 0_{ny\times nu} & 0_{ny\times nu} & 0_{ny\times nu} & \cdots & 0_{ny\times nu} \\
0_{ny} & 0_{ny} & I_{ny} & 0_{ny} & \cdots & 0_{ny} & 0_{ny} & 0_{ny\times nu} & 0_{ny\times nu} & 0_{ny\times nu} & \cdots & 0_{ny\times nu} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0_{ny} & 0_{ny} & 0_{ny} & 0_{ny} & \cdots & I_{ny} & 0_{ny} & 0_{ny\times nu} & 0_{ny\times nu} & 0_{ny\times nu} & 0_{ny\times nu} & 0_{ny\times nu} \\
0_{nu\times ny} & 0_{nu\times ny} & 0_{nu\times ny} & 0_{nu\times ny} & \cdots & 0_{nu\times ny} & 0_{nu\times ny} & 0_{nu} & 0_{nu} & 0_{nu} & 0_{nu} & 0_{nu} \\
0_{nu\times ny} & 0_{nu\times ny} & 0_{nu\times ny} & 0_{nu\times ny} & \cdots & 0_{nu\times ny} & 0_{nu\times ny} & I_{nu} & 0_{nu} & 0_{nu} & 0_{nu} & 0_{nu} \\
0_{nu\times ny} & 0_{nu\times ny} & 0_{nu\times ny} & 0_{nu\times ny} & \cdots & 0_{nu\times ny} & 0_{nu\times ny} & 0_{nu} & I_{nu} & 0_{nu} & 0_{nu} & 0_{nu} \\
0_{nu\times ny} & 0_{nu\times ny} & 0_{nu\times ny} & 0_{nu\times ny} & \cdots & 0_{nu\times ny} & 0_{nu\times ny} & 0_{nu} & 0_{nu} & I_{nu} & 0_{nu} & 0_{nu} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0_{nu\times ny} & 0_{nu\times ny} & 0_{nu\times ny} & 0_{nu\times ny} & \cdots & 0_{nu\times ny} & 0_{nu\times ny} & 0_{nu} & 0_{nu} & 0_{nu} & I_{nu} & 0_{nu}
\end{bmatrix}
\begin{bmatrix} y(k) \\ y(k-1) \\ \vdots \\ y(k-na+1) \\ y(k-na) \\ \Delta u(k-1) \\ \vdots \\ \Delta u(k-nb+2) \\ \Delta u(k-nb+1) \end{bmatrix}
+
\begin{bmatrix} b_1 \\ 0_{ny\times nu} \\ 0_{ny\times nu} \\ 0_{ny\times nu} \\ \cdots \\ 0_{ny\times nu} \\ I_{nu} \\ 0_{nu} \\ 0_{nu} \\ 0_{nu} \\ \cdots \\ 0_{nu} \end{bmatrix}
\Delta u(k)
$$

MODELO INCREMENTAL

$$\underline{x}(k+1) = \underline{\underline{A}}\,\underline{x}(k) + \underline{\underline{B}}\,\Delta\underline{u}(k)$$

PREDIÇÃO DO MODELO EM ESPAÇO DE ESTADO

MPC com Modelo em Espaço de Estados na forma incremental

- Predição para o horizonte *np*

$$\begin{bmatrix} \underline{y}(k+1) \\ \underline{y}(k+2) \\ \underline{y}(k+3) \\ \vdots \\ \underline{y}(k+np) \end{bmatrix} = \begin{bmatrix} \underline{\underline{C}}\,\underline{\underline{A}} \\ \underline{\underline{C}}\,\underline{\underline{A}}^2 \\ \underline{\underline{C}}\,\underline{\underline{A}}^3 \\ \vdots \\ \underline{\underline{C}}\,\underline{\underline{A}}^{np} \end{bmatrix} \underline{x}(k) + \begin{bmatrix} \underline{\underline{C}}\,\underline{\underline{B}} & 0 & 0 & \cdots & 0 \\ \underline{\underline{C}}\,\underline{\underline{A}}\,\underline{\underline{B}} & \underline{\underline{C}}\,\underline{\underline{B}} & 0 & \cdots & 0 \\ \underline{\underline{C}}\,\underline{\underline{A}}^2\,\underline{\underline{B}} & \underline{\underline{C}}\,\underline{\underline{A}}\,\underline{\underline{B}} & \underline{\underline{C}}\,\underline{\underline{B}} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \underline{\underline{C}}\,\underline{\underline{A}}^{np-1}\,\underline{\underline{B}} & \underline{\underline{C}}\,\underline{\underline{A}}^{np-2}\,\underline{\underline{B}} & \underline{\underline{C}}\,\underline{\underline{A}}^{np-3}\,\underline{\underline{B}} & \cdots & \underline{\underline{C}}\,\underline{\underline{A}}^{np-m}\,\underline{\underline{B}} \end{bmatrix} \Delta\underline{u}$$

MPC com Modelo em Espaço de Estados na forma incremental

- Predição para o horizonte *np*

$$\begin{bmatrix} \underline{y}(k+1) \\ \underline{y}(k+2) \\ \underline{y}(k+3) \\ \vdots \\ \underline{y}(k+np) \end{bmatrix} = \begin{bmatrix} \underline{\underline{C}}\,\underline{\underline{A}} \\ \underline{\underline{C}}\,\underline{\underline{A}}^2 \\ \underline{\underline{C}}\,\underline{\underline{A}}^3 \\ \vdots \\ \underline{\underline{C}}\,\underline{\underline{A}}^{np} \end{bmatrix} \underline{x}(k) + \begin{bmatrix} \underline{\underline{C}}\,\underline{\underline{B}} & 0 & 0 & \cdots & 0 \\ \underline{\underline{C}}\,\underline{\underline{A}}\,\underline{\underline{B}} & \underline{\underline{C}}\,\underline{\underline{B}} & 0 & \cdots & 0 \\ \underline{\underline{C}}\,\underline{\underline{A}}^2\,\underline{\underline{B}} & \underline{\underline{C}}\,\underline{\underline{A}}\,\underline{\underline{B}} & \underline{\underline{C}}\,\underline{\underline{B}} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \underline{\underline{C}}\,\underline{\underline{A}}^{np-1}\,\underline{\underline{B}} & \underline{\underline{C}}\,\underline{\underline{A}}^{np-2}\,\underline{\underline{B}} & \underline{\underline{C}}\,\underline{\underline{A}}^{np-3}\,\underline{\underline{B}} & \cdots & \underline{\underline{C}}\,\underline{\underline{A}}^{np-m}\,\underline{\underline{B}} \end{bmatrix} \Delta\underline{u}$$

Melhorias no MPC

- Redução dos parâmetros de sintonia
- Robustez quanto a estabilidade
  - o modelo real da planta não coincidir com o modelo previsto no controlador, que é considerado o caso nominal;
  - quando uma saída do processo se tornar ativa ou inativa no controle de faixas;
  - quando uma entrada do processo comutar da condição de restrição para a condição de não restrição, ou vice-versa;

Melhorias no MPC

- Portanto, a robustez quanto à estabilidade deve ser analisada em 3 condições distintas
  - Chaveamento das variáveis controladas da situação ativa para a situação inativa, ou da condição inativa para a condição ativa
  - Chaveamento das entradas da situação disponível para a situação indisponível ou da situação indisponível para a situação disponível
  - Incerteza de modelo – um controlador sintonizado para a condição nominal é robusto para variações em torno de 20% do modelo esperado pelo controlador. Além disso, a estabilidade pode ficar comprometida;

Controladores Nominalmente Estáveis

- A literatura fornece diversos controladores nominalmente estáveis, mas, devido a incertezas de modelo ou restrições nas entradas de processo, tornam-se instáveis;

Como a estabilidade de um controlador pode ser garantida para o caso nominal?

Controladores Nominalmente Estáveis

- Introdução de restrições que garantam que o estado final do sistema seja nulo.

$$\begin{cases} \underline{x}(k+1) = \underline{\underline{A}}\,\underline{x}(k) + \underline{\underline{B}}\,\underline{u}(k) \\ \underline{y}(k) = \underline{\underline{C}}\,\underline{x}(k) \end{cases}$$

⇩

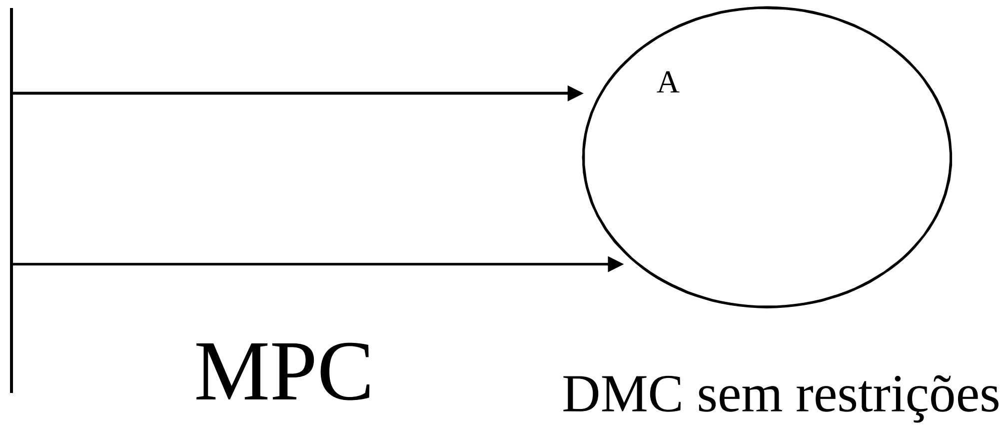
$$\underline{x}(k+np) = 0$$

Contração de estados

VÁLIDA APENAS PARA O PROBLEMA REGULATÓRIO

Controladores Nominalmente Estáveis

- Introdução de restrições que levem o estado final a um conjunto de estados onde existe um controlador estável.



ESSE CONTROLADOR ESTÁVEL PRESSUPÕE NÃO TER RESTRIÇÃO O QUE NÃO ACONTECE NA PRÁTICA

ESTABILIDADE VIA FUNÇÃO DE LYAPUNOV

Função de Lyapunov

- Seja uma função F tal que:

$$x(k+1) = f\left(x(k)\right)$$

- A função *V(x)* >0 é dita função de Lyapunov se:

$$\|x(1)\| \geq \|x(2)\| \geq \|x(3)\| \ldots$$
$$\downarrow$$
$$V(x_1) \geq V(x_2) \geq V(x_3) \ldots$$

$$V(x) = 0 \Leftrightarrow x = 0$$

Função de Lyapunov

- Nesse caso, pode-se afirmar que:

$$k \rightarrow \infty$$

$$\Downarrow$$

$$x(k) \rightarrow 0$$

Controladores Nominalmente Estáveis

- Uso do controlador de horizonte infinito – Rawlings (1993) provou que o controlador de horizonte infinito é estável para o caso nominal, ou seja, o modelo da planta é equivalente ao modelo utilizado para a predição do MPC.

MPC de Horizonte Infinito

$$\begin{cases} \underline{x}(k+1) = \underline{\underline{A}}\,\underline{x}(k) + \underline{\underline{B}}\,\underline{u}(k) \\ \underline{y}(k) = \underline{\underline{C}}\,\underline{x}(k) \end{cases}$$

C é a matriz identidade, ou, em outras palavras, o estado é medido

x e *u* representam variáveis incrementais
Portanto, para um sistema estável, onde x(infinito) tente da zero, o que traz, como consequência, **desde que não hajam perturbações desconhecidas**, ou simplesmente,

$$u(k+m) = 0$$

Função Objetivo do MPC de Horizonte Infinito

$$\min_{u(k),u(k+1),\ldots u(k+m-1)} J_k = \sum_{j=1}^{\infty} \underline{x}^T(k+j)\underline{\underline{Q}}\underline{x}(k+j) + \sum_{j=0}^{m-1} \underline{u}^T(k+j)\underline{\underline{R}}\underline{u}(k+j)$$

$$\sum_{j=1}^{\infty} \underline{x}^T(k+j)\underline{\underline{Q}}\underline{x}(k+j) = \sum_{j=1}^{m-1} \underline{x}^T(k+j)\underline{\underline{Q}}\underline{x}(k+j) + \sum_{j=m}^{\infty} \underline{x}^T(k+j)\underline{\underline{Q}}\underline{x}(k+j)$$

$$\sum_{j=m}^{\infty} \underline{x}^T(k+j)\underline{\underline{Q}}\underline{x}(k+j) = \sum_{j=0}^{\infty} \underline{x}^T(k+m+j)\underline{\underline{Q}}\underline{x}(k+m+j)$$

Função Objetivo do MPC de Horizonte Infinito

$$\sum_{j=m}^{\infty} \underline{x}^T(k+j)\underline{\underline{Q}}\,\underline{x}(k+j) = \sum_{j=0}^{\infty} \underline{x}^T(k+m+j)\underline{\underline{Q}}\,\underline{x}(k+m+j)$$

$$\underline{x}(k+1) = \underline{\underline{A}}\,\underline{x}(k) + \underline{\underline{B}}\,\underline{u}(k)$$

$$\underline{x}(k+m+1) = \underline{\underline{A}}\,\underline{x}(k+m) + \underline{\underline{B}}\,\underline{u}(k+m) = \underline{\underline{A}}\,\underline{x}(k+m)$$

$$\underline{x}(k+m+2) = \underline{\underline{A}}\,\underline{x}(k+m+1) + \underline{\underline{B}}\,\underline{u}(k+m+1) = \underline{\underline{A}}\,\underline{x}(k+m+1) = \underline{\underline{A}}^2\,\underline{x}(k+m)$$

$$\vdots$$

$$\underline{x}(k+m+j) = \underline{\underline{A}}^j\,\underline{x}(k+m)$$

Função Objetivo do MPC de Horizonte Infinito

$$\sum_{j=m}^{\infty} \underline{x}^T(k+j)\underline{\underline{Q}}\,\underline{x}(k+j) = \sum_{j=0}^{\infty} \underline{x}^T(k+m+j)\underline{\underline{Q}}\,\underline{x}(k+m+j)$$

⇓

$$\sum_{j=m}^{\infty} \underline{x}^T(k+j)\underline{\underline{Q}}\,\underline{x}(k+j) = \sum_{j=0}^{\infty} \left[\underline{\underline{A}}^j \underline{x}(k+m)\right]^T \underline{\underline{Q}}\,\underline{\underline{A}}^j \underline{x}(k+m)$$

⇓

$$\sum_{j=m}^{\infty} \underline{x}^T(k+j)\underline{\underline{Q}}\,\underline{x}(k+j) = \sum_{j=0}^{\infty} \underline{x}(k+m)^T \left[\underline{\underline{A}}^j\right]^T \underline{\underline{Q}}\,\underline{\underline{A}}^j \underline{x}(k+m)$$

Função Objetivo do MPC de Horizonte Infinito

$$\sum_{j=m}^{\infty} \underline{x}^T(k+j)\underline{\underline{Q}}\,\underline{x}(k+j) = \sum_{j=0}^{\infty} \underline{x}(k+m)^T \left[\underline{\underline{A}}^j\right]^T \underline{\underline{Q}}\,\underline{\underline{A}}^j\, \underline{x}(k+m)$$

$$\sum_{j=m}^{\infty} \underline{x}^T(k+j)\underline{\underline{Q}}\,\underline{x}(k+j) = \underline{x}^T(k+m)\left(\sum_{j=0}^{\infty}\left[\underline{\underline{A}}^j\right]^T \underline{\underline{Q}}\,\underline{\underline{A}}^j\right)\underline{x}(k+m)$$

Matlab comando dlyap

$$\underline{\underline{P}} = \left(\sum_{j=0}^{\infty}\left[\underline{\underline{A}}^j\right]^T \underline{\underline{Q}}\,\underline{\underline{A}}^j\right)$$

Função Objetivo do MPC de Horizonte Infinito

$$P = \left(\sum_{j=0}^{\infty} \left[A^j \right]^T Q A^j \right)$$

$$P = \left(\sum_{j=0}^{\infty} \left[A^j \right]^T Q A^j \right) = Q + \left[A \right]^T Q A + \left[A^2 \right]^T Q A^2 + \left[A^3 \right]^T Q A^3 + \ldots + \left[A^\infty \right]^T Q A^\infty$$

$\Uparrow \Delta$

$$A^T P A = A^T Q A + \left[A^2 \right]^T Q A^2 + \left[A^3 \right]^T Q A^3 + \left[A^4 \right]^T Q A^4 + \ldots + \left[A^{\infty+1} \right]^T Q A^{\infty+1}$$

$$A^T P A - P = \left[A^{\infty+1} \right]^T Q A^{\infty+1} - Q \quad \Rightarrow \quad A^T P A - P = -Q$$

Formulação do MPC de Horizonte Infinito

$$\min_{\underline{u}(k),\underline{u}(k+1),\ldots\underline{u}(k+m-1)} J_k = \sum_{j=1}^{m-1} \underline{x}^T(k+j)\underline{\underline{Q}}\,\underline{x}(k+j) + \underline{x}^T(k+m)\underline{\underline{P}}\,\underline{x}(k+m) + \sum_{j=0}^{m-1} \underline{u}^T(k+j)\underline{\underline{R}}\,\underline{u}(k+j)$$

sujeito a

$$\underline{u}_{\min} \le \underline{u}(k+j) \le \underline{u}_{\max}, \quad j = 0,1,2\ldots m-1$$

$$\underline{u}(k+j) = 0 \quad j \ge m$$

Formulação do MPC de Horizonte Infinito

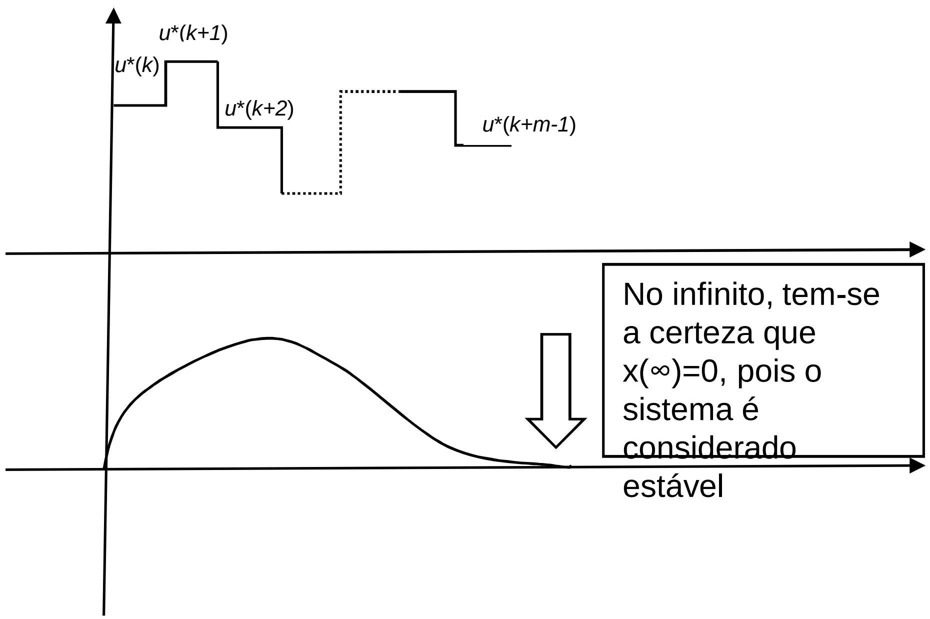
Este controlador é estável para quaisquer valores de horizonte de controle *m*, fator de supressão *R* e peso das variáveis controladas *W*

PROVA DA ESTABILIDADE DO CONTROLADOR DE HORIZONTE INFINITO CASO NOMINAL

Formulação do MPC de Horizonte Infinito

- Instante k

$$\left[\underline{u}^*(k), \underline{u}^*(k+1), \ldots, \underline{u}^*(k+m-1)\right]$$



Formulação do MPC de Horizonte Infinito

- Instante k+1

$$\left[\underline{u}^*(k+1), \underline{u}^*(k+2) \ldots, \underline{u}^*(k+m-1), 0\right]$$

SOLUÇÃO VIÁVEL MAS NÃO NECESSARIAMENTE ÓTIMA

Formulação do MPC de Horizonte Infinito

- Função objetivo do Instante k+1

$$J_k = \underbrace{\sum_{j=1}^{m-1} \underline{x}^T(k+j)\underline{\underline{Q}}\,\underline{x}(k+j)}_{\text{Retira-se o instante } k} + \underline{x}^T(k+m)\underline{\underline{P}}\,\underline{x}(k+m) + \underbrace{\sum_{j=0}^{m-1} \underline{u}^T(k+j)\underline{\underline{R}}\,\underline{u}(k+j)}_{\text{Retira-se o instante } k}$$

$$J_{k+1} = J_k^* - \underline{x}^T(k)\underline{\underline{Q}}\,\underline{x}(k) - \underline{u}^T(k)\underline{\underline{R}}\,\underline{u}(k)$$

Q E R >0

$$J_{k+1} < J_k^*$$

LIMITAÇÕES DO MPC PROPOSTO POR RAWLINGS

Formulação do MPC de Horizonte Infinito

- Se houver perturbações desconhecidas, o fato do estado contrai para ZERO não significa que as saídas voltarão para o estado zero.
- A formulação prevê um modelo posicional. Se o modelo for convertido para incremental, aparecem autovalores em 1 na nova matriz de estados, o que não garante que $\lim_{j \to \infty} \underline{\underline{A}}^{j} = 0$

IHMPC COM MODELO INCREMENTAL

IHMPC COM MODELO INCREMENTAL

$$\begin{cases} \underline{x}(k+1) = \underline{\underline{A}}\,\underline{x}(k) + \underline{\underline{B}}\,\underline{u}(k) \\ \underline{y}(k) = \underline{\underline{C}}\,\underline{x}(k) \end{cases}$$

$$\downarrow$$

$$\begin{cases} \underline{\tilde{x}}(k+1) = \underline{\underline{\tilde{A}}}\,\underline{\tilde{x}}(k) + \underline{\underline{\tilde{B}}}\,\Delta\underline{u}(k) \\ \underline{y}(k) = \underline{\underline{\tilde{C}}}\,\underline{\tilde{x}}(k) \end{cases}$$

GERAM POLOS EM +1

INSTABILIDADE

IDEIA PARA TRATAR OS POLOS INSTÁVEIS:

SEPARAR OS ESTÁVEIS DOS INSTÁVEIS E ADICIONAR RESTRIÇÕES QUE GARANTAM A CONTRAÇÃO DE ESTADOS

TRANSFORMAÇÃO

$$\underline{\underline{A}} = \underline{\underline{V}}\,\underline{\underline{D}}\,\underline{\underline{V}}^{-1}$$

TRANSFORMAÇÃO NA MATRIZ A PARA SEPARAR POLOS ESTÁVEIS DOS INSTÁVEIS

$$\underline{\underline{D}} = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \lambda_n & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots & 1 \end{bmatrix}$$

NOVO ESTADO

$$\underline{x}(k) = \underline{\underline{V}}\,\underline{z}(k)$$

NOVO ESTADO Z

$$\underline{\underline{V}}\,\underline{z}(k+1) = \underline{\underline{A}}\,\underline{\underline{V}}\,\underline{z}(k) + \underline{\underline{B}}\Delta\underline{u}(k)$$

$$\times \underline{\underline{V}}^{-1}$$

$$\underline{z}(k+1) = \underline{\underline{V}}^{-1}\underline{\underline{A}}\,\underline{\underline{V}}\,\underline{z}(k) + \underline{\underline{V}}^{-1}\underline{\underline{B}}\Delta\underline{u}(k)$$

$$\underline{\underline{A}} = \underline{\underline{V}}\,\underline{\underline{D}}\,\underline{\underline{V}}^{-1}$$

EQUAÇÃO EM ESPAÇO DE ESTADO

$$\underline{z}(k+1)=\underline{\underline{V}}^{-1}\underline{\underline{A}}\,\underline{\underline{V}}\,\underline{z}(k)+\underline{\underline{V}}^{-1}\underline{\underline{B}}\Delta\underline{u}(k)$$

$$\underline{\underline{A}}=\underline{\underline{V}}\,\underline{\underline{D}}\,\underline{\underline{V}}^{-1}$$

$$\underline{z}(k+1)=\underline{\underline{D}}\,\underline{z}(k)+\underline{\underline{V}}^{-1}\underline{\underline{B}}\Delta\underline{u}(k)$$

FUNÇÃO OBJETIVO DO IHMPC

$$\min_{\Delta u(k),\Delta u(k+1),\ldots\Delta u(k+m-1)} J_k = \sum_{j=1}^{\infty} \underline{x}^T(k+j)\underline{\underline{Q}}\,\underline{x}(k+j) + \sum_{j=0}^{m-1} \Delta\underline{u}^T(k+j)\underline{\underline{R}}\Delta\underline{u}(k+j)$$

$$\sum_{j=1}^{\infty} \underline{x}^T(k+j)\underline{\underline{Q}}\,\underline{x}(k+j) = \sum_{j=1}^{m-1} \underline{x}^T(k+j)\underline{\underline{Q}}\,\underline{x}(k+j) + \sum_{j=m}^{\infty} \underline{x}^T(k+j)\underline{\underline{Q}}\,\underline{x}(k+j)$$

$$\underline{x}(k) = \underline{\underline{V}}\,\underline{z}(k)$$

$$\sum_{j=m}^{\infty} \underline{x}^T(k+j)\underline{\underline{Q}}\,\underline{x}(k+j) = \sum_{j=m}^{\infty} \underline{z}(k+j)^T \underline{\underline{V}}^T \underline{\underline{Q}}\,\underline{\underline{V}}\,\underline{z}(k+j)$$

FUNÇÃO OBJETIVO DO IHMPC

$$\sum_{j=m}^{\infty} \underline{x}^T(k+j)\underline{\underline{Q}}\,\underline{x}(k+j) = \sum_{j=m}^{\infty} \underline{z}(k+j)^T \underline{\underline{V}}^T \underline{\underline{Q}}\,\underline{\underline{V}}\,\underline{z}(k+j)$$

$$\sum_{j=0}^{\infty} \underline{z}(k+j+m)^T \underline{\underline{V}}^T \underline{\underline{Q}}\,\underline{\underline{V}}\,\underline{z}(k+j+m)$$

PREDIÇÃO A PARTIR DO INSTANTE M

PREDIÇÃO A PARTIR DO INSTANTE M

$$\underline{z}(k+m+j) = \underline{\underline{D}}\,\underline{z}(k+m+j-1) + \underline{\underline{V}}^{-1}\underline{\underline{B}}\Delta\underline{u}(k)$$

PARA K>M

$$\Delta\underline{u}(k) = 0$$

$$\underline{z}(k+j) = \underline{\underline{D}}\,\underline{z}(k+j-1)$$

PREDIÇÃO EM FUNÇÃO DO INSTANTE K

PARA K>M

$$\underline{z}(k+m+1) = \underline{\underline{D}}\underline{z}(k+m) + \underline{\underline{V}}^{-1}\underline{\underline{B}}\Delta\underline{u}(k+m) = \underline{\underline{D}}\underline{z}(k+m)$$

$$\underline{z}(k+m+2) = \underline{\underline{D}}^{2}\underline{z}(k+m)$$

$$\vdots$$

$$\underline{z}(k+m+j) = \underline{\underline{D}}^{j}\underline{z}(k+m)$$

FUNÇÃO OBJETIVO DO IHMPC

$$\sum_{j=m}^{\infty} \underline{x}^T(k+j)\,\underline{\underline{Q}}\,\underline{x}(k+j) = \sum_{j=m}^{\infty} \underline{z}(k+j)^T\,\underline{\underline{V}}^T\,\underline{\underline{Q}}\,\underline{\underline{V}}\,\underline{z}(k+j)$$

$$\sum_{j=0}^{\infty} \underline{z}(k+j+m)^T\,\underline{\underline{V}}^T\,\underline{\underline{Q}}\,\underline{\underline{V}}\,\underline{z}(k+j+m)$$

$$\underline{\underline{D}}^j\,\underline{z}(k+m)$$

$$\sum_{j=0}^{\infty} \underline{z}(k+m)^T\,\underline{\underline{D}}^{j^T}\,\underline{\underline{V}}^T\,\underline{\underline{Q}}\,\underline{\underline{V}}\,\underline{\underline{D}}^j\,\underline{z}(k+m)$$

FUNÇÃO OBJETIVO DO IHMPC

$$\sum_{j=0}^{\infty} \underline{z}(k+m)^T \underline{\underline{D}}^{j^T} \underline{\underline{V}}^T \underline{\underline{Q}}\underline{\underline{V}}\,\underline{\underline{D}}^j \underline{z}(k+m)$$

$$\downarrow$$

$$\underline{z}(k+m)^T \left[\sum_{j=0}^{\infty} \underline{\underline{D}}^{j^T} \underline{\underline{V}}^T \underline{\underline{Q}}\underline{\underline{V}}\,\underline{\underline{D}}^j \right] \underline{z}(k+m)$$

FUNÇÃO OBJETIVO DO IHMPC

$$\underline{z}(k+m)^T \left[\sum_{j=0}^{\infty} \underline{\underline{D}}^{j^T} \underline{\underline{V}}^T \underline{\underline{Q}} \underline{\underline{V}} \underline{\underline{D}}^j \right] \underline{z}(k+m)$$

↓

MATRIZ D POLOS EM +1
SOMA ILIMITADA

SEPARAÇÃO MODOS INSTÁVEIS

$$\underline{z} = \begin{bmatrix} \underline{z}_e & \underline{z}_i \end{bmatrix}^T$$

SEPARAÇÃO DOS MODOS ESTÁVEIS DOS MODOS INSTÁVEIS

$$\underline{z}_e = [n_e \times 1] \qquad \underline{z}_i = [n_i \times 1]$$

SEPARAÇÃO MODOS INSTÁVEIS

DEFININDO

$$\underline{\underline{N}} = diag\left[0_{ne \times ne} \quad I_{ni \times ni}\right]$$

$$\underline{\underline{N}}\,\underline{z} = diag\left[0_{ne \times ne} \quad I_{ni \times ni}\right] \times \left[\underline{z}_e \quad \underline{z}_i\right]^T$$

CAPTURA OS MODOS INSTÁVEIS

SEPARAÇÃO MODOS INSTÁVEIS

ACRESCENDO A RESTRIÇÃO AO PROBLEMA DE OTIMIZAÇÃO

$$\underline{\underline{N}}\,\underline{z}\left(k+m\right)=0$$

PREDIÇÃO DA SOMA FINITA

$$\sum_{j=1}^{\infty} \underline{x}^T(k+j)\underline{\underline{Q}}\,\underline{x}(k+j) = \sum_{j=1}^{m-1} \underline{x}^T(k+j)\underline{\underline{Q}}\,\underline{x}(k+j) + \sum_{j=m}^{\infty} \underline{x}^T(k+j)\underline{\underline{Q}}\,\underline{x}(k+j)$$

$$\underline{z}(k+1) = \underline{\underline{D}}\,\underline{z}(k) + \underline{\underline{V}}^{-1}\underline{\underline{B}}\Delta\underline{u}(k)$$

$$\underline{z}(k+2) = \underline{\underline{D}}\,\underline{z}(k+1) + \underline{\underline{V}}^{-1}\underline{\underline{B}}\Delta\underline{u}(k+1)$$

$$\underline{z}(k+2) = \underline{\underline{D}}\left[\underline{\underline{D}}\,\underline{z}(k) + \underline{\underline{V}}^{-1}\underline{\underline{B}}\Delta\underline{u}(k)\right] + \underline{\underline{V}}^{-1}\underline{\underline{B}}\Delta\underline{u}(k+1)$$

$$\underline{z}(k+2) = \underline{\underline{D}}^2\,\underline{z}(k) + \underline{\underline{D}}\,\underline{\underline{V}}^{-1}\underline{\underline{B}}\Delta\underline{u}(k) + \underline{\underline{V}}^{-1}\underline{\underline{B}}\Delta\underline{u}(k+1)$$

PREDIÇÃO DA SOMA FINITA

PARA O INSTANTE M

$$\underline{z}(k+m)=\underline{\underline{D}}^{m}\underline{z}(k)+\underline{\underline{D}}^{m-1}\underline{\underline{V}}^{-1}\underline{\underline{B}}\Delta\underline{u}(k)+\underline{\underline{D}}^{m-2}\underline{\underline{V}}^{-1}\underline{\underline{B}}\Delta\underline{u}(k+1)+\ldots+\underline{\underline{V}}^{-1}\underline{\underline{B}}\Delta\underline{u}(k+m-1)$$

$$\underline{z}(k+m)=\underline{\underline{D}}^{m}\underline{z}(k)+\overline{B}\Delta\overline{\underline{u}}$$

$$\overline{B}=\left[\underline{\underline{D}}^{m-1}\underline{\underline{V}}^{-1}\underline{\underline{B}} \quad \underline{\underline{D}}^{m-2}\underline{\underline{V}}^{-1}\underline{\underline{B}} \quad \ldots \quad \underline{\underline{V}}^{-1}\underline{\underline{B}}\right]$$

$$\Delta\overline{\underline{u}}=\left[\Delta\underline{u}(k) \quad \Delta\underline{u}(k+1) \quad \ldots \quad \Delta\underline{u}(k+m-1)\right]^{T}$$

RESTRIÇÃO PARA OS MODOS INSTÁVEIS EM FUNÇÃO DO ESTADO ORIGINAL

$$\underline{\underline{N}}\,\underline{z}(k+m)=0$$

$$\underline{\underline{N}}\,\underline{\underline{D}}^{m}\,\underline{z}(k)+\underline{\underline{N}}\,\overline{B}\Delta\overline{\underline{u}}=0$$

$$\underline{z}(k)=\underline{\underline{V}}^{-1}\underline{x}(k)$$

$$\underline{\underline{N}}\,\underline{\underline{D}}^{m}\underline{\underline{V}}^{-1}\underline{x}(k)+\underline{\underline{N}}\,\overline{B}\Delta\overline{\underline{u}}=0$$

SOMA INFINITA COM RESTRIÇÃO SATISFEITA

$$\underline{z}(k+m)^T\left[\sum_{j=0}^{\infty}\underline{\underline{D}}^{j^T}\underline{\underline{V}}^T\underline{\underline{Q}}\,\underline{\underline{V}}\,\underline{\underline{D}}^j\right]\underline{z}(k+m)$$

$$\begin{bmatrix}\underline{z}_e(k+m)^T & 0\end{bmatrix}\left(\sum_{j=0}^{\infty}\left\{\begin{bmatrix}\underline{\underline{\widetilde{D}}} & 0\\ 0 & 0\end{bmatrix}^j\right\}^T\underline{\underline{V}}^T\underline{\underline{Q}}\,\underline{\underline{V}}\begin{bmatrix}\underline{\underline{\widetilde{D}}} & 0\\ 0 & 0\end{bmatrix}^j\right)\begin{bmatrix}\underline{z}_e(k+m)\\ 0\end{bmatrix}$$

$$\underline{\underline{\widetilde{D}}}=\begin{bmatrix}\lambda_1 & 0 & \cdots & 0\\ 0 & \lambda_2 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \lambda_n\end{bmatrix}$$

$$\underline{\underline{\overline{P}}}$$

OS MODOS INSTÁVEIS SÃO REMOVIDOS DA SOMA

SOMA INFINITA COM RESTRIÇÃO SATISFEITA

$$\underline{z}(k+m)^T \left[\sum_{j=0}^{\infty} \underline{\underline{D}}^{j^T} \underline{\underline{V}}^T \underline{\underline{Q}}\,\underline{\underline{V}}\,\underline{\underline{D}}^j \right] \underline{z}(k+m)$$

$$\overline{\underline{\underline{P}}}$$

$$\underline{z}(k+m)^T \overline{\underline{\underline{P}}}\,\underline{z}(k+m) = \underline{x}(k+m)^T \left[\underline{\underline{V}}^{-1}\right]^T \overline{\underline{\underline{P}}}\,\underline{\underline{V}}^{-1} \underline{x}(k+m)$$

SOMA INFINITA COM RESTRIÇÃO SATISFEITA

$$\overline{\underline{\underline{P}}} = \sum_{j=0}^{\infty} \underline{\underline{D}}^{j^T} \underline{\underline{V}}^T \underline{\underline{Q}}\,\underline{\underline{V}}\,\underline{\underline{D}}^j$$

$$\overline{\underline{\underline{P}}} = \sum_{j=0}^{\infty} \underline{\underline{D}}^{j^T} \underline{\underline{V}}^T \underline{\underline{Q}}\,\underline{\underline{V}}\,\underline{\underline{D}}^j = \underline{\underline{V}}^T \underline{\underline{Q}}\,\underline{\underline{V}} + \left[\underline{\underline{D}}\right]^T \underline{\underline{V}}^T \underline{\underline{Q}}\,\underline{\underline{V}}\,\underline{\underline{D}} + \left[\underline{\underline{D}}^2\right]^T \underline{\underline{V}}^T \underline{\underline{Q}}\,\underline{\underline{V}}\,\underline{\underline{D}}^2 + \left[\underline{\underline{A}}^3\right]^T \underline{\underline{Q}}\,\underline{\underline{A}}^3 + \ldots + \left[\underline{\underline{D}}^{\infty}\right]^T \underline{\underline{V}}^T \underline{\underline{Q}}\,\underline{\underline{V}}\,\underline{\underline{D}}^{\infty}$$

$$\times\left(\underline{\underline{D}}^T \underline{\underline{D}}\right)$$

$$\underline{\underline{D}}^T \overline{\underline{\underline{P}}}\,\underline{\underline{D}} = \underline{\underline{D}}^T \underline{\underline{V}}^T \underline{\underline{Q}}\,\underline{\underline{V}}\,\underline{\underline{D}} + \left[\underline{\underline{D}}^2\right]^T \underline{\underline{V}}^T \underline{\underline{Q}}\,\underline{\underline{V}}\,\underline{\underline{D}}^2 + \left[\underline{\underline{D}}^3\right]^T \underline{\underline{V}}^T \underline{\underline{Q}}\,\underline{\underline{V}}\,\underline{\underline{D}}^3 + \left[\underline{\underline{D}}^4\right]^T \underline{\underline{V}}^T \underline{\underline{Q}}\,\underline{\underline{V}}\,\underline{\underline{D}}^4 + \ldots + \left[\underline{\underline{D}}^{\infty+1}\right]^T \underline{\underline{V}}^T \underline{\underline{Q}}\,\underline{\underline{V}}\,\underline{\underline{D}}^{\infty+1}$$

Δ

$$\underline{\underline{D}}^T \overline{\underline{\underline{P}}}\,\underline{\underline{D}} - \overline{\underline{\underline{P}}} = \left[\underline{\underline{D}}^{\infty+1}\right]^T \underline{\underline{V}}^T \underline{\underline{Q}}\,\underline{\underline{V}}\,\underline{\underline{D}}^{\infty+1} - \underline{\underline{V}}^T \underline{\underline{Q}}\,\underline{\underline{V}}$$

EQUAÇÃO DE LYAPUNOV

$$\underline{\underline{D}}^T \underline{\underline{\overline{P}}}\,\underline{\underline{D}} - \underline{\underline{\overline{P}}} = \left[\underline{\underline{D}}^{\infty+1}\right]^T \underline{\underline{V}}^T \underline{Q}\,\underline{\underline{V}}\,\underline{\underline{D}}^{\infty+1} - \underline{\underline{V}}^T \underline{Q}\,\underline{\underline{V}}$$

TENDE A ZERO COM A RESTRIÇÃO SATISFEITA

$$\underline{\underline{D}}^T \underline{\underline{\overline{P}}}\,\underline{\underline{D}} - \underline{\underline{\overline{P}}} = -\underline{\underline{V}}^T \underline{Q}\,\underline{\underline{V}}$$

FUNÇÃO OBJETIVO DO IHMPC

$$\min_{\Delta u(k),\Delta u(k+1),\ldots\Delta u(k+m-1)} J_k = \sum_{j=1}^{m-1} \underline{x}^T(k+j)\underline{\underline{Q}}\,\underline{x}(k+j) + \sum_{j=m}^{\infty} \underline{x}^T(k+j)\underline{\underline{Q}}\,\underline{x}(k+j)$$

$$+\sum_{j=0}^{m-1} \Delta\underline{u}^T(k+j)\underline{\underline{R}}\Delta\underline{u}(k+j)$$

$$\underline{x}(k+m)^T\left[\underline{\underline{V}}^{-1}\right]^T \overline{\underline{\underline{P}}}\,\underline{\underline{V}}^{-1}\underline{x}(k+m)$$

$$\underline{\underline{D}}^T\,\overline{\underline{\underline{P}}}\,\underline{\underline{D}} - \overline{\underline{\underline{P}}} = -\underline{\underline{V}}^T\underline{\underline{Q}}\,\underline{\underline{V}}$$

FUNÇÃO OBJETIVO DO IHMPC

$$\underline{x}(k+m)^T\left[\underline{\underline{V}}^{-1}\right]^T\overline{\underline{\underline{P}}}\,\underline{\underline{V}}^{-1}\underline{x}(k+m)$$

$$\underline{z}(k+m)=\underline{\underline{D}}^m\underline{z}(k)+\overline{B}\Delta\overline{\underline{u}}$$

$$\underline{z}(k)=\underline{\underline{V}}^{-1}\underline{x}(k)$$

$$\underline{x}(k+m)=\underline{\underline{V}}\,\underline{\underline{D}}^m\underline{\underline{V}}^{-1}\underline{x}(k)+\underline{\underline{V}}\,\overline{B}\Delta\overline{\underline{u}}$$

FUNÇÃO OBJETIVO DO IHMPC

$$\min_{\Delta u(k),\Delta u(k+1),\ldots\Delta u(k+m-1)} J_k = \sum_{j=1}^{m-1} \underline{x}^T(k+j)\underline{\underline{Q}}\,\underline{x}(k+j) +$$

$$\left[\underline{\underline{V}}\,\underline{\underline{D}}^m \underline{\underline{V}}^{-1}\underline{x}(k) + \underline{\underline{V}}\,\overline{B}\Delta\overline{\underline{u}}\right]^T \left[\underline{\underline{V}}^{-1}\right]^T \overline{P}\,\underline{\underline{V}}^{-1}\left[\underline{\underline{V}}\,\underline{\underline{D}}^m \underline{\underline{V}}^{-1}\underline{x}(k) + \underline{\underline{V}}\,\overline{B}\Delta\overline{\underline{u}}\right] + \sum_{j=0}^{m-1}\Delta\underline{u}^T(k+j)\underline{\underline{R}}\Delta\underline{u}(k+j)$$

sa:

$$\underline{u}_{\min} \le \underline{u}(k+j) \le \underline{u}_{\max}, \quad j = 0,1,2\ldots m-1$$

$$-\Delta\underline{u}_{\max} \le \Delta\underline{u}(k+j) \le \Delta\underline{u}_{\max}, \quad j = 0,1,2\ldots m-1$$

$$\underline{\underline{N}}\,\underline{\underline{D}}^m \underline{\underline{V}}^{-1}\underline{x}(k) + \underline{\underline{N}}\,\overline{B}\Delta\overline{\underline{u}} = 0$$

MODELO OPOM

Output Predictive Oriented Module

CASO SISO

MODELO OPOM

$$G_P(s) = \frac{Y(s)}{U(s)} = \frac{b_0 + b_1 s + b_2 s^2 + \ldots + b_{nb} s^{nb}}{(s-p_1)(s-p_2)\ldots(s-p_{na})}$$

$$Y(s) = G_P(s)U(s) = \frac{b_0 + b_1 s + b_2 s^2 + \ldots + b_{nb} s^{nb}}{s(s-p_1)(s-p_2)\ldots(s-p_{na})}$$

$$y(t) = s(t) = \mathcal{L}^{-1}\left[\frac{b_0 + b_1 s + b_2 s^2 + \ldots + b_{nb} s^{nb}}{s(s-p_1)(s-p_2)\ldots(s-p_{na})}\right]$$

MODELO OPOM

$$y(t) = s(t) = \mathcal{L}^{-1}\left[\frac{b_0 + b_1 s + b_2 s^2 + \ldots + b_{nb} s^{nb}}{s(s-p_1)(s-p_2)\ldots(s-p_{na})}\right]$$

$$s(t) = \underbrace{\underbrace{C_0 + C_1 e^{p_1 t}}_{\text{1a ordem}} + C_2 e^{p_2 t}}_{\text{2a ordem}} + \ldots + C_{na} e^{p_{na} t}$$

EXEMPLO

$$G_P(s)=\frac{Y(s)}{U(s)}=\frac{1+2s}{s^2+1,7s+0,72}=\frac{1+2s}{(s+0,8)(s+0,9)}$$

$$s(t)=\mathcal{L}^{-1}\left[\frac{1+2s}{s(s+0,8)(s+0,9)}\right]=L^{-1}\left[\frac{1,389}{s}+\frac{7,5}{(s+0,8)}-\frac{8,889}{(s+0,9)}\right]$$

$$s(t)=1,389+7,5e^{-0,8t}-8,889e^{-0,9t}$$

PREDIÇÃO NO OPOM

$$s(t) = C_0 + C_1 e^{p_1 t} + C_2 e^{p_2 t} + \ldots + C_{na} e^{p_{na} t}$$

$$\left[y(kT+t) \right]_k = [P_0]_k + [P_1]_k e^{p_1 t} + [P_2]_k e^{p_2 t} + \ldots [P_{na}]_k e^{p_{na} t}$$

PREDIÇÃO DO INSTANTE ATUAL SEM CONSIDERAR AÇÕES FUTURAS MAS TRAZ AS INFORMAÇÕES DAS AÇÕES DE CONTROLE PASSADAS

PREDIÇÃO NO OPOM

PREDIÇÃO DO INSTANTE ATUAL INCLUINDO AS AÇÕES DE CONTROLE FUTURAS

$$\left[y(kT+t)\right]_k^C = [P_0]_k + [P_1]_k e^{p_1 t} + [P_2]_k e^{p_2 t} + \ldots [P_{na}]_k e^{p_{na} t} +$$
$$+ \left[[C_0]_k + [C_1]_k e^{p_1 t} + [C_2]_k e^{p_2 t} + \ldots [C_{na}]_k e^{p_{na} t}\right] \Delta u(k)$$

COMPONENTE ESTÁTICA

RESPOSTA AO DEGRAU

EQUAÇÃO ESPAÇO DE ESTADOS OPOM

$$\left[y(kT+t)\right]_k^C = [P_0]_k + [P_1]_k e^{p_1 t} + [P_2]_k e^{p_2 t} + \ldots [P_{na}]_k e^{p_{na} t} + \\ + \left[[C_0]_k + [C_1]_k e^{p_1 t} + [C_2]_k e^{p_2 t} + \ldots [C_{na}]_k e^{p_{na} t}\right] \Delta u(k)$$

$$\underbrace{\begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ \vdots \\ P_{na} \end{bmatrix}_{k+1}}_{x(k+1)} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & e^{p_1 T} & 0 & \cdots & 0 \\ 0 & 0 & e^{p_2 T} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & e^{p_{na} T} \end{bmatrix}}_{\underline{\underline{A}}} \underbrace{\begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ \vdots \\ P_{na} \end{bmatrix}_k}_{x(k)} + \underbrace{\begin{bmatrix} C_0 \\ C_1 e^{p_1 T} \\ C_2 e^{p_2 T} \\ \vdots \\ C_{na} e^{p_{na} T} \end{bmatrix}}_{B} \Delta u(k)$$

MATRIZ DINÂMICA F

EQUAÇÃO ESPAÇO DE ESTADOS OPOM

$$y(k) = Cx(k) = \begin{bmatrix} 1 & e^{p_1 t} & e^{p_2 t} & \cdots & e^{p_{na} t} \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ \vdots \\ P_{na} \end{bmatrix}_K$$

MATRIZ DE SÁIDA

MODELO OPOM

Output Predictive Oriented Module

CASO MIMO

MODELO OPOM

$$\underline{\underline{G}}_P(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) & \cdots & G_{1nu}(s) \\ G_{21}(s) & G_{22}(s) & \cdots & G_{2nu}(s) \\ \vdots & \ddots & \cdots & \vdots \\ G_{ny1}(s) & G_{ny2}(s) & \cdots & G_{nynu}(s) \end{bmatrix}$$

PARA CADA PAR:

$$s_{i,j}(t) = C^0_{i,j} + C^d_{i,j,1} e^{p_{i,j,1}t} + C^d_{i,j,2} e^{p_{i,j,2}t} + \ldots + C^d_{i,j,na} e^{p_{i,j,na}t} = C^0_{i,j} + \sum_{l=1}^{na} C^d_{i,j,l} e^{p_{i,j,l}t}$$

DEFINIÇÃO DO ESTADO OPOM

$$\left[y(kT+t)\right]_k = [P_0]_k + [P_1]_k e^{p_1 t} + [P_2]_k e^{p_2 t} + \ldots [P_{na}]_k e^{p_{na} t}$$

O ESTADO SERÁ:

$$[P_0]_k = [x_1]_k$$

$$[P_1]_k e^{p_1 t} = [x_2]_k$$

$$[P_2]_k e^{p_1 t} = [x_3]_k$$

$$\vdots$$

$$[P_{na}]_k e^{p_{na} t} = [x_{na+1}]_k$$

VANTAGEM DA ESCOLHA DO ESTADO OPOM

SE O SISTEMA É ESTÁVEL:

$$\lim_{t\to\infty}\left[y(kT+t)\right]_k = \left[P_0\right]_k = \left[x_1\right]_k$$

ESTADO MENSURÁVEL

MODELO MIMO OPOM

SIMILAR AO DMC MIMO

$$\underline{\underline{C}}^0 = \begin{bmatrix} C^0_{11} & C^0_{12} & \cdots & C^0_{1nu} \\ C^0_{21} & C^0_{22} & \cdots & C^0_{2nu} \\ \vdots & \vdots & \ddots & \vdots \\ C^0_{ny1} & C^0_{ny2} & \cdots & C^0_{nynu} \end{bmatrix}$$

$$\underline{\underline{C}}^0 \in \mathbb{R}^{ny \times nu}$$

PARCELA ESTÁTICA

MODELO MIMO OPOM

$$
\underline{\underline{C}}^{d}=\left[\begin{array}{cccccccccccccccccccccc}
C_{111}^{d} & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0\\
0 & C_{112}^{d} & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0\\
0 & 0 & \ddots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0\\
0 & 0 & \cdots & C_{11na}^{d} & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0\\
0 & 0 & \cdots & 0 & C_{121}^{d} & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0\\
0 & 0 & \cdots & 0 & 0 & C_{122}^{d} & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0\\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & C_{12na}^{d} & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0\\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & C_{1nu1}^{d} & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0\\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & C_{1nu2}^{d} & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0\\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & C_{1nuna}^{d} & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0\\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 & C_{211}^{d} & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0\\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & C_{212}^{d} & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0\\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & C_{21na}^{d} & \cdots & 0 & 0 & \cdots & 0\\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & C_{nynu1}^{d} & 0 & \cdots & 0\\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & C_{nynu2}^{d} & \cdots & 0\\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & C_{nynuna}^{d}
\end{array}\right]
$$

SIMILAR AO DMC MIMO

PARCELA DINÂMICA

MODELO MIMO OPOM

NOTAÇÃO MAIS COMPACTA

$$\underline{\underline{C}}^d = diag \begin{pmatrix} C_{111}^d, C_{112}^d, \cdots, C_{11na}^d, \\ C_{121}^d, C_{122}^d, \cdots C_{12na}^d \cdots \\ \vdots \\ C_{211}^d, C_{212}^d \cdots C_{21na}^d \\ \vdots \\ C_{ny11}^d \cdots C_{ny1na}^d, \\ \vdots \\ C_{nynu1}^d \cdots C_{nynuna}^d \end{pmatrix}$$

$$\underline{\underline{C}}^d \in \mathbb{C}^{ny.nu.na \times ny.nu.na}$$

MODELO MIMO OPOM

EQUAÇÃO ESPAÇO DE ESTADOS OPOM

$$\begin{bmatrix} \underline{x}^s(k+1) \\ \underline{x}^d(k+1) \end{bmatrix} = \begin{bmatrix} \underline{\underline{I}}_{ny} & \underline{\underline{0}} \\ \underline{\underline{0}} & \underline{\underline{F}} \end{bmatrix} \begin{bmatrix} \underline{x}^s(k) \\ \underline{x}^d(k) \end{bmatrix} + \begin{bmatrix} \underline{\underline{C}}^0 \\ \underline{\underline{C}}^d \underline{\underline{F}}\,\underline{\underline{N}} \end{bmatrix} \Delta \underline{u}(k)$$

$$\underline{x}^s(k+1) = \underline{x}^s(k) + \underline{\underline{C}}^0 \Delta \underline{u}(k)$$

$$\underline{x}^d(k+1) = \underline{\underline{F}}\,\underline{x}^d(k) + \underline{\underline{C}}^d \underline{\underline{F}}\,\underline{\underline{N}} \Delta \underline{u}(k)$$

MODELO MIMO OPOM

PARTE ESTÁTICA DO ESTADO

$$\underline{x}^s(k)$$

$$\underline{x}^s(k) = \begin{bmatrix} x_1 & x_2 & \cdots & x_{ny} \end{bmatrix}^T$$

PARTE DINÂMICA DO ESTADO

$$\underline{x}^d(k)$$

$$\underline{x}^d(k) = \begin{bmatrix} x_{ny+1} & x_{ny+2} & \cdots & x_{ny.nu.na+ny} \end{bmatrix}^T$$

MODELO MIMO OPOM

EQUAÇÃO ESPAÇO DE ESTADOS OPOM

$$\begin{bmatrix} \underline{x}^s(k+1) \\ \underline{x}^d(k+1) \end{bmatrix} = \begin{bmatrix} \underline{\underline{I}}_{ny} & \underline{\underline{0}} \\ \underline{\underline{0}} & \underline{\underline{F}} \end{bmatrix} \begin{bmatrix} \underline{x}^s(k) \\ \underline{x}^d(k) \end{bmatrix} + \begin{bmatrix} \underline{\underline{C}}^0 \\ \underline{\underline{C}}^d \underline{\underline{F}}\,\underline{\underline{N}} \end{bmatrix} \Delta \underline{u}(k)$$

$$\underline{\underline{F}} = \begin{bmatrix} e^{p_{111}T} & 0 & \cdots & 0 \\ 0 & e^{p_{112}T} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{p_{ny.nu.na}T} \end{bmatrix}$$

MOD

$$\begin{bmatrix} \underline{x}^{s}(k+1) \\ \underline{x}^{d}(k+1) \end{bmatrix} = \begin{bmatrix} \underline{\underline{I}}_{ny} & \cdots \\ \underline{\underline{0}} & \cdots \end{bmatrix} \cdots (k)$$

$$\underline{\underline{N}} = \left.\begin{bmatrix} \underline{\underline{J}} \\ \underline{\underline{J}} \\ \vdots \\ \underline{\underline{J}} \end{bmatrix}\right\} ny$$

$$\underline{\underline{J}} = \left.\begin{bmatrix} \left.\begin{matrix} 1 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \cdots & 0 \end{matrix}\right\| nu \\ \left.\begin{matrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 1 & 0 & \cdots & 0 \end{matrix}\right\| nu \\ \vdots \\ \left.\begin{matrix} 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{matrix}\right\| nu \end{bmatrix}\right\} na$$

MODELO MIMO OPOM

EQUAÇÃO ESPAÇO DE ESTADOS OPOM

$$\underline{y} = \underline{\underline{C}}\,\underline{x}(k)$$

$$\underline{y}(k) = \begin{bmatrix} \underline{\underline{I}}_{ny} & \underline{\underline{\varphi}} \end{bmatrix} \begin{bmatrix} \underline{x}^s(k) \\ \underline{x}^d(k) \end{bmatrix}$$

$$\underline{\underline{\phi}} = \underbrace{\begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}}_{nu.na}$$

$$\underline{\underline{\varphi}} = \begin{bmatrix} \underline{\phi} & 0 & 0 & \cdots & 0 \\ 0 & \phi & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \phi \end{bmatrix}, \left[\underline{\underline{\varphi}}\right] = ny \times nu.na.ny$$

IHMPC COM OPOM

FUNÇÃO OBJETIVO

$$\min_{\Delta u(k),\Delta u(k+1),\ldots\Delta u(k+m-1)} J_k = \sum_{j=1}^{\infty} \underline{e}^T(k+j)\underline{\underline{Q}}\underline{e}(k+j) + \sum_{j=0}^{m-1} \Delta\underline{u}^T(k+j)\underline{\underline{R}}\Delta\underline{u}(k+j)$$

$$\underline{e}(k+j) = \underline{y}(k+j) - \underline{y}^{SP} = \underline{\underline{C}}x(k+j) - \underline{y}^{SP}$$

FUNÇÃO OBJETIVO

$$\min_{\Delta u(k),\Delta u(k+1),\ldots\Delta u(k+m-1)} J_k = \sum_{j=1}^{\infty} \underline{e}^T(k+j)\underline{\underline{Q}}\underline{e}(k+j) + \sum_{j=0}^{m-1} \Delta\underline{u}^T(k+j)\underline{\underline{R}}\Delta\underline{u}(k+j)$$

$$\sum_{j=1}^{m-1} \underline{e}^T(k+j)\underline{\underline{Q}}\underline{e}(k+j) + \sum_{j=m}^{\infty} \underline{e}^T(k+j)\underline{\underline{Q}}\underline{e}(k+j)$$

FUNÇÃO OBJETIVO

$$\sum_{j=1}^{m-1} \underline{e}^T(k+j)\underline{\underline{Q}}\underline{e}(k+j) + \sum_{j=m}^{\infty} \underline{e}^T(k+j)\underline{\underline{Q}}\underline{e}(k+j)$$

$$\sum_{j=m}^{\infty} \left[\underline{y}(k+j) - \underline{y}^{SP}\right]^T \underline{\underline{Q}} \left[\underline{y}(k+j) - \underline{y}^{SP}\right]$$

FUNÇÃO OBJETIVO

$$\underline{y}(k) = \begin{bmatrix} \underline{\underline{I}}_{ny} & \underline{\underline{\varphi}} \end{bmatrix} \begin{bmatrix} \underline{x}^{s}(k) \\ \underline{x}^{d}(k) \end{bmatrix}$$

$$\underline{y}(k) = \underline{\underline{I}}_{ny}\,\underline{x}^{s}(k) + \underline{\underline{\varphi}}\,\underline{x}^{d}(k) = \underline{x}^{s}(k) + \underline{\underline{\varphi}}\,\underline{x}^{d}(k)$$

$$\sum_{j=m}^{\infty} \left[\underline{y}(k+j) - \underline{y}^{SP}\right]^{T} \underline{\underline{Q}} \left[\underline{y}(k+j) - \underline{y}^{SP}\right]$$

$$\sum_{j=m}^{\infty} \left[\underline{x}^{s}(k+j) + \underline{\underline{\varphi}}\,\underline{x}^{d}(k+j) - \underline{y}^{SP}\right]^{T} \underline{\underline{Q}} \left[\underline{x}^{s}(k+j) + \underline{\underline{\varphi}}\,\underline{x}^{d}(k+j) - \underline{y}^{SP}\right]$$

FUNÇÃO OBJETIVO

$$\sum_{j=m}^{\infty}\left[\underline{x}^{s}(k+j)+\underline{\underline{\varphi}}\,\underline{x}^{d}(k+j)-\underline{y}^{SP}\right]^{T}\underline{\underline{Q}}\left[\underline{x}^{s}(k+j)+\underline{\underline{\varphi}}\,\underline{x}^{d}(k+j)-\underline{y}^{SP}\right]$$

$$\sum_{j=0}^{\infty}\left[\underline{x}^{s}(k+m+j)+\underline{\underline{\varphi}}\,\underline{x}^{d}(k+m+j)-\underline{y}^{SP}\right]^{T}\underline{\underline{Q}}\left[\underline{x}^{s}(k+m+j)+\underline{\underline{\varphi}}\,\underline{x}^{d}(k+m+j)-\underline{y}^{SP}\right]$$

$$j=1\rightarrow\underline{x}^{s}(k+m+1)=\underline{x}^{s}(k+m)+\underline{\underline{C}}^{0}\Delta\underline{u}(k+m)=\underline{x}^{s}(k+m)+\underline{\underline{C}}^{0}\times\underline{0}=\underline{x}^{s}(k+m)$$

$$j=2\rightarrow\underline{x}^{s}(k+m+2)=\underline{x}^{s}(k+m+1)=\underline{x}^{s}(k+m)$$

$$j=1\rightarrow\underline{x}^{d}(k+m+1)=\underline{\underline{F}}\,\underline{x}^{d}(k+m)+\underline{\underline{C}}^{d}\,\underline{\underline{F}}\,\underline{\underline{N}}\Delta\underline{u}(k+m)=\underline{\underline{F}}\,\underline{x}^{d}(k+m)+\underline{\underline{C}}^{d}\,\underline{\underline{F}}\,\underline{\underline{N}}\underline{0}=\underline{\underline{F}}\,\underline{x}^{d}(k+m)$$

$$j=2\rightarrow\underline{x}^{d}(k+m+2)=\underline{\underline{F}}\,\underline{x}^{d}(k+m+1)=\underline{\underline{F}}^{2}\,\underline{x}^{d}(k+m)$$

$$\vdots$$

$$j\rightarrow\underline{x}^{d}(k+m+j)=\underline{\underline{F}}^{j}\,\underline{x}^{d}(k+m)$$

FUNÇÃO OBJETIVO

$$\sum_{j=0}^{\infty}\left[\underline{x}^{s}(k+m+j)+\underline{\underline{\varphi}}\,\underline{x}^{d}(k+m+j)-\underline{y}^{SP}\right]^{T}\underline{\underline{Q}}\left[\underline{x}^{s}(k+m+j)+\underline{\underline{\varphi}}\,\underline{x}^{d}(k+m+j)-\underline{y}^{SP}\right]$$

$$\underline{\underline{F}}^{j}\,\underline{x}^{d}(k+m)$$

$$\underline{x}^{s}(k+m)$$

$$\sum_{j=0}^{\infty}\left[\underline{x}^{s}(k+m)+\underline{\underline{\varphi}}\,\underline{\underline{F}}^{j}\,\underline{x}^{d}(k+m)-\underline{y}^{SP}\right]^{T}\underline{\underline{Q}}\left[\underline{x}^{s}(k+m)+\underline{\underline{\varphi}}\,\underline{\underline{F}}^{j}\,\underline{x}^{d}(k+m)-\underline{y}^{SP}\right]$$

TENDE A ZERO (SISTEMA ESTÁVEL) LEVA A SOMA A INFINITO

RESTRIÇÃO

PARA LIMITAR A SOMA:

$$\underline{x}^{s}\left(k+m\right)-\underline{y}^{SP}=0$$

A parte estática, que representa o valor final da variável, deve tender ao *set point*

RESTRIÇÃO EM FUNÇÃO DO ESTADO ATUAL

$$\underline{x}^{s}(k+m)-\underline{y}^{SP}=0$$

$$\begin{aligned}
&\underline{x}^{s}(k+1)=\underline{x}^{s}(k)+\underline{\underline{C}}^{0}\Delta\underline{u}(k)\\
&\underline{x}^{s}(k+2)=\underline{x}^{s}(k+1)+\underline{\underline{C}}^{0}\Delta\underline{u}(k+1)=\underline{x}^{s}(k)+\underline{\underline{C}}^{0}\Delta\underline{u}(k)+\underline{\underline{C}}^{0}\Delta\underline{u}(k+1)\\
&\underline{x}^{s}(k+3)=\underline{x}^{s}(k+2)+\underline{\underline{C}}^{0}\Delta\underline{u}(k+2)=\underline{x}^{s}(k)+\underline{\underline{C}}^{0}\Delta\underline{u}(k)+\underline{\underline{C}}^{0}\Delta\underline{u}(k+1)+\underline{\underline{C}}^{0}\Delta\underline{u}(k+2)\\
&\vdots\\
&\underline{x}^{s}(k+m)=\underline{x}^{s}(k)+\underline{\underline{C}}^{0}\Delta\underline{u}(k)+\underline{\underline{C}}^{0}\Delta\underline{u}(k+1)+\underline{\underline{C}}^{0}\Delta\underline{u}(k+2)\ldots+\underline{\underline{C}}^{0}\Delta\underline{u}(k+m-1)
\end{aligned}$$

RESTRIÇÃO EM FUNÇÃO DO ESTADO ATUAL

$$\underline{x}^{s}\left(k+m\right)-\underline{y}^{SP}=0$$

$$\underline{x}^{s}\left(k+m\right)=\underline{x}^{s}\left(k\right)+\left[\underline{\underline{C}}^{0}\quad \underline{\underline{C}}^{0}\quad \underline{\underline{C}}^{0}\quad \cdots\quad \underline{\underline{C}}^{0}\right]\begin{bmatrix}\Delta\underline{u}\left(k\right)\\ \Delta\underline{u}\left(k+1\right)\\ \Delta\underline{u}\left(k+2\right)\\ \vdots\\ \Delta\underline{u}\left(k+m-1\right)\end{bmatrix}$$

$$\underline{x}^{s}\left(k+m\right)=\underline{x}^{s}\left(k\right)+\underline{\underline{\widetilde{C}}}^{0}\Delta\underline{\overline{u}}$$

RESTRIÇÃO EM FUNÇÃO DO ESTADO ATUAL

$$\underline{x}^s(k+m) - \underline{y}^{SP} = 0$$

$$\underline{x}^s(k) + \underline{\underline{\widetilde{C}}}^0 \Delta\overline{\underline{u}} - \underline{y}^{SP} = 0$$

$$\underline{e}^s + \underline{\underline{\widetilde{C}}}^0 \Delta\overline{\underline{u}} = 0$$

VOLTANDO A SOMA INFINITA

$$\sum_{j=0}^{\infty}\left[\underline{x}^{s}(k+m)+\underline{\underline{\varphi}}\,\underline{\underline{F}}^{j}\,\underline{x}^{d}(k+m)-\underline{y}^{SP}\right]^{T}\underline{\underline{Q}}\left[\underline{x}^{s}(k+m)+\underline{\underline{\varphi}}\,\underline{\underline{F}}^{j}\,\underline{x}^{d}(k+m)-\underline{y}^{SP}\right]$$

ZERO

$$\sum_{j=0}^{\infty}\left[\underline{\underline{\varphi}}\,\underline{\underline{F}}^{j}\,\underline{x}^{d}(k+m)\right]^{T}\underline{\underline{Q}}\left[\underline{\underline{\varphi}}\,\underline{\underline{F}}^{j}\,\underline{x}^{d}(k+m)\right]$$

VOLTANDO A SOMA INFINITA

$$\sum_{j=0}^{\infty}\left[\underline{\underline{\varphi}}\,\underline{\underline{F}}^{j}\,\underline{x}^{d}(k+m)\right]^{T}\underline{\underline{Q}}\left[\underline{\underline{\varphi}}\,\underline{\underline{F}}^{j}\,\underline{x}^{d}(k+m)\right]$$

$$\sum_{j=0}^{\infty}\left[\underline{x}^{d}(k+m)\right]^{T}\left[\underline{\underline{F}}^{j}\right]^{T}\underline{\underline{\varphi}}^{T}\underline{\underline{Q}}\,\underline{\underline{\varphi}}\,\underline{\underline{F}}^{j}\,\underline{x}^{d}(k+m)$$

$$\left[\underline{x}^{d}(k+m)\right]^{T}\left[\sum_{j=0}^{\infty}\left[\underline{\underline{F}}^{j}\right]^{T}\underline{\underline{\varphi}}^{T}\underline{\underline{Q}}\,\underline{\underline{\varphi}}\,\underline{\underline{F}}^{j}\right]\underline{x}^{d}(k+m)$$

P

VOLTANDO A SOMA INFINITA

$$\left[\underline{x}^d(k+m)\right]^T \left[\sum_{j=0}^{\infty}\left[\underline{\underline{F}}^j\right]^T \underline{\underline{\varphi}}^T \underline{\underline{Q}}\,\underline{\underline{\varphi}}\,\underline{\underline{F}}^j\right]\underline{x}^d(k+m)$$

$\triangle$

$\times \underline{\underline{F}}^T \underline{\underline{F}}$

$$\underline{\underline{F}}^T \underline{\underline{P}}\,\underline{\underline{F}} = \underline{\underline{F}}^T \underline{\underline{\varphi}}^T \underline{\underline{Q}}\,\underline{\underline{\varphi}}\,\underline{\underline{F}} + \left(\underline{\underline{F}}^2\right)^T \underline{\underline{\varphi}}^T \underline{\underline{Q}}\,\underline{\underline{\varphi}}\,\underline{\underline{F}}^2 + \left(\underline{\underline{F}}^3\right)^T \underline{\underline{\varphi}}^T \underline{\underline{Q}}\,\underline{\underline{\varphi}}\,\underline{\underline{F}}^3 + \ldots$$

$$\underline{\underline{F}}^T \underline{\underline{P}}\,\underline{\underline{F}} - \underline{\underline{P}} = -\underline{\underline{\varphi}}^T \underline{\underline{Q}}\,\underline{\underline{\varphi}}$$

EQUAÇÃO DE LYAPUNOV

FUNÇÃO OBJETIVO

$$\min_{\Delta u(k),\Delta u(k+1),\ldots\Delta u(k+m-1)} J_k = \sum_{j=1}^{m-1} \underline{e}^T(k+j)\underline{\underline{Q}}\underline{e}(k+j) +$$

$$\left[\underline{x}^d(k+m)\right]^T \underline{\underline{P}}\underline{x}^d(k+m) + \sum_{j=0}^{m-1} \Delta\underline{u}^T(k+j)\underline{\underline{R}}\Delta\underline{u}(k+j)$$

SOMA INFINITA

$$\underline{e}(k+j) = \underline{x}^s(k+j) + \underline{\underline{\varphi}}\underline{x}^d(k+j) - \underline{y}^{SP}$$

FUNÇÃO OBJETIVO

$$\underline{e}(k+j) = \underline{x}^{s}(k+j) + \underline{\underline{\varphi}}\,\underline{x}^{d}(k+j) - \underline{y}^{SP}$$

$$\underline{x}^{s}(k+j) = \underline{x}^{s}(k) + \underline{\underline{C}}^{0}\Delta\underline{u}(k) + \underline{\underline{C}}^{0}\Delta\underline{u}(k+1) + \underline{\underline{C}}^{0}\Delta\underline{u}(k+2)\ldots + \underline{\underline{C}}^{0}\Delta\underline{u}(k+j-1)$$

$$\underline{\overline{x}}^{s} = \underline{\underline{\overline{I}}}\,\underline{x}^{s}(k) + \underline{\underline{C}}^{0}_{m}\Delta\underline{\overline{u}}$$

FUNÇÃO OBJETIVO

$$\overline{\underline{x}}^{s} = \overline{\underline{\underline{I}}}\,\underline{x}^{s}(k) + \underline{\underline{C}}_{m}^{0}\,\overline{\underline{\Delta u}}$$

$$\overline{\underline{\underline{I}}} = \begin{bmatrix} \underline{\underline{I}} & \underline{\underline{I}} & \ldots & \underline{\underline{I}} \end{bmatrix}^{T}$$

$$\underline{\underline{C}}_{m}^{0} = \begin{bmatrix} \underline{\underline{C}}^{0} & 0 & 0 & \cdots & 0 \\ \underline{\underline{C}}^{0} & \underline{\underline{C}}^{0} & 0 & \cdots & 0 \\ \underline{\underline{C}}^{0} & \underline{\underline{C}}^{0} & \underline{\underline{C}}^{0} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \underline{\underline{C}}^{0} & \underline{\underline{C}}^{0} & \underline{\underline{C}}^{0} & \cdots & \underline{\underline{C}}^{0} \end{bmatrix}$$

FUNÇÃO OBJETIVO

$$\underline{e}(k+j)=\underline{x}^{s}(k+j)+\underline{\underline{\varphi}}\,\underline{x}^{d}(k+j)-\underline{y}^{SP}$$

$$\underline{x}^{d}(k+1)=\underline{\underline{F}}\,\underline{x}^{d}(k)+\underline{\underline{C}}^{d}\,\underline{\underline{F}}\,\underline{\underline{N}}\Delta\underline{u}(k)$$

$$\underline{x}^{d}(k+j)=\underline{\underline{F}}^{j}\,\underline{x}^{d}(k)+\underline{\underline{C}}^{d}\,\underline{\underline{F}}^{j}\,\underline{\underline{N}}\Delta\underline{u}(k)+$$
$$\underline{\underline{C}}^{d}\,\underline{\underline{F}}^{j-1}\,\underline{\underline{N}}\Delta\underline{u}(k+1)+\ldots+\underline{\underline{C}}^{d}\,\underline{\underline{F}}\,\underline{\underline{N}}\Delta\underline{u}(k+j-1)$$

FUNÇÃO OBJETIVO

$$\underline{e}(k+j) = \underline{x}^{s}(k+j) + \underline{\underline{\varphi}}\,\underline{x}^{d}(k+j) - \underline{y}^{SP}$$

$$\underline{x}^{d}(k+j) = \underline{\underline{F}}^{j}\underline{x}^{d}(k) + \underline{\underline{C}}^{d}\underline{\underline{F}}^{j}\underline{\underline{N}}\Delta\underline{u}(k) + \underline{\underline{C}}^{d}\underline{\underline{F}}^{j-1}\underline{\underline{N}}\Delta\underline{u}(k+1) + \ldots + \underline{\underline{C}}^{d}\underline{\underline{F}}\,\underline{\underline{N}}\Delta\underline{u}(k+j-1)$$

$$\underline{\overline{x}}^{d} = \underline{\underline{\overline{F}}}_{x}\underline{x}^{d}(k) + \underline{\underline{\overline{F}}}_{u}\Delta\underline{\overline{u}}$$

FUNÇÃO OBJETIVO

$$\overline{\underline{x}}^{d} = \overline{\underline{\underline{F}}}_{x}\,\underline{x}^{d}(k) + \overline{\underline{\underline{F}}}_{u}\,\Delta\overline{\underline{u}}$$

$$\overline{\underline{x}}^{d} = \left[\underline{x}^{d}(k+1) \quad \underline{x}^{d}(k+2) \quad \ldots \quad \underline{x}^{d}(k+m) \right]^{T}$$

$$\overline{\underline{\underline{F}}}_{x} = \left[\underline{\underline{F}} \quad \underline{\underline{F}}^{2} \quad \ldots \quad \underline{\underline{F}}^{m} \right]^{T}$$

FUNÇÃO OBJETIVO

$$\overline{\underline{x}}^{d} = \overline{\underline{\underline{F}}}_{x}\, \underline{x}^{d}(k) + \overline{\underline{\underline{F}}}_{u}\, \Delta \overline{\underline{u}}$$

$$\overline{\underline{\underline{F}}}_{u} = \begin{bmatrix} \underline{\underline{F}} & 0 & 0 & \cdots & 0 \\ \underline{\underline{F}}^{2} & \underline{\underline{F}} & 0 & \cdots & 0 \\ \underline{\underline{F}}^{3} & \underline{\underline{F}}^{2} & \underline{\underline{F}} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \underline{\underline{F}}^{m} & \underline{\underline{F}}^{m-1} & \underline{\underline{F}}^{m-2} & \cdots & \underline{\underline{F}} \end{bmatrix} \begin{bmatrix} \underline{\underline{C}}^{d}\underline{\underline{N}} & 0 & 0 & \cdots & 0 \\ 0 & \underline{\underline{C}}^{d}\underline{\underline{N}} & 0 & \vdots & 0 \\ 0 & 0 & \underline{\underline{C}}^{d}\underline{\underline{N}} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \underline{\underline{C}}^{d}\underline{\underline{N}} \end{bmatrix}$$

RESTRIÇÕES

$$\underline{e}^{s} + \underline{\underline{\widetilde{C}}}^{0} \Delta \overline{\underline{u}} = 0$$

$$\underline{u}_{\min} \leq \underline{u} \leq \underline{u}_{\max}$$

$$-\Delta \underline{u}_{\max} \leq \Delta \underline{u} \leq \Delta \underline{u}_{\max}$$

Interface Operação

Usuario 200A: ADMINISTRATOR Ultimo Faceplate: Ultimas Telas: Tela_626 Usuario 200 **NONE**

T-2051 T-2052 T-2151 T-2057 CARGA | TELA | Carga T-2001 T-2002 T-2101 T-2007

F-2052 F-2051 F-2151 U-340A F-2151C | Amostra F-2002 F-2001 F-2101 U-260

U-200A — **CMV_T-2052** — 10 Feb 06 10:34:17

F-2051A F-2051B T-2052

Variaveis Controladas

Descritivo	TAG	Atrib.	Lim. Infer.	Valor	Lim. Sup.	Target	Previsao	Lig./Desl.	
90% querosene	AI-20534	PV	206.00	209.76	209.00	210.47	210.47	D	Liga
Fulgor do Quero	AI-20535	PV	42.0	56.1	65.0	56.65	56.65	L	Desl
80% diesel pesa	AI-20566	PV	390.0	393.3	395.0	393.6	393.6	L	Desl
DIFER TI-20826	TDI-20826	PV	1.0	1.9	28.0	1.9	1.9	L	Desl
Ctrl nivel da T	LIC-20519	AO	60.0	71.9	70.0	71.5	71.5	L	Desl
O2 CHAMINE F-20	AI-20505	PV	3.2	3.4	5.0	3.4	3.4	L	Desl
O2 CHAMINE F-20	AI-20506	PV	3.2	3.5	5.0	3.5	3.5	L	Desl
CARGA TERMICA F	QIC-20586	PV	25.0	37.4	45.0	37.4	37.4	L	Desl
CARGA TERMICA F	QIC-20587	PV	25.0	36.3	45.0	36.3	36.3	L	Desl
TOPO T-2052	TI-20506	PV	100.0	103.8	102.0	104.0	104.0	L	Desl

Variaveis Manipuladas

Descritivo	TAG	Atrib.	Lim. Infer.	Valor	Lim. Sup.	Target	MODO	Lig./Desl.
REFLUXO TOPO T-	FIC-20538	SP	2316.0	2414.9	2516.0	2420.1	CAS	Liga
QUEROSENE PRODU	FIC-20508	SP	1600.0	1700.0	1800.0	1700.0	AUTO	Liga
DL PRODUCAO	FIC-20509	SP	3650.0	3650.0	3850.0	3650.0	RCAS	Desl.
CRU PRE-VAP SAI	TIC-20504	SP	365.0	365.0	370.0	365.0	RCAS	Desl.
CRU PRE-VAP SAI	TIC-20505	SP	365.0	365.0	370.0	365.0	RCAS	Desl.

Perturbacao

Descritivo	TAG	Atrib.	Valor	Lig./Desl.
GLOBAL CRU U-200A	FIC-20667	PV	29908.2	Desl.
Inf. Volat. Pet	AI-20577A	PV	455.7	Desl.

B-4016B | B-2070B | B-2152B | TI-21798 | TI-21776

200A

CMV_T-2051/56

22-Jan-09

T-2051 | T-2056 | F-2052

Variaveis Controladas

Descritivo	TAG	Atrib.	Lim. Infer.	Valor	Lim. Sup.	Target	Previsao	Lig./De	
Raz Ref T2051	RFIC-20514	PV	0.48	0.48	0.62	0.48	0.48	L	De
Intemp. GLP	AI-20522	PV	-0.5	-0.3	0.5	-0.40	-0.31	L	De
90% nafta leve	AI-20542	PV	125.0	126.9	135.0	126.9	126.9	L	De
T TOPO T-2056	TIC-20507	PV	68.0	68.2	70.0	68.2	68.1	L	De
Pr. do V-2058	PIC-20510	AO	40.0	61.1	85.0	62.4	62.4	L	De
O2 CH F-2052	AI-20503	PV	3.0	4.6	3.8	4.7	4.7	L	De
C TERM F-2052A	QIC-20588	PV	11.0	12.0	16.0	11.8	12.0	L	De
C TERM F-2052B	QIC-20589	PV	11.0	14.5	18.0	14.3	14.5	L	De

Variaveis Manipuladas

Descritivo	TAG	Atrib.	Lim. Infer.	Valor	Lim. Sup.	Target	MODO	Lig./D
REF TOP T-2056	FIC-20545	SP	800.0	1094.5	1100.0	1094.5	RCAS	Des
Pr. do V-2058	PIC-20510	SP	7.5	7.7	8.5	7.7	RCAS	Des
TOPO T-2051	TIC-20501	SP	116.0	116.0	117.0	116.0	RCAS	Des
T SAIDA F2052A	TIC-20502	SP	340.0	342.0	342.0	340.8	RCAS	Des
T SAIDA F2052B	TIC-20503	SP	340.0	342.0	342.0	340.8	RCAS	Des

Perturbacao

Descritivo	TAG	Atrib.	Valor	Lig./Desl

GUT	56