Because the number of evaporators represents an integer-valued variable, and because many engineers use tables and graphs as well as equations for evaporator calculations, some of the methods outlined in Chapters 9 and 10 can be applied for the optimization of multi-effect evaporator cascades.

EXAMPLE 11.4 BOILER/TURBO-GENERATOR SYSTEM OPTIMIZATION

Linear programming is often used in the design and operation of steam systems in the chemical industry. Figure E11.4 shows a steam and power system for a small power house fired by wood pulp. To produce electric power, this system contains two turbo-generators whose characteristics are listed in Table E11.4A. Turbine 1 is a double-extraction turbine with two intermediate streams leaving at 195 and 62 psi; the final stage produces condensate that is used as boiler feed water. Turbine 2 is a single-

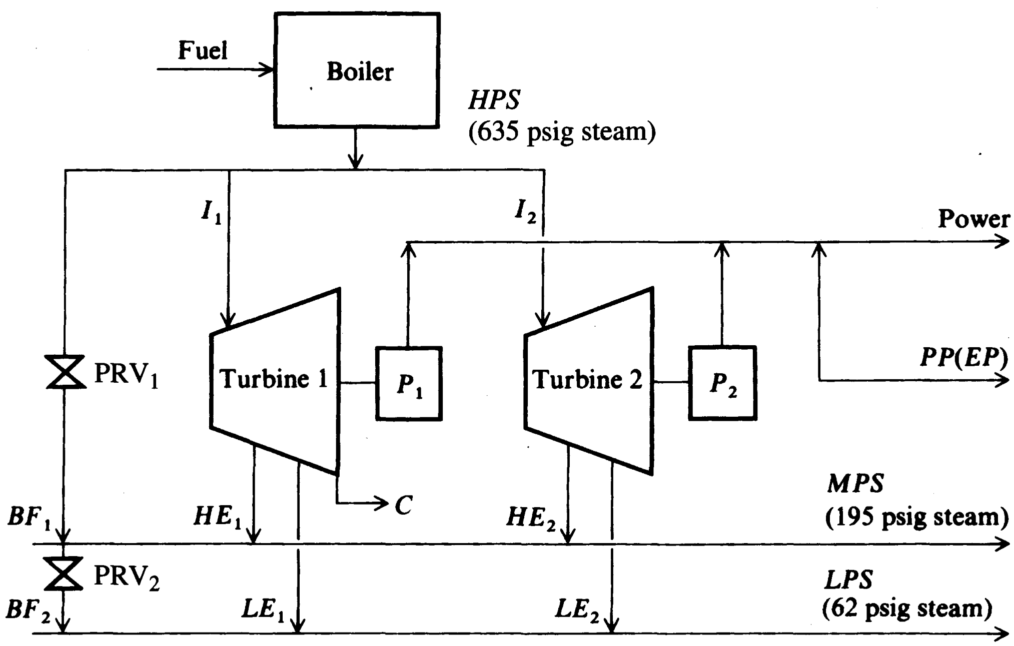

FIGURE E11.4
Boiler/turbo-generator system.
Key: I_i = inlet flow rate for turbine i [lb$_m$/h]
HE_i = exit flow rate from turbine i to 195 psi header [lb$_m$/h]
LE_i = exit flow rate from turbine i to 62 psi header [lb$_m$/h]
C = condensate flow rate from turbine 1 [lb$_m$/h]
P_i = power generated by turbine i [kW]
BF_1 = bypass flow rate from 635 psi to 195 psi header [lb$_m$/h]
BF_2 = bypass flow rate from 195 psi to 62 psi header [lb$_m$/h]
HPS = flow rate through 635 psi header [lb$_m$/h]
MPS = flow rate through 195 psi header [lb$_m$/h]
LPS = flow rate through 62 psi header [lb$_m$/h]
PP = purchased power [kW]
EP = excess power [kW] (difference of purchased power from base power)
PRV = pressure-reducing valve

extraction turbine with one intermediate stream at 195 psi and an exit stream leaving at 62 psi with no condensate being formed. The first turbine is more efficient due to the energy released from the condensation of steam, but it cannot produce as much power as the second turbine. Excess steam may bypass the turbines to the two levels of steam through pressure-reducing valves.

Table E11.4B lists information about the different levels of steam, and Table E11.4C gives the demands on the system. To meet the electric power demand, electric power may be purchased from another producer with a minimum base of 12,000 kW. If the electric power required to meet the system demand is less than this base, the power that is not used will be charged at a penalty cost. Table E11.4D gives the costs of fuel for the boiler and additional electric power to operate the utility system.

The system shown in Figure E11.4 may be modeled as linear constraints and combined with a linear objective function. The objective is to minimize the operating cost of the system by choice of steam flow rates and power generated or purchased, subject to the demands and restrictions on the system. The following objective function is the cost to operate the system per hour, namely, the sum of steam produced *HPS*, purchased power required *PP*, and excess power *EP*:

TABLE 11.4A
Turbine data

Turbine 1		Turbine 2	
Maximum generative capacity	6,250 kW	Maximum generative capacity	9,000 kW
Minimum load	2,500 kW	Minimum load	3,000 kW
Maximum inlet flow	192,000 lb_m/h	Maximum inlet flow	244,000 lb_m/h
Maximum condensate flow	62,000 lb_m/h	Maximum 62 psi exhaust	142,000 lb_m/h
Maximum internal flow	132,000 lb_m/h	High-pressure extraction at	195 psig
High-pressure extraction at	195 psig	Low-pressure extraction at	62 psig
Low-pressure extraction at	62 psig		

TABLE 11.4B
Steam header data

Header	Pressure (psig)	Temperature (°F)	Enthalpy (Btu/lb_m)
High-pressure steam	635	720	1359.8
Medium-pressure steam	195	130 superheat	1267.8
Low-pressure steam	62	130 superheat	1251.4
Feedwater (condensate)			193.0

TABLE 11.4C
Demands on the system

Resource	Demand
Medium-pressure steam (195 psig)	271,536 lb_m/h
Low-pressure steam (62 psig)	100,623 lb_m/h
Electric power	24,550 kW

TABLE 11.4D
Energy data

Fuel cost	$\$1.68/10^6$ Btu
Boiler efficiency	0.75
Steam cost (635 psi)	$\$2.24/10^6$ Btu
	$= \$2.24\ (1359.8 - 193)/10^6$
	$= \$0.002614/\text{lb}_m$
Purchased electric power	\$0.0239/kWh average
Demand penalty	\$0.009825/kWh
Base-purchased power	12,000 kW

$$\text{Minimize:} \quad f = 0.00261\,HPS + 0.0239\,PP + 0.00983\,EP \tag{a}$$

The constraints are gathered into the following specific subsets:

Turbine 1

$$\begin{aligned} P_1 &\le 6250 \\ P_1 &\ge 2500 \\ HE_1 &\le 192{,}000 \\ C &\le 62{,}000 \\ I_1 - HE_1 &\le 132{,}000 \end{aligned} \tag{b}$$

Turbine 2

$$\begin{aligned} P_2 &\le 9000 \\ P_2 &\ge 3000 \\ I_2 &\le 244{,}000 \\ LE_2 &\le 142{,}000 \end{aligned} \tag{c}$$

Material balances

$$\begin{aligned} HPS - I_1 - I_2 - BF_1 &= 0 \\ I_1 + I_2 + BF_1 - C - MPS - LPS &= 0 \\ I_1 - HE_1 - LE_1 - C &= 0 \\ I_2 - HE_2 - LE_2 &= 0 \\ HE_1 + HE_2 + BF_1 - BF_2 - MPS &= 0 \\ LE_1 + LE_2 + BF_2 - LPS &= 0 \end{aligned} \tag{d}$$

Power purchased

$$EP + PP \ge 12{,}000 \tag{e}$$

Demands

$$MPS \geq 271{,}536$$

$$LPS \geq 100{,}623 \qquad (f)$$

$$P_1 + P_2 + PP \geq 24{,}550$$

Energy balances

$$1359.8I_1 - 1267.8HE_1 - 1251.4LE_1 - 192C - 3413P_1 = 0$$

$$1359.8\, I_2 - 1267.8\, I_2 - 1251.4\, LE_2 - 3413\, P_2 = 0 \qquad (g)$$

TABLE E11.4E

Optimal solution to steam system LP

Variable	Name	Value	Status
1	I_1	136,329	BASIC
2	I_2	244,000	BOUND
3	HE_1	128,158	BASIC
4	HE_2	143,377	BASIC
5	LE_1	0	ZERO
6	LE_2	100,623	BASIC
7	C	8,170	BASIC
8	BF_1	0	ZERO
9	BF_2	0	ZERO
10	HPS	380,329	BASIC
11	MPS	271,536	BASIC
12	LPS	100,623	BASIC
13	P_1	6,250	BOUND
14	P_2	7,061	BASIC
15	PP	11,239	BASIC
16	EP	761	BASIC

Value of objective function = 1268.75 \$/h
BASIC = basic variable
ZERO = 0
BOUND = variable at its upper bound

Table E11.4E lists the optimal solution to the linear program posed by Equations (a)–(g). Basic and nonbasic (zero) variables are identified in the table; the minimum cost is \$1268.75/h. Note that $EP + PP$ must sum to 12,000 kWh; in this case the excess power is reduced to 761 kWh.

REFERENCES

Athier, G.; P. Floquet; L. Pibouleau; et al. "Process Optimization by Simulated Annealing and NLP Procedures. Application to Heat Exchanger Network Synthesis." *Comput Chem Eng* **21 (Suppl):** S475–S480 (1997).