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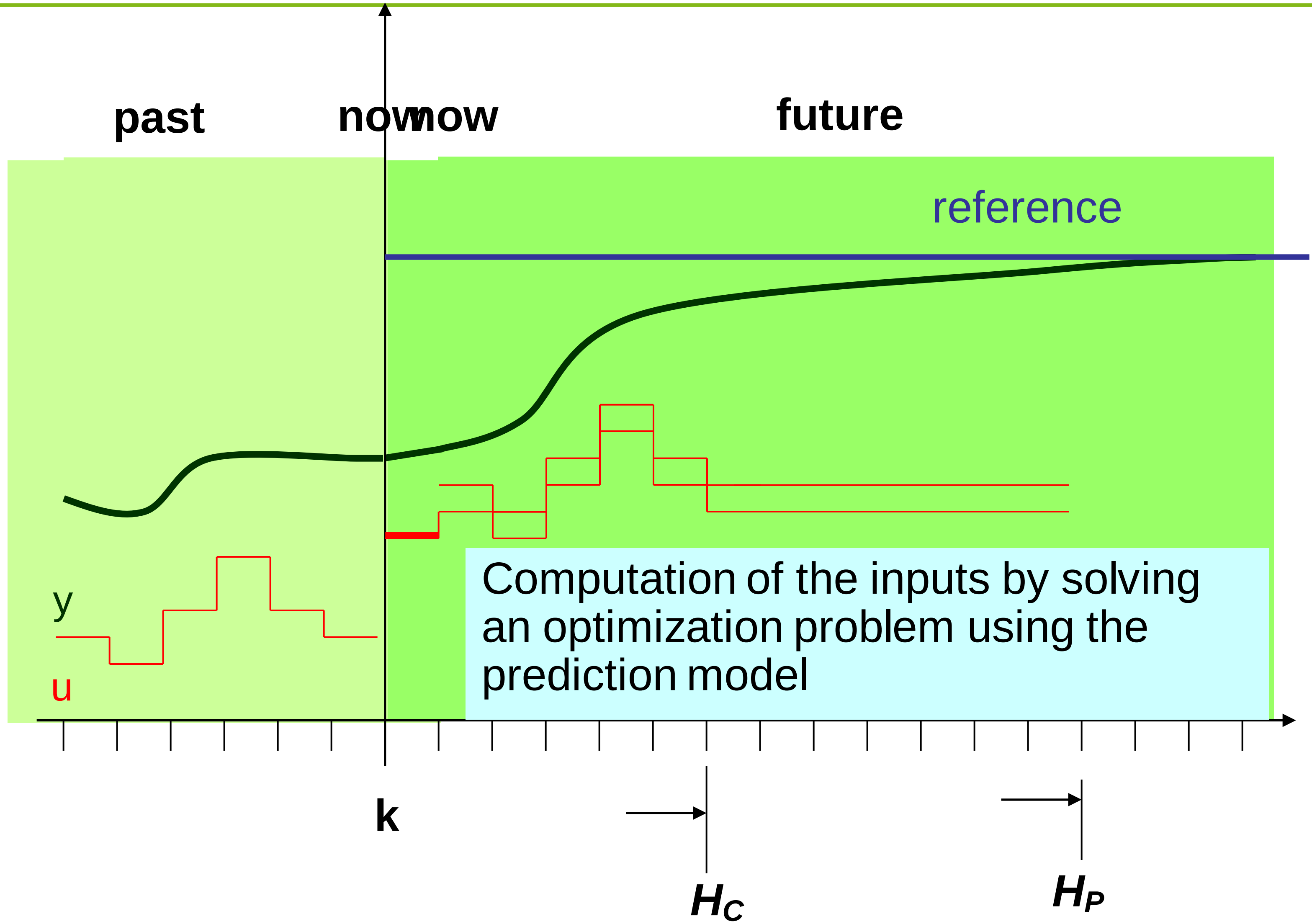
Department of Biochemical and Chemical Engineering
Process Dynamics and Operations Group (DYN)

Model Predictive Control

PSE Summer School
Salvador da Bahia 2011

Process Dynamics
and Operations

Model Predictive Control



Features

- Open-loop optimization instead of feedback control
- Based upon a plant model, a number of future inputs are optimized such that the performance over a finite (prediction) horizon is optimized
- Number of inputs considered can be smaller than the prediction horizon, following values are kept constant

- Ingredients:
  - Model
  - Cost function
  - Optimizer
  - Error feedback

History (1)

- The fathers:
  - Jacques Richalet et al. (1976, 1978)
    Model-predictive heuristic control
  - Charles Cutler (Shell, first application 1973)
    DMC
  - Linear impulse response or step response input/output models obtained from plant tests
  - Quadratic performance criterion
  - No constraints
- The second generation
  - **QDMC** (1983)
  - As before, but: **WITH CONSTRAINTS** on inputs and outputs
  - Online solution of a quadratic program

Typical MPC Cost Function

General cost function

$$\min_{k} J = \sum_{j=1}^{N_p} \underline{e}_{k+j}^{T}\, \underline{Q}_j\, \underline{e}_{k+j} \qquad \underline{e}_{k+j} = \underline{y}^{ref}_{k+j} - \hat{\underline{y}}_{k+j}$$

$$+ \sum_{j=0}^{N_u} \Delta\underline{u}_{k+j}^{T}\, \underline{S}_j\, \Delta\underline{u}_{k+j}$$

$$+ \sum_{j=0}^{N_u} \left(\underline{u}_{k+j} - \underline{u}_{ss}^{opt}\right)^{T} \underline{R}_j \left(\underline{u}_{k+j} - \underline{u}_{ss}^{opt}\right)$$

$\underline{u}_{ss}^{opt}$: optimal steady state input.

while respecting the constraints on $\underline{u}_{k+j}$, $\hat{\underline{y}}_{k+j}$

- Bias-update: Actual prediction error is subtracted from the reference → steady-state accuracy for constant model error, integrating controller

More parameters / variants

- Filtering of the reference moves → desired trajectory more realistic
  - Effect similar to weights on the control moves
- Variant: Minimize the error to the reference trajectory at one point rather than minimizing the overall error
- Range control: no reference tracking but only constraints for some variables (if not critical for performance)
- Many tuning parameters, experience required
- Handling of infeasibility: dropping or softening of constraints

Stability

- Stability of classical input/output MPC schemes is not guaranteed
- Heuristic rules
  - Prediction horizon should be sufficiently long
  - Less inputs give more robust behaviour
- Plant identification is the key to success

- More recent developments
  - System identification rather than data-based models
  - Stability guarantees by use of state space models and terminal constraints / penalties of an infinite prediction horizon

Current Developments

- MPC with nonlinear models
  - Straightforward generalization from the linear case
  - Nonconvex nonlinear online optimization required
- MPC with economic cost function rather than tracking
- Robustness against plant-model mismatch
  - Minmax MPC: minimize the cost function for the worst case disturbance
    - Disturbance replaces model mismatch
    - High effort, conservative