4.2 Nonlinear State Estimation

In the following section nonlinear dynamic systems of this kind are considered:

$$
\begin{aligned}
\dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{u}) + \xi(t) \\
\mathbf{x}(0) &= \mathbf{x}_0 + \xi_0 \\
\mathbf{y}(t) &= \mathbf{h}(\mathbf{x}) + \varphi(t).
\end{aligned}
\tag{4.15}
$$

The stochastic disturbances ξ, ξ_0 and φ are zero mean and will be considered when stochastic state estimators are discussed. For deterministic observers their consideration is not necessary.

Similar to linear observers the nonlinear observer can be described as a simulator with an additive correction. [SZ95].

$$
\dot{\hat{\mathbf{x}}} = \mathbf{f}(\hat{\mathbf{x}}, \mathbf{u}) + \mathbf{K}(\hat{\mathbf{x}}, \mathbf{u}, \mathbf{y}), \qquad \hat{\mathbf{x}}(0) = \hat{\mathbf{x}}_0 \tag{4.16}
$$

It is obvious that the nonlinear observer has to fulfil the simulation and convergence conditions similar to the linear case [Zei77]. From the combination of the simulation condition and equation (4.16) it follows that the correction term tends to zero, i. e. $\lim\limits_{t \to \infty} \mathbf{K}(\hat{\mathbf{x}}, \mathbf{u}, \mathbf{y}) = 0$. The differential equation of the estimation error reads as:

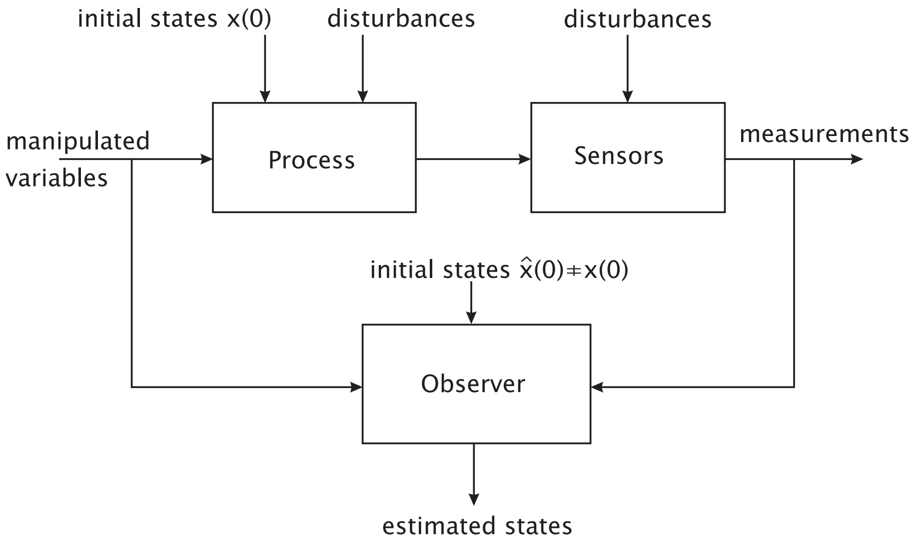

Figure 4.1: Block diagram of a nonlinear observer

$$\dot{\tilde{\mathbf{x}}} = \mathbf{f}(\hat{\mathbf{x}}, \mathbf{u}) - \mathbf{f}(\mathbf{x}, \mathbf{u}) + \mathbf{K}(\hat{\mathbf{x}}, \mathbf{u}, \mathbf{y}), \qquad \tilde{\mathbf{x}}(0) = \hat{\mathbf{x}}_0 - \mathbf{x}_0. \tag{4.17}$$

This equation demonstrates that the estimation error is not described by an autonomous differential equation. The difficulty in the design of nonlinear observers is to determine the correction in dependency of the states and manipulated variables. Furthermore, the choice of the initial conditions $\hat{\mathbf{x}}(t = 0) = \hat{\mathbf{x}}_0$ influences the convergence of the observer if an approximation such as linearization is used in the observer design.

Figure 4.1 depicts the general presentation of a nonlinear state estimator. Similar to linear estimators the approaches to nonlinear estimator/observer design differ in the determination of the correction function. In recent years many different techniques were developed. They can be classified in approaches using differential geometry, approaches with guaranteed stability and methods based on approximations such as linearizations.

In the following section different approaches to nonlinear state estimation are treated. Firstly the *Extended Kalman Filter (EKF)* as a stochastic estimator is discussed. The *Moving Horizon Estimator (MHE)* is also designed for systems with disturbances given in (4.15) but as the stochastics of these disturbances are treated as deterministical disturbances it is not a stochastic state estimator. Further deterministic observers described are the *Extended Luenberger Observer* and the *Sliding Mode Observer (SMO)* .

Observers for specially structured systems [SZ95] are not considered as their use due to the restrictions of the system structure make them seldom applicable in chemical engineering problems.

4.2.1 Extended Kalman Filter

The **E**xtended **K**alman **F**ilter (**EKF**) is an extension of the linear Kalman Filter to nonlinear systems. It is assumed that the disturbances in equation (4.15) (ξ_0, ξ and φ) are stochastic variables with the following properties:

- the disturbances are zero mean, i. e. for the expectations it holds: $E\{\xi_0\} = 0, E\{\xi\} = 0, E\{\varphi\} = 0$
- ξ and φ are not correlated $cov\{\xi, \varphi^T\} = 0$
- the disturbances are described by white noise with the given covariance matrices: $\mathbf{P}_0$ for the initial estimation error, $\mathbf{Q}$ for the model error and $\mathbf{R}$ for the measurement disturbances.

The EKF minimizes the diagonal elements of the covariance matrix of the estimation error $\mathbf{P} = E\{(\hat{\mathbf{x}} - \mathbf{x})(\hat{\mathbf{x}} - \mathbf{x})^T\}$. The mathematical derivation of the EKF can be found in detail in [Jaz70, Gel74]. For continuous time systems the algorithm of the EKF is given by the following equations: [Gel74]:

$$\begin{aligned} \dot{\hat{\mathbf{x}}} &= \mathbf{f}(\hat{\mathbf{x}}, \mathbf{u}) + \mathbf{K}(\mathbf{y} - \mathbf{h}(\hat{\mathbf{x}})) \\ \dot{\mathbf{P}}(t) &= \mathbf{A}(t)\mathbf{P}(t) + \mathbf{P}(t)\mathbf{A}(t)^T + \mathbf{Q} - \mathbf{P}(t)\mathbf{H}(t)^T\mathbf{R}^{-1}\mathbf{H}(t)\mathbf{P}(t) \\ \mathbf{K}(t) &= \mathbf{P}(t)\mathbf{H}(t)^T\mathbf{R}^{-1} \qquad \mathbf{P}(t=0) = \mathbf{P_0} \end{aligned} \tag{4.18}$$

with:

$$\mathbf{A}(t) = \left.\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right|_{\hat{\mathbf{x}}(t), \mathbf{u}(t)} \qquad \text{and} \qquad \mathbf{H}(t) = \left.\frac{\partial \mathbf{h}}{\partial \mathbf{x}}\right|_{\hat{\mathbf{x}}(t)} .$$

Free (and therefore tuning-) parameters are the elements of the constant matrices $\mathbf{P}_0$, $\mathbf{Q}$ and $\mathbf{R}$. It has to be considered that the matrices $\mathbf{P}_0$ and $\mathbf{Q}$ are symmetric, positive semi-definite and $\mathbf{R}$ is symmetric and positive definite. Usually only the diagonal elements are used to tune the EKF, i. e. each state and measurement is not correlated with the other states and measurements, respectively.

Hence, the EKF is determined by a large number of parameters. The disadvantage is the difficulty to isolate the effect of single parameters. For the choice of the parameters are no fixed rules available but some hints on general relations and effects are given in the following paragraph.

As $\mathbf{R}$ is the covariance matrix of the measurements the values of the elements can be taken from the information given for the sensor. Decreasing values for the elements in $\mathbf{R}$ yield in an increasing weight of the concerning measurements, i. e. the measurement is assumed to be more reliable. For noisy measurements this choice may lead to a large gain of the noise and therefore to unsatisfactory estimation results.

The covariance matrix of the initial error $\mathbf{P}_0$ has a large influence on the initial convergence behavior of the EKF [AB89]. In order to get a large space of feasible initial estimates, the elements of $\mathbf{P}_0$ have to be chosen larger for worth initial conditions. The initial convergence of the EKF stays constant if the ratio of the elements of $\mathbf{P}_0$ and $\mathbf{R}$ remains constant. For the choice of $\mathbf{P}_0$ by simulation different initial values should be considered as a large $\mathbf{P}_0$ for good initial condition may also yield in bad estimates [AB89].

The covariance matrix $\mathbf{Q}$ effects the behavior of the estimator opposite to $\mathbf{R}$. Decreasing values of the elements of $\mathbf{Q}$ yield a larger weight of the model equations, i. e. the model is assumed to be more accurate.

Even though, the tuning of an EKF is most often highly dependent on the experience of the control engineer and is usually done by simulations. Valappil et al. [VG99] developed a method to adapt the covariance matrix $\mathbf{Q}$. For systems with not too strong nonlinearities an approach based on a linearization is derived. The second approach uses Monte-Carlo-Simulations. Both approaches are derived for the online application. Papastratos et al. [PHSH99] use an equation derived from an equation based on stationary states for improving the elements of $\mathbf{Q}$. Indeed, the derivation of this formula is not clear. Morad [MSM99] proposes a method based on the use of mean values for stationary points of operation. For batch or semi-batch processes this method can not be applied.

Nevertheless, the EKF is the most often applied state estimation technique in chemical engineering problems. In most cases a discrete EKF is applied as usually the measurements are available only at certain sample times. In [Gel74] the equations for mixed continuous/discrete EKF and for a discrete EKF are derived. The time continuous system (4.15) can be transformed into a discrete form by integration between two sample times [RLR96]:

$$\begin{aligned}
\mathbf{x}_{k+1} &= \mathbf{x}_k + \int_{t_k}^{t_{k+1}} (\mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) + \xi(t))dt \qquad k = 0, 1, \ldots \\
&=: \mathbf{F}(\hat{\mathbf{x}}_k, \mathbf{u}_k) + \xi_k \\
\mathbf{x}(0) &= \bar{\mathbf{x}}_0 + \xi_0 \\
\mathbf{y}_k &= \mathbf{h}(\mathbf{x}_k) + \varphi_k.
\end{aligned} \tag{4.19}$$

Here the manipulated variables are assumed to be constant for the integration (zero order hold). $\bar{\mathbf{x}}_0$ is an a priori estimate of the states. The equations of the discrete EKF are similar to the continuous ones, but the continuous Matrix-Riccati-Equation (4.18) is transformed into the algebraic Matrix-Riccati-Equation which can easily be calculated. Therefore, the algorithm of the discrete EKF can be divided into two steps [RLR96]:

1. Correction:

$$\begin{aligned}
\mathbf{K}_k &= \mathbf{P}_{k,k-1}\mathbf{H}_{k,k-1}^T\left(\mathbf{H}_{k,k-1}\mathbf{P}_{k,k-1}\mathbf{H}_{k,k-1}^T+\mathbf{R}\right)^{-1}\\
\hat{\mathbf{x}}_{k,k} &= \hat{\mathbf{x}}_{k,k-1}+\mathbf{K}_k(\mathbf{y}_k-\mathbf{h}(\hat{\mathbf{x}}_{k,k-1}))\\
\mathbf{P}_{k,k} &= (\mathbf{I}-\mathbf{K}_k\mathbf{H}_{k,k-1})\mathbf{P}_{k,k-1}
\end{aligned} \tag{4.20}$$

2. Prediction:
   Based on the last corrected estimate (filtered state) $\hat{\mathbf{x}}_{k,k}$, the states for the next step are predicted by the model equations without considering disturbances:

   $$\hat{\mathbf{x}}_{k+1,k} = \mathbf{F}(\hat{\mathbf{x}}_{k,k},\mathbf{u}_k)$$

   Furthermore, the covariance matrix of the estimation error is predicted for the next sample time:

   $$\mathbf{P}_{k+1,k} = \mathbf{A}_{k,k}\mathbf{P}_{k,k}\mathbf{A}_{k,k}^T+\mathbf{Q}.$$

In this equation, $\mathbf{A}_{k,k} = \frac{\partial \mathbf{F}}{\partial \mathbf{x}}\Big|_{\hat{\mathbf{x}}_{k,k},\mathbf{u}_k}$ and $\mathbf{H}_{k,k-1} = \frac{\partial \mathbf{h}}{\partial \mathbf{x}}\Big|_{\hat{\mathbf{x}}_{k,k-1}}$, and $\hat{\mathbf{x}}_{k,k-1}$ is the estimated state at time t_k, that is calculated from available measurements up to time t_{k-1}.

4.2.2 Moving Horizon Estimation

Principally the EKF is an estimator which is developed by solving an optimization problem analytically. In the sequel optimization based observers are observers based on numerical optimization. These types of deterministic observers allow for including constraints to states and disturbances. For nonlinear time discrete systems of the form:

$$\begin{aligned}
\mathbf{x}_{k+1} &= \mathbf{F}(\mathbf{x}_k,\mathbf{u}_k)+\xi_k\\
\mathbf{x}(0) &= \mathbf{x}_0+\xi_0\\
\mathbf{y}_k &= \mathbf{h}(\mathbf{x}_k)+\varphi_k.
\end{aligned} \tag{4.21}$$

Muske et al. [MRL93, MR94] and Robertson et al. [RLR96] developed an optimization based observer which uses current and past measurements to calculate the estimates. The problem of infinite dimensional estimation due to the accumulation of measurements as it occurs for the Batch-Least-Squares-Estimator [Jaz70] is avoided by a recursive formulation on a finite horizon.

Although disturbances similar to the EKF are considered in (4.21) the MHE is an deterministic observer as no assumptions on the statistics of the noise are made. Nevertheless, the approach is derived from a statistical point of view. It is assumed that the process can be described as a Markov-process, i. e. the ξ_i are independent. The measurement equation (4.21) maps the states onto the measurements. Therefore, the conditioned probability density function of the states $\{\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_t\}$ for the given measurements $\{\mathbf{y}_0, \mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_{t-1}\}$ has to be maximized in the estimation problem. Following Jazwinski [Jaz70] the probability density function is noted

$$p(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_t | \mathbf{y}_0, \mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_{t-1}). \tag{4.22}$$

As described in [Jaz70] or [Rao00] the maximization of this function can be formulated as an optimization problem and results in the so called **B**atch-**L**east-**S**quares-(**BLS**) Estimator

$$\min_{\hat{\xi}_{-1,k}\ldots\hat{\xi}_{k-1,k},\,\hat{\varphi}_{0,k}\ldots\hat{\varphi}_{k,k}} \Psi_k^N = \hat{\xi}_{-1,k}^T \mathbf{P}_0^{-1} \hat{\xi}_{-1,k} + \sum_{j=0}^{k-1} \hat{\xi}_{j,k}^T \mathbf{Q}^{-1} \hat{\xi}_{j,k} + \sum_{j=0}^{k} \hat{\varphi}_{j,k}^T \mathbf{R}^{-1} \hat{\varphi}_{j,k}. \tag{4.23}$$

s. t.

$$\begin{aligned} \hat{\mathbf{x}}_{j+1,k} &= \mathbf{F}(\hat{\mathbf{x}}_{j,k}, \mathbf{u}_j) + \hat{\xi}_{j,k} \qquad j = 0 \ldots k-1 \\ \hat{\mathbf{x}}_{0,k} &= \bar{\mathbf{x}}_0 + \hat{\xi}_{-1,k} \\ \mathbf{y}_j &= \mathbf{h}(\hat{\mathbf{x}}_{j,k}) + \hat{\varphi}_{j,k} \qquad j = 0 \ldots k. \end{aligned} \tag{4.24}$$

The state vector $\bar{\mathbf{x}}_0$ is an a priori estimate of the initial state. Disadvantageous in this approach is the increasing size of the optimization problem with every new measurement. Consequently the size of the optimization problem has to be bounded for practical use. Jang et al. [JJM86] proposed restarting the BLS-estimator if a certain size of the optimization problem is reached.

Alternatively Muske et al. in [MRL93, MR94] derived an approach on a moving horizon. They devide the cost function (4.23) into 2 time intervals $t_1 = \{j : 0 \leq j \leq k-N-1\}$ and $t_2 = \{j : k-N \leq j \leq k\}$:

$$\begin{aligned} \min_{\hat{\xi}_{-1,k}\ldots\hat{\xi}_{k-1,k},\,\hat{\varphi}_{0,k}\ldots\hat{\varphi}_{k,k}} \Psi_k^N \;=\; & \hat{\xi}_{-1,k}^T \mathbf{P}_0^{-1} \hat{\xi}_{-1,k} + \sum_{j=1}^{k-N-1} \hat{\xi}_{j,k}^T \mathbf{Q}^{-1} \hat{\xi}_{j,k} + \sum_{j=0}^{k-N-1} \hat{\varphi}_{j,k}^T \mathbf{R}^{-1} \hat{\varphi}_{j,k} + \\ & \sum_{j=k-N}^{k-1} \hat{\xi}_{j,k}^T \mathbf{Q}^{-1} \hat{\xi}_{j,k} + \sum_{j=k-N}^{k} \hat{\varphi}_{j,k}^T \mathbf{R}^{-1} \hat{\varphi}_{j,k}. \end{aligned} \tag{4.25}$$

Due to the assumption of a Markov-process

$$\sum_{j=k-N}^{k-1} \hat{\xi}_{j,k}^T \mathbf{Q}^{-1} \hat{\xi}_{j,k} + \sum_{j=k-N}^{k} \hat{\varphi}_{j,k}^T \mathbf{R}^{-1} \hat{\varphi}_{j,k} \tag{4.26}$$

depends only on the state $\hat{\mathbf{x}}_{k-N,k}$, the measurements $\{\mathbf{y}_{k-N}, \ldots, \mathbf{y}_k\}$ and the disturbances $\{\hat{\xi}_{k-N,k}, \ldots, \hat{\xi}_{k-1,k}\}$.

Applying the optimality principle of Bellmann [Rao00] equations (4.23)-(4.24) can be rewritten as:

$$\min_{\mathbf{x}_{k-N},\hat{\xi}_{k-N,k}\ldots\hat{\xi}_{k-1,k},\,\hat{\varphi}_{k-N,k}\ldots\hat{\varphi}_{k,k}} \Psi_k^N = Z_{k-N}(x_{k-N}) + \sum_{j=k-N}^{k-1} \hat{\xi}_{j,k}^T \mathbf{Q}^{-1} \hat{\xi}_{j,k} + \sum_{j=k-N}^{k} \hat{\varphi}_{j,k}^T \mathbf{R}^{-1} \hat{\varphi}_{j,k}. \tag{4.27}$$

The function $Z_{k-N}(\mathbf{x}_{k-N})$ describes the *arrival cost* and summarizes the information of the data in the past. This enables the formulation of the BLS-estimator as an equivalent problem of finite size. Rawlings et al. [MRL93, MR94, RLR96, ABQ$^+$99] use the following approach:

$$Z_{k-N}(x_{k-N}) = (\mathbf{x}_{k-N} - \hat{\mathbf{x}}_{k-N,k})^T \, \mathbf{P}_{k-N,k-N-1}^{-1} \, (\mathbf{x}_{k-N} - \hat{\mathbf{x}}_{k-N,k}) \,. \tag{4.28}$$

$\mathbf{P}_{k-N,k-N-1}$ is taken as the covariance matrix of the estimation error known from the time discrete Extended Kalman Filter (4.20)-(4.21) which can be calculated from the algebraic Matrix-Riccati-equation.

$$\mathbf{P}_{k+1,k} = \mathbf{A}_{k,k} \left(\mathbf{P}_{k,k-1} - \mathbf{P}_{k,k-1} H_{k,k-1}^T \left(\mathbf{H}_{k,k-1} \mathbf{P}_{k,k-1} \mathbf{H}_{k,k-1}^T + \mathbf{R} \right)^{-1} \mathbf{H}_{k,k-1} \mathbf{P}_{k,k-1} \right) \mathbf{A}_{k,k}^T + \mathbf{Q} \tag{4.29}$$

The matrices $\mathbf{A}_{k,k}$ and $\mathbf{H}_{k,k-1}$ are the Jacobians at the states $\hat{\mathbf{x}}_{k,k}$ and $\hat{\mathbf{x}}_{k,k-1}$, respectively.

$$\mathbf{A}_{k,k} = \left.\frac{\partial \mathbf{F}}{\partial \mathbf{x}_k}\right|_{\hat{\mathbf{x}}_{k,k},\mathbf{u}_k} \qquad \mathbf{H}_{k,k-1} = \left.\frac{\partial \mathbf{h})}{\partial \mathbf{x}_k}\right|_{\hat{\mathbf{x}}_{k,k-1}} \tag{4.30}$$

The mathematical formulation of the Moving Horizon Estimator reads as:

$$\begin{aligned} \min_{\hat{\xi}_{k-N-1,k}\ldots\hat{\xi}_{k-1,k},\,\hat{\varphi}_{k-N,k}\ldots\hat{\varphi}_{k,k}} \Psi_k^N \quad = \quad & (\mathbf{x}_{k-N} - \hat{\mathbf{x}}_{k-N,k})^T \mathbf{P}_{k-N}^{-1} (\mathbf{x}_{k-N} - \hat{\mathbf{x}}_{k-N,k}) + \\ & \sum_{j=k-N}^{k-1} \hat{\xi}_{j,k}^T \mathbf{Q}^{-1} \hat{\xi}_{j,k} + \sum_{j=k-N}^{k} \hat{\varphi}_{j,k}^T \mathbf{R}^{-1} \hat{\varphi}_{j,k} \end{aligned} \tag{4.31}$$

subject to the equality constraints:

$$\begin{aligned} \hat{\mathbf{x}}_{j+1,k} &= \mathbf{F}(\hat{\mathbf{x}}_{j,k}, \mathbf{u}_j) + \hat{\xi}_{j,k} \qquad j = k-N, \ldots, k-1 \\ \hat{\mathbf{x}}_{k-N,k} &= \hat{\mathbf{x}}_{k-N,k-N-1} + \hat{\xi}_{k-N-1,k} \\ \mathbf{y}_j &= \mathbf{h}(\hat{\mathbf{x}}_{j,k}) + \hat{\varphi}_{j,k} \qquad j = k-N, \ldots, k \end{aligned} \tag{4.32}$$

and the inequality constraints:

$$\begin{aligned}
\hat{\mathbf{x}}_{min} &\leq \quad \hat{\mathbf{x}}_{j,k} \quad \leq \hat{\mathbf{x}}_{max} \\
\hat{\xi}_{min} &\leq \quad \hat{\xi}_{j-1,k} \quad \leq \hat{\xi}_{max} \\
\hat{\varphi}_{min} &\leq \quad \hat{\varphi}_{j,k} \quad \leq \hat{\varphi}_{max} \\
j &= k-N,\ldots,k.
\end{aligned}$$

Following the notation of Muske et al. N is the length of the horizon and the number of measurements considered in the optimization equals $N+1$. $\hat{\mathbf{x}}_{j,k}$ determines the estimate at time $t = t_j$ based on the measurements up to time $t = t_k$. The matrices $\mathbf{P}_{k-N}$, $\mathbf{Q}$ and $\mathbf{R}$ are weighting matrices for the estimation error, the model error and the measurement noise and are usually chosen similar to the EKF.

The formulation of the estimation problem by an optimization problem allows the consideration of constraints of the estimated states and the disturbances. For the unconstrained case it can be shown that the MHE results in the iterated EKF [Rao00]. For $N = 0$ (i. e. only the current measurements are taken into account) the unconstrained MHE equals the EKF. For linear systems the unconstrained MHE is similar to the Kalman Filter [MRL93].

Similar to the algorithm of the EKF the calculation of the estimates by the MHE can be devided into two steps:

1. Calculation of the estimated disturbances of the model ($\hat{\xi}_{j-1,k}, \quad j = k - N,\ldots,k$), the measurements ($\hat{\varphi}_{j,k}, \quad j = k-N,\ldots,k$) and the corrected state estimates ($\hat{\mathbf{x}}_{j,k}, \quad j = k-N,\ldots,k$) by solving the optimization problem (4.31)-(4.33)

2. Prediction of the covariance matrix of the estimation error by equation (4.29) and the states for the next sample time $\hat{\mathbf{x}}_{k+1,k} = \mathbf{F}(\hat{\mathbf{x}}_{k,k}, \mathbf{u}_k)$ without considering disturbances.

Figure 4.2 depicts the principle of the MHE for an horizon length $N = 2$.

Up to the third measurement the MHE performs like the BLS-estimator as the size of the optimization problem increases with every new measurements. Based on the current filtered state the prediction of the states for the next point of time is realized. When the fourth measurement becomes available the horizon moves and the state $\hat{\mathbf{x}}_{k-N,k-N-1}$ is used as an initial value for the optimization as shown by the arrow in figure 4.2.

The inequality constraints on states and disturbances have mostly a physical reason, as e. g. concentrations or levels in tanks never can be less than zero. Robertson et al. [RLR96] as well as Gesthuisen and Engell [GE98] and Rao [Rao00] showed that the constrained MHE has a larger region of feasible initial conditions than the comparable EKF by different examples. Furthermore, the MHE provides a better convergence

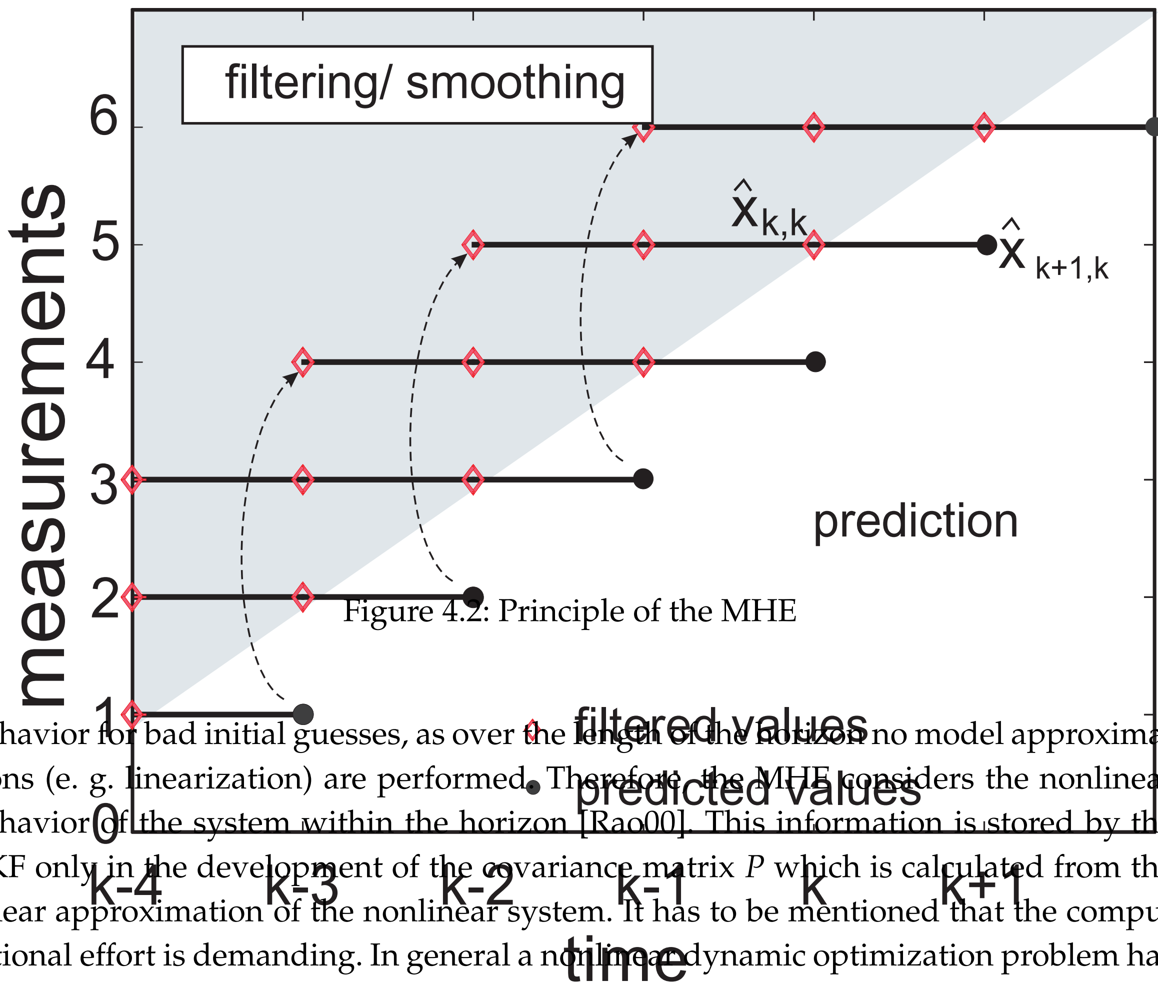

Figure 4.2: Principle of the MHE

behavior for bad initial guesses, as over the length of the horizon no model approximations (e. g. linearization) are performed. Therefore, the MHE considers the nonlinear behavior of the system within the horizon [Rao00]. This information is stored by the EKF only in the development of the covariance matrix P which is calculated from the linear approximation of the nonlinear system. It has to be mentioned that the computational effort is demanding. In general a nonlinear dynamic optimization problem has to be solved.

If only the current measurement is considered and the measurement equation is linear the problem to be solved is quadratic and called Constrained Extended Kalman Filter (CEKF). For quadratic problems effective solvers are available. Therefore the CEKF can easily be applied online [GE98].

4.2.3 Extended Luenberger Observer

The Extended Luenberger Observer (ELB) is the extension of the Luenberger Observer to nonlinear systems, basically based on the approximation of the nonlinear system at each and every point of time and the pole placement for the linearized differential equation of the estimation error. Therefore, the design equations read as:

$$\det\left[s\mathbf{I} - \left(\left.\frac{\partial \mathbf{F}}{\partial \mathbf{x}}\right|_{\hat{\mathbf{x}}_{k,k},\mathbf{u}_k} + \mathbf{K}\left.\frac{\partial \mathbf{h}}{\partial \mathbf{x}}\right|_{\hat{\mathbf{x}}_{k,k-1}}\right)\right] = \prod_{\nu=1}^{n}(s-\lambda_{K,\nu}) \tag{4.33}$$

and has always to be calculated using the current estimates. As linearization is applied for the design it is obvious that for the Luenberger Observer only local stability/convergence can be proved.