TECHNISCHE UNIVERSITÄT DORTMUND
Heiko Brandt, Yi Yu
Department of Bio- and Chemical Engineering
Chair of Process Dynamics and Operations
Prof. Dr. Sebastian Engell

Advanced Process Control - Tutorial 7 **WS 10/11**

Model Predictive Control
Solutions

Quickies

1. A model predictive control scheme is mainly composed of the following four steps:

   (i) It uses a plant model to predict the plant output(s) over a certain prediction horizon Hp and a control horizon Hm.

   (ii) It minimizes a quadratic cost function to compute the optimal control moves over the control horizon.

   (iii) From the obtained control moves, only the first one is used and the rest is discarded.

   (iv) At the next time instant, both the horizons are shifted by one time step and the above steps are repeated to obtain a new set of control moves. This is called the *receding horizon* strategy.

   The quadratic cost function is usually of the form:

$$J = \sum_{i=1}^{Hp} [\hat{y}(k+i|k) - y_{ref}(k+i)]^T Q [\hat{y}(k+i|k) - y_{ref}(k+i)] + \sum_{j=0}^{Hm-1} u(k+j)^T R u(k+j)$$

   where $\hat{y}(k+i|k), u(k+j)$ are the predicted outputs and the future control inputs respectively; Q, R are the penalty matrices; $\Delta u(k+j)$ refer to the change in control inputs between two successive time instants, that is $\Delta u(k+j) = u(k+j) - u(k+j-1)$.

2. In commercial applications, the reasons for the popularity of the MPC scheme can be cited as below:

   (i) MPC is an optimization based control scheme. It computes the optimal control moves considering a certain prediction horizon as well as a control horizon.

   (ii) Constraints on the control input(s) as well as the plant output(s) can be easily incorporated as opposed to the classical control methods. Moreover, other types of constraints can also be considered.

(iii) MPC is a multivariable control scheme which means that it can handle multiple inputs and multiple outputs simultaneously as opposed to the classical control methods.

Still MPC is not wildly used in industry as running a plant with a MP controller from industrial point of view demands several points:

- A proof that a new control scheme (MPC) does really improve the productivity: difficult to proof
- Stability must be guaranteed, i.e. it must be assured that the optimization solver always converges
- The biggest problem is, that a plant operator cannot understand the control steps. For them it's more or less a black box. It follows that they will not longer watch the process attentively, errors may not be detected or important alarms ignored.

Exercises

1. The smallest time constant of the system is $T_{min} = 0.2$ as can be seen by the eigenvalues. The sampling time should be smaller than T_{min} as otherwise certain dynamics may not be represented well by a step response model. This is illustrated in Figure 1 and Figure 2 Of course, in reality it is probably impossible to measure concentra-

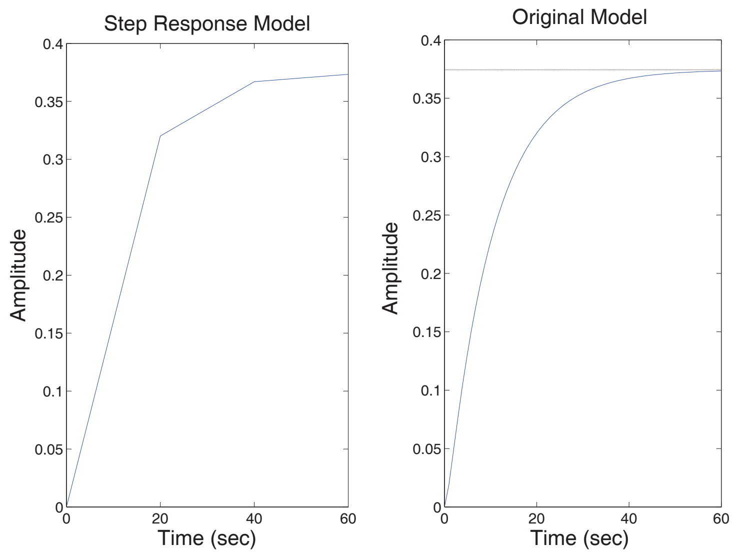

Figure 1: Step response model with a sampling time of 20 seconds

tions every 0.1 seconds.

2. With different weights *ywt*, i.e. Q and *uwt*, i.e. R (cf. quickies). In principle higher values for R cause smaller changes in u, therewith you avoid oscillations but suffer this with slow convergence to the desired state (cf. Figure 3 and 4). However,

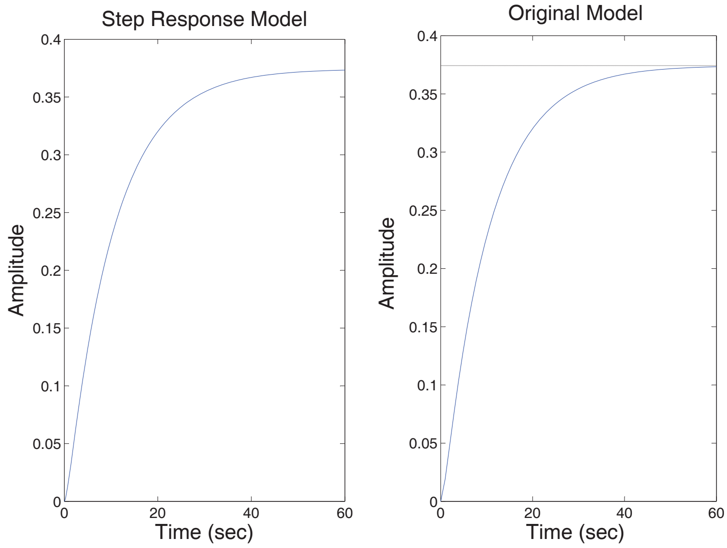

Figure 2: Step response model with a sampling time of 0.1 seconds

the reason for R being present in the optimization problem is to penalize sharp, unphysical jumps in u. The Q part of the cost function works as a plant feedback and is the reason why MPC is a control scheme and not only an optimal feed-forward scheme. In order to let Q have an effect on the control, R must be nonzero. With Q you can penalize slow convergence, thus contradict the control penalty part (cf. Figure 4 and 5). Therefore a tuning of an MPC controller is always necessary.

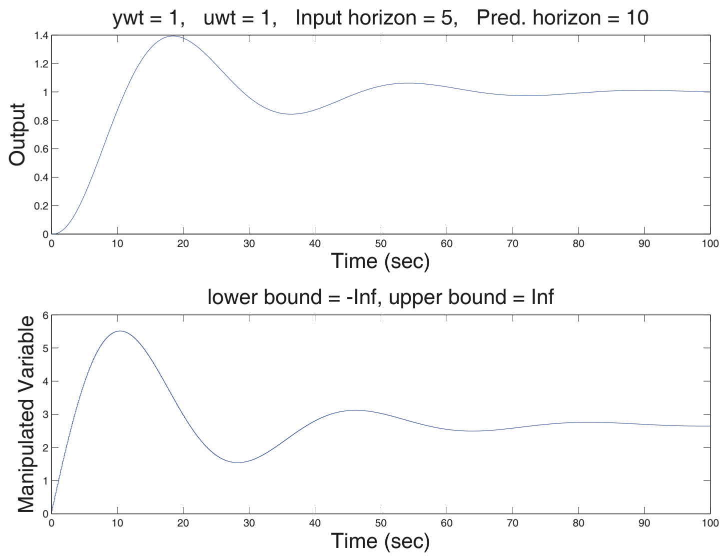

Figure 3: Small weight on the control penalty function

The control as well as the prediction horizon have an effect on the cost function as longer horizons of course gives more insight in the plant behavior but demand higher computing costs and requires an accurate model. This is especially problematic when MPC is used for nonlinear problems, as the linearization may describe the process behavior not very precise for long horizons. Smaller horizon length should therefore be used when the model is not that accurate. Please note, even when the model is well known is does not make sense to increase the horizon length arbitrarily, as at a certain horizon length the solution of the optimization problem will not change anymore. This length is of course determined by the time the plant needs to converge to the desired state.

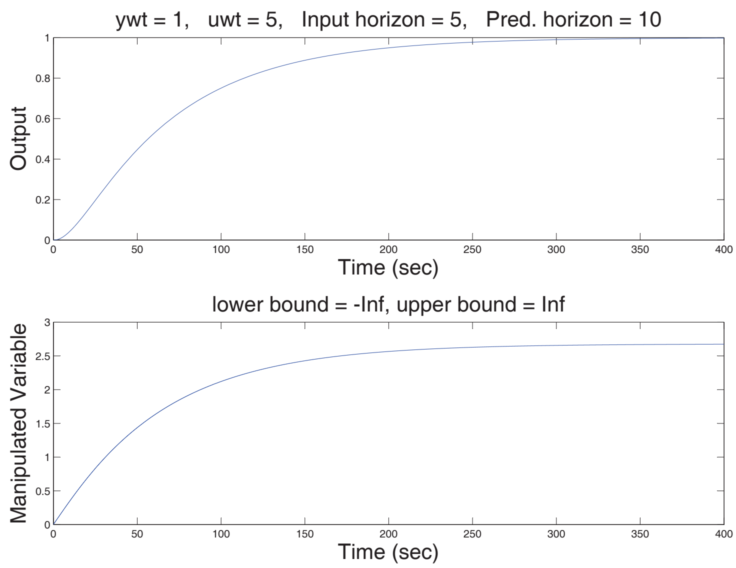

Figure 4: Big weight on the control penalty function

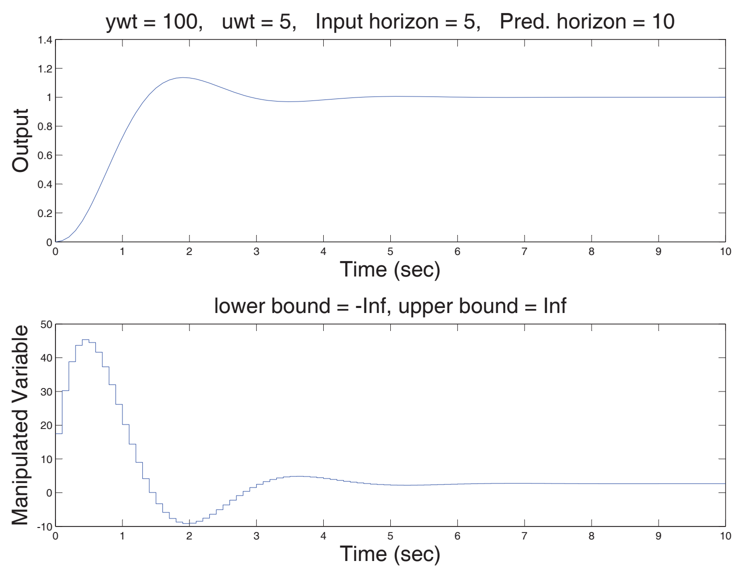

Figure 5: Huge weight on the feedback

3. In order to see the real effect of imposed constraints, we set $R = 0$. As a consequence of the upper bound, the rise time is increased as unreasonable high values for the input concentration are not possible anymore. The lower bound on the other hand may lead to a significant overshoot, that can only be corrected by the dynamics of the system, since no negative input is available.

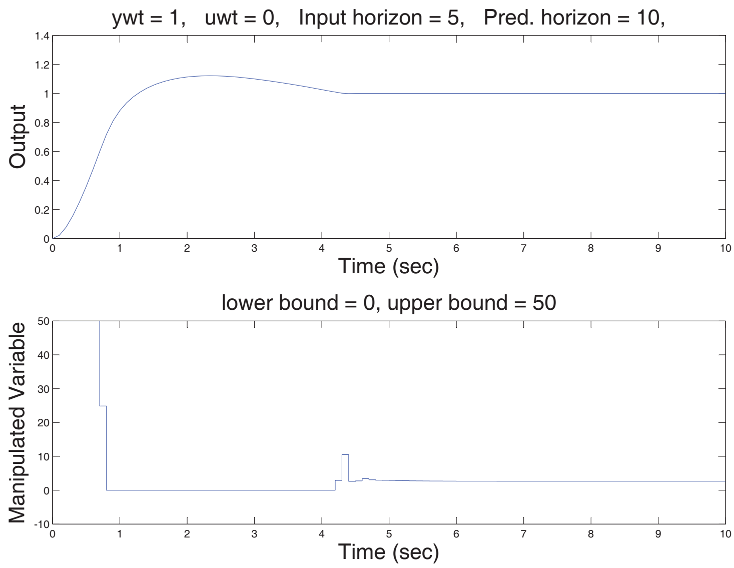

Figure 6: Imposing constraints

4. Changing the reactor volume in the controller model to a smaller value (here: $V_R = 5$), simply makes the time constant of the system smaller. Therewith the model predicts a faster dynamic as the real plant. Its quicker reaction leads to a larger overshoot (comparing Figure 6 and Figure). Considering the large difference between the models, the performance of MPC controller is satisfactory. Even with V_R smaller than 5, the controller behaves well. However, oscillations occur in the output due to the wrong predictions which must be corrected by feedback.

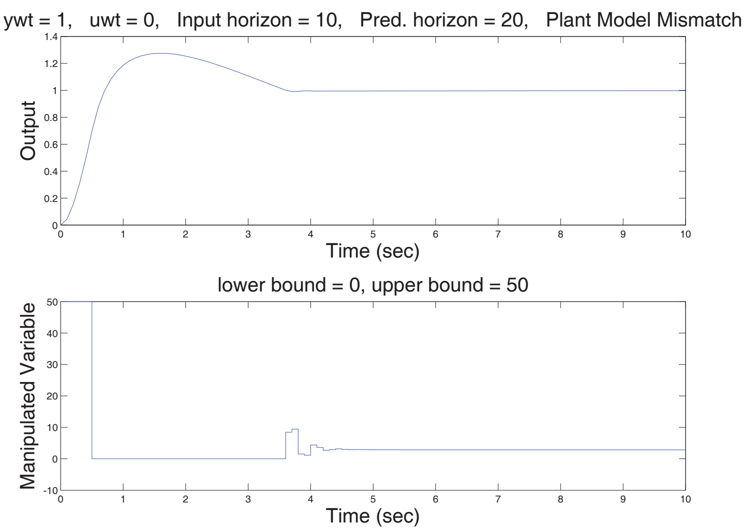

Figure 7: Plant Model Mismatch, $V_R = 5$