FRTN10 Multivariable Control — Lecture 1

Anton Cervin

Automatic Control LTH, Lund University

Department of Automatic Control

- Founded 1965 by Karl Johan Åström (IEEE Medal of Honor)
- Approx. 50 employees
- Education for B, BME, C, D, E, F, I, K, M, N, Pi, W
- Research in complex systems, robotics, real-time systems, process control, biomedicine, . . .

Department of Automatic Control's 50th anniversary

Lecture 1

- Course program
- Examples/introduction
- Signals and systems
  - Review of system representations
  - Norm of signals
  - Gain of systems

Administration

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and lecturer

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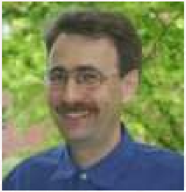
Anders Robertsson
Course responsible
and lecturer



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Course administrator

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Prerequisites

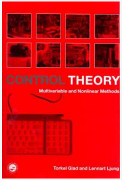
FRT010 Automatic Control, Basie Course or FRTN25 Automatic Process Control is required prior knowledge.

It is assumed that you have taken the compulsory courses in mathematics, including linear algebra, calculus in several variables, and systems & transforms or linear systems.

Course material

All course material is available in English. Most lectures are covered by the following textbook sold by KFS AB:

- Glad & Ljung: *Reglerteori – Flervariabla och olinjära metoder, 2 uppl.* Studentlitteratur, 2004.
- English translation: Glad & Ljung: *Control Theory – Multivariable and Nonlinear Methods*, Taylor & Francis



All other material on the homepage:

- Lecture notes
- Lecture slides
- Exercise problems with solutions
- Laboratory exercises
- English–Swedish control dictionary

`http://www.control.lth.se/course/FRTN10`

Lectures

The lectures (30 hours) are given by Anton Cervin and Anders Robertsson as follows:

Mondays		M:B	8.15–10.00
Wednesdays	until Oct 7	MA:2	8.15–10.00
Thursdays	Sep 3 and Sep 10	M:B	8.15–10.00

Exercise sessions and TAs

The exercise sessions (28 hours) are arranged in three groups:

Group	*Times*	*Room*	*Teaching Assistant*
1	Mon 13–15, Thu 13–15	Lab A	Mattias Fält
2	Mon 13–15, Thu 13–15	Lab B	Gabriel Ingesson
3	Mon 15–17, Fri 13–15	Lab A	Jonas Dürango

Booking lists for the exercise groups are available on the homepage.

Mattias Fält

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Jonas Dürango

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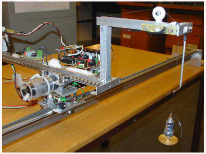
Laboratory experiments

The laboratory experiments (12 hours) are mandatory. Booking lists are posted on the course homepage. Before each lab session some home assignments have to be completed. No reports are required.

Lab	*Weeks*	*Booking*	*Room*	*Responsible*	*Process*
1	38–39	Aug 31	Lab C	Jonas Dürango	Flexible servo
2	40–41	Sep 14	Lab B	Gabriel Ingesson	Quadruple tank
3	42–43	Sep 28	Lab B	Mattias Fält	Crane



Exam

The exam is given on Thursday Oct 29 at 14.00–19.00.

A second occasion is on January 8, 2016.

Lecture notes, lecture slides, and the textbook are allowed on the exam, but no exercise materials or hand-written notes.

Use of computers in the course

- In our lab rooms, use your personal student account or a common course account
- Matlab is used in both exercise sessions and laboratory sessions
  - Control System Toolbox
  - Simulink
  - Q-Tools (available on the homepage—requires additional solvers)
  - (Symbolic Math Toolbox)

Feedback and Q&A

For each course LTH uses the following feedback mechanisms

- CEQ (reporting / longer time scale)
- Student representatives (fast feedback)
  - Election of student representative ("kursombud")

We will be using Piazza for Q&A:

`https://piazza.com/lu.se/fall2015/frtn10/home`

Please post your questions here!

Course registration

Course registration in LADOK will be performed on Thursday September 3.

Put a mark next to your name on the registration list (or fill in your details on an empty row at the end).

If you decide to drop out during the first three weeks of the course, you should notify us so that we can unregister you in LADOK.

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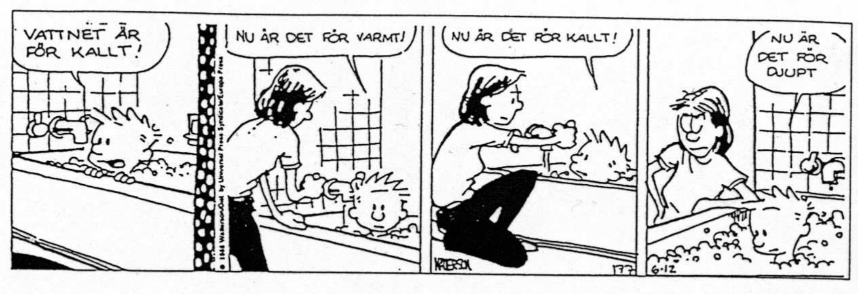
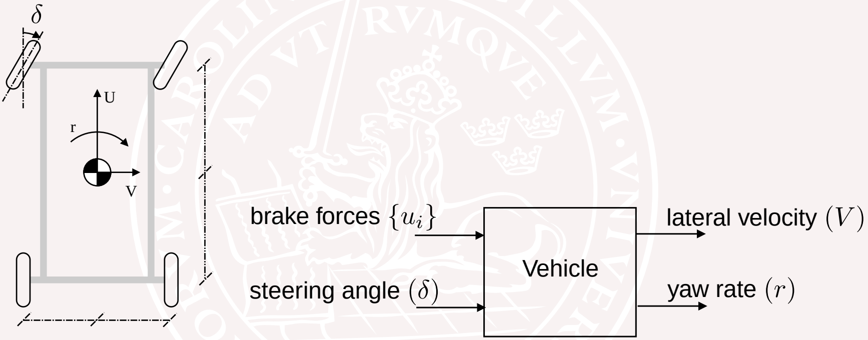
Multivariable control – Example 1

– The water is too cold! – Now it is too hot! – Now it is too cold! – Now it is too deep!

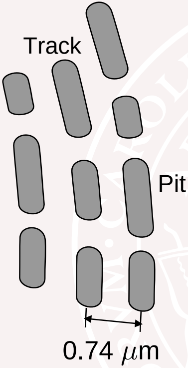


Example 2: Rollover control

Rollover control



Example 3: DVD reader control

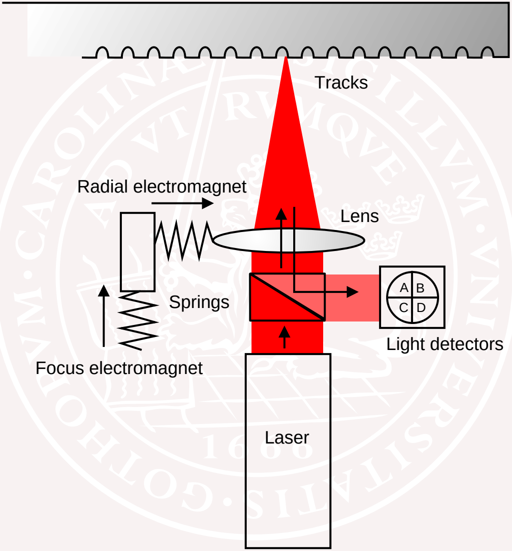


- 3.5 m/s speed along track
- 0.022 μm tracking tolerance
- 100 μm deviations at ~23 Hz due to asymmetric discs

DVD Digital Versatile Disc, 4.7 Gb

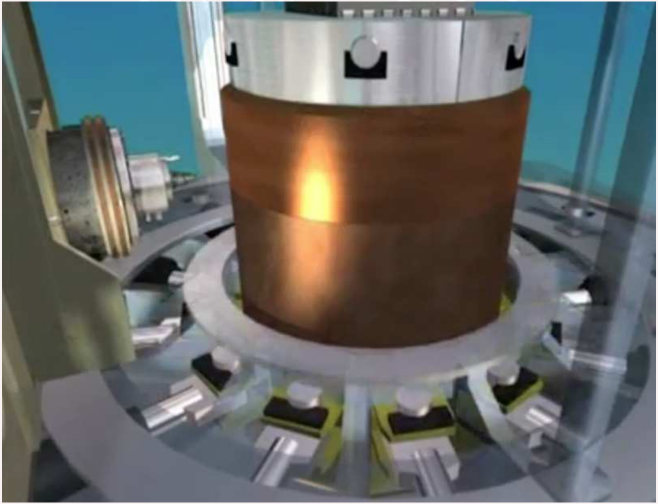
CD Compact Disc, 650 Mb, mostly audio and software

Focus and tracking control



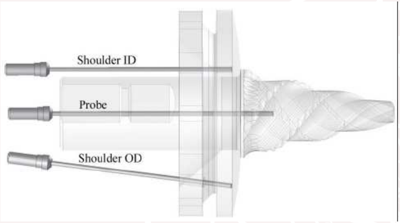
Example 4: Control of friction stir welding

Prototype FSW machine at Swedish Nuclear Fuel and Waste Management Co (SKB) in Oskarshamn

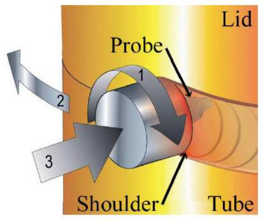


Control of friction stir welding

Measurement variables:



Control variables:



- Temperatures (3 sensors)
- Motor torque
- Shoulder depth

- Tool rotation speed
- Weld speed
- Axial force

Control objectives:

- Keep weld temperature at 845 °C
- Keep shoulder depth at 1 mm

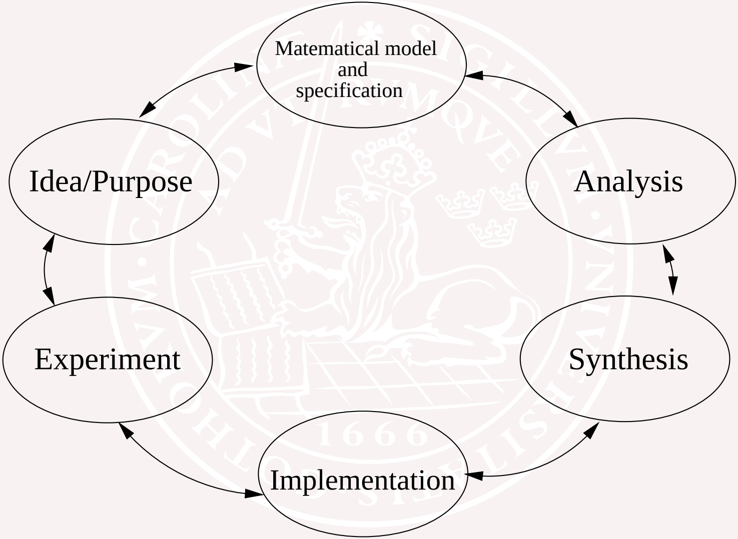
Contents of the course

Despite its name, this couse is **not only about multivariable control**. You will also learn about:

- sensitivity and robustness
- design trade-offs and fundamental limitations
- stochastic control
- optimization of controllers

Contents of the course

The design process



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Systems

A **system** is a mapping from the input signal $u(t)$ to the output signal $y(t)$, $-\infty < t < \infty$:

$$y = \mathcal{S}(u)$$

System properties

A system $\mathcal{S}$ is

- **causal** if $y(t_1)$ only depends on $u(t)$, $-\infty < t \leq t_1$; **non-causal** otherwise
- **static** if $y(t_1)$ only depends on $u(t_1)$; **dynamic** otherwise
- **discrete-time** if $u(t)$ and $y(t)$ are only defined for a countable set of discrete time instances $t = t_k,\ k = 0, \pm 1, \pm 2, \ldots$; **continuous-time** otherwise
- **single-variable** or **scalar** if $u(t)$ and $y(t)$ are scalar signals; **multivariable** otherwise
- **time-invariant** if $y(t) = \mathcal{S}(u(t))$ implies $y(t+\tau) = \mathcal{S}(u(t+\tau))$; **time-varying** otherwise
- **linear** if $\mathcal{S}(\alpha_1 u_1 + \alpha_2 u_2) = \alpha_1 \mathcal{S}(u_1) + \alpha_2 \mathcal{S}(u_2)$; **nonlinear** otherwise

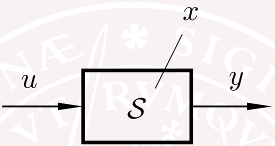
LTI system representations

We will mainly deal with continuous-time **linear time-invariant** (LTI) systems in this course

For LTI systems, the same input–output mapping $\mathcal{S}$ can be represented in a number of equivalent ways:

- linear ordinary differential equation
- linear state-space model
- impulse response
- step response
- transfer function
- frequency response
- ...

State-space models



Linear state-space model:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

Solution:

$$y(t) = Ce^{At}x(0) + \int_0^t Ce^{A(t-\tau)}Bu(\tau)d\tau + Du(t)$$

Mini-problem 1

$$
\begin{aligned}
\dot{x}_1 &= -x_1 + 2x_2 + u_1 + u_2 - u_3 \\
\dot{x}_2 &= -5x_2 + 3u_2 + u_3 \\
y_1 &= x_1 + x_2 + u_3 \\
y_2 &= 4x_2 + 7u_1
\end{aligned}
$$

How many state variables, inputs and outputs?

Determine the matrices A, B, C, D to write the system as

$$
\begin{cases}
\dot{x} = Ax + Bu \\
y = Cx + Du
\end{cases}
$$

Mini-problem 2

Write the following system in state-space form:

$$\ddot{y} + 3\dot{y} + 2y = 5u$$

What if derivatives of the input signal appears?

- Superposition
- Canonical forms
- Collection of formulae
- ...

Mini-problem 2

Write the following system in state-space form:

$$\ddot{y} + 3\dot{y} + 2y = 5u$$

What if derivatives of the input signal appears?

- Superposition
- Canonical forms
- Collection of formulae
- ...

Change of coordinates

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

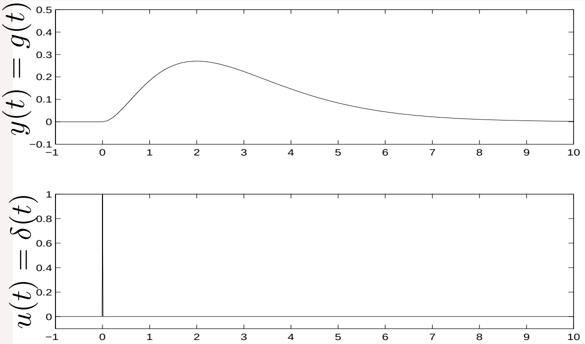
Change of coordinates

$$z = Tx$$

$$\begin{cases} \dot{z} = T\dot{x} = T(Ax + Bu) & = T(AT^{-1}z + Bu) = TAT^{-1}z + TBu \\ y = Cx + Du & = CT^{-1}z + Du \end{cases}$$

Note: There are infinitely many different state-space representations of the same system $\mathcal{S}$

Impulse response

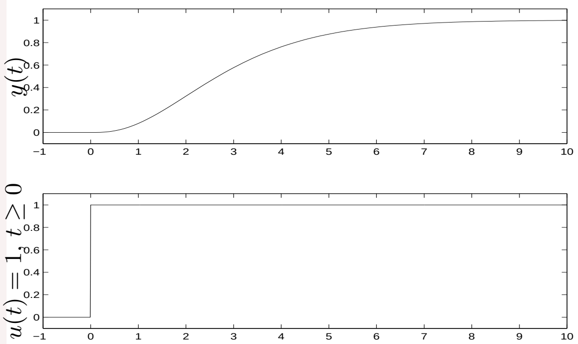


Common experiment in medicin and biology

$$g(t) = \int_0^t Ce^{A(t-\tau)} B\delta(\tau) d\tau + D\delta(t) = Ce^{At}B + D\delta(t)$$

$$y(t) = \int_0^t g(t-\tau)u(\tau)d\tau = (g * u)(t)$$

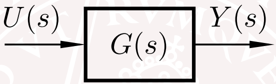
Step response



Common experiment in process industry

$$y(t) = \int_0^t g(t-\tau)u(\tau)d\tau = \int_0^t g(\tau)d\tau$$

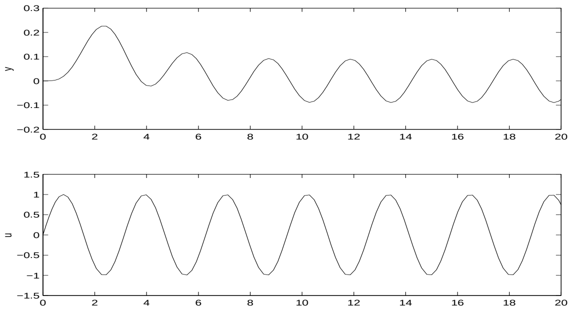
Transfer function



$$G(s) = \mathcal{L}\{g(t)\}$$

$$y(t) = (g * u)(t) \quad \Leftrightarrow \quad Y(s) = G(s)U(s)$$

Frequency response



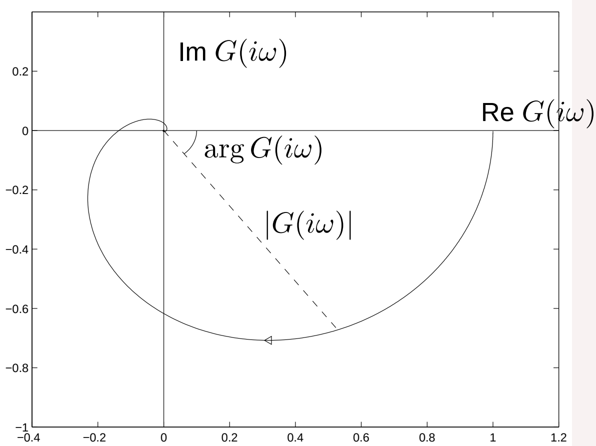
Assume scalar transfer function $G = \mathcal{L}g$. Input $u(t) = \sin \omega t$ gives

$$y(t) = \int_0^t g(\tau)u(t-\tau)d\tau = \text{Im}\left[\int_0^t g(\tau)e^{-i\omega\tau}d\tau \cdot e^{i\omega t}\right]$$

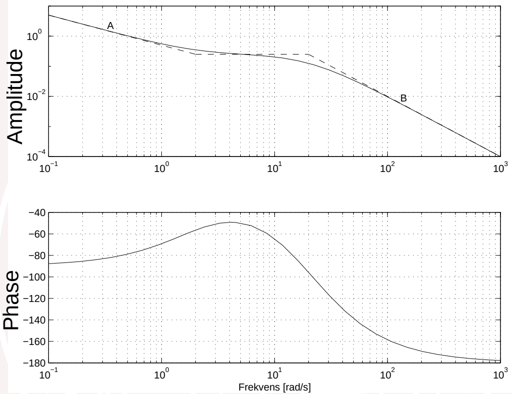
$$[t \to \infty] = \text{Im}\left(G(i\omega)e^{i\omega t}\right) = |G(i\omega)| \sin\left(\omega t + \arg G(i\omega)\right)$$

After a transient, also the output becomes sinusoidal

The Nyquist diagram



The Bode diagram



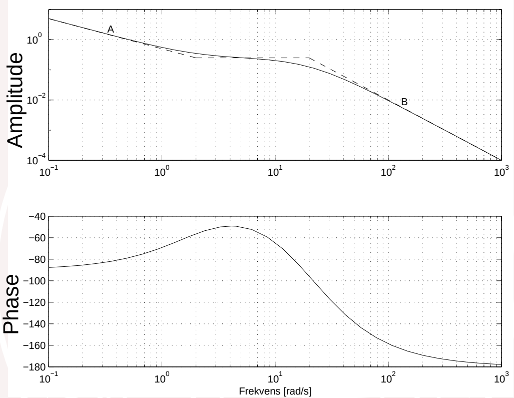
$$G = G_1 G_2 G_3 \qquad \begin{cases} \log|G| = \log|G_1| + \log|G_2| + \log|G_3| \\ \arg G = \arg G_1 + \arg G_2 + \arg G_3 \end{cases}$$

Each new factors enter additively!

Hint: Set Matlab scales

» `ctrlpref`

The Bode diagram



$$G = G_1 G_2 G_3 \qquad \begin{cases} \log|G| = \log|G_1| + \log|G_2| + \log|G_3| \\ \arg G = \arg G_1 + \arg G_2 + \arg G_3 \end{cases}$$

Each new factors enter additively!

Hint: Set Matlab scales

» `ctrlpref`

Signal norm and system gain

How to quantify

- the "size" of the signals u and y
- the "maximum amplification" between u and y

for multivariable systems?

Signal norm and system gain

The L_2-norm of a signal $y(t) \in \mathbf{R}^n$ is defined as

$$\|y\|_2 := \sqrt{\int_0^\infty |y(t)|^2 dt} = \sqrt{\frac{1}{2\pi}\int_{-\infty}^\infty |Y(i\omega)|^2 d\omega}$$

(The equality is known as Parseval's theorem)

The L_2-gain of a system $\mathcal{S}$ with input u and output $\mathcal{S}(u)$ is defined as

$$\|\mathcal{S}\| := \sup_u \frac{\|\mathcal{S}(u)\|_2}{\|u\|_2}$$

Mini-problem 3

What are the gains of the following scalar LTI systems?

1. $y(t) = -u(t)$ (a sign shift)
2. $y(t) = u(t-T)$ (a time delay)
3. $y(t) = \int_0^t u(\tau)d\tau$ (an integrator)
4. $y(t) = \int_0^t e^{-(t-\tau)}u(\tau)d\tau$ (a first order filter)

L_2-gain for LTI systems

Consider a stable LTI system $\mathcal{S}$ with input u and output $\mathcal{S}(u)$ having the transfer function $G(s)$. Then

$$\|\mathcal{S}\| := \sup_u \frac{\|\mathcal{S}(u)\|_2}{\|u\|_2} = \sup_\omega |G(i\omega)| := \|G\|_\infty$$

Proof. Let $y = \mathcal{S}(u)$. Then

$$\|y\|^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} |Y(i\omega)|^2 d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} |G(i\omega)|^2 \cdot |U(i\omega)|^2 d\omega \leq \|G\|_\infty^2 \|u\|^2$$

The inequality is arbitrarily tight when $u(t)$ is a sinusoid near the maximizing frequency.

(How to interpret $|G(i\omega)|$ for matrix transfer functions will be explained in Lecture 2.)

Summary of today's most important concepts

- $\mathcal{L}_2$-norm of signals
  - Definition: $\|y\|_2 := \sqrt{\int_0^\infty |y(t)|^2 dt}$
- $\mathcal{L}_2$-gain of systems
  - Definition: $\|\mathcal{S}\| := \sup_u \frac{\|\mathcal{S}(u)\|_2}{\|u\|_2}$
  - Special case—LTI systems: $\|\mathcal{S}\| = \sup_\omega |G(i\omega)|$

Course outline

L1–L5 Specifications, models and loop-shaping by hand

1. Introduction
2. Stability and robustness
3. Disturbance models
4. Control synthesis in frequency domain
5. Case study

L6–L8 Limitations on achievable performance

L9–L11 Controller optimization: Analytic approach

L12–L14 Controller optimization: Numerical approach