$TOL := 10^{-12}$ Primeiro exercício sobre integração

1. O fluxo, $q(\lambda,T)d\lambda$, com que a energia radiante é emitida da superfície de um corpo negro com comprimento de onda entre λ e $\lambda+d\lambda$ é dada pela equação de Planck:

$$q(\lambda, T)\cdot d\lambda = \frac{2\cdot\pi\cdot h\cdot c^2}{\lambda^5\cdot\left[\exp\left(\frac{h\cdot c}{k\cdot\lambda\cdot T}\right)-1\right]}\cdot d\lambda$$

onde: c: velocidade da luz: = $2.997925.\ 10^{10}$ cm/s;
h: constante de Planck = $6.6256.\ 10^{-27}$ erg. s
k: constante de Boltzmann = $1.38054\ .\ 10^{-16}$ erg /K
T: temperatura [K];
λ : comprimento de onda [cm].

Calcule o fluxo total da energia emitida [em erg/cm^2/s] de um corpo negro entre os comprimentos de onde : λ_1 = 3933.666 Angstrom e λ_2 = 5895.923 Angstrom às temperaturas de 2000 e 6000 K.

$c := 2.997925\cdot10^{10}\cdot\frac{cm}{sec}$ $h := 6.6256\cdot10^{-27}\cdot erg\cdot sec$ $k := 1.38054\cdot10^{-16}\cdot\frac{erg}{K}$ $\lambda_1 := 3933.666\cdot10^{-8}\cdot cm$ $\lambda_2 := 5895.923\cdot10^{-8}\cdot cm$

$$q(\lambda, T) := \frac{2\cdot\pi\cdot h\cdot c^2}{\lambda^5\cdot\left(e^{\frac{h\cdot c}{k\cdot\lambda\cdot T}}-1\right)}$$ Integrando

$T := 2000\cdot K$ $Q := \int_{\lambda_1}^{\lambda_2} q(\lambda, T)\,d\lambda$ $Q = 1.631961\times10^{6}\cdot\frac{erg}{cm^2\cdot sec}$ Fluxo de Energia a 2000 K

$T := 6000\cdot K$ $Q := \int_{\lambda_1}^{\lambda_2} q(\lambda, T)\,d\lambda$ $Q = 1.90054\times10^{10}\cdot\frac{erg}{cm^2\cdot sec}$ Fluxo de Energia a 6000 K

Adimensionamento das Variáveis Comprimento para o Adimensionamento de λ $L(T) := \frac{h\cdot c}{k\cdot T}$

Fluxo Padrão para o Adimensionamento de Q $Q_{padrão}(T) := \frac{2\cdot\pi\cdot h\cdot c^2}{L(T)^4}$

INTEGRAÇÃO PELO MÉTODO DE SIMPSON

$$\text{Simpson}(f,a,b,n,\delta) := \begin{array}{|l} S0 \leftarrow f(a)+f(b) \\ h \leftarrow \dfrac{b-a}{2\cdot n} \\ S1 \leftarrow \displaystyle\sum_{j=1}^{n} f[a+(2\cdot j-1)\cdot h] \\ S2 \leftarrow \displaystyle\sum_{j=1}^{n-1} f(a+2\cdot j\cdot h) \\ I \leftarrow \dfrac{h}{3}\cdot(S0+4\cdot S1+2\cdot S2) \\ \Delta \leftarrow 1 \\ \text{while } \Delta > \delta \\ \quad \begin{array}{|l} n \leftarrow n+n \\ h \leftarrow \dfrac{h}{2} \\ S2 \leftarrow S1+S2 \\ S1 \leftarrow \displaystyle\sum_{j=1}^{n} f[a+(2\cdot j-1)\cdot h] \\ I_{prox} \leftarrow \dfrac{h}{3}\cdot(S0+4\cdot S1+2\cdot S2) \\ \Delta \leftarrow \left|I-I_{prox}\right| \\ I \leftarrow I_{prox} \end{array} \\ R \leftarrow \begin{pmatrix} \dfrac{16\cdot I_{prox}-I}{15} \\ h \end{pmatrix} \end{array}$$

$\delta := TOL \quad n := 20$

$T := 2000\cdot K \qquad a := \dfrac{\lambda_1}{L(T)} \qquad b := \dfrac{\lambda_2}{L(T)} \qquad f(x) := \dfrac{q(x\cdot L(T),T)\cdot L(T)}{Q_{padrão}(T)} \qquad Re := \text{Simpson}(f,a,b,n,\delta)$

$Re^T = (0.011682 \quad 0.000085) \qquad Q := Re_0\cdot Q_{padrão}(T) \qquad Q = 1.631961\times 10^6\cdot\dfrac{erg}{cm^2\cdot sec}$ Fluxo de Energia a 2000 K

$T := 6000\cdot K \qquad a := \dfrac{\lambda_1}{L(T)} \qquad b := \dfrac{\lambda_2}{L(T)} \qquad f(x) := \dfrac{q(x\cdot L(T),T)\cdot L(T)}{Q_{padrão}(T)} \qquad Re := \text{Simpson}(f,a,b,n,\delta)$

$Re^T = (1.679629 \quad 0.000128) \qquad Q := Re_0\cdot Q_{padrão}(T) \qquad Q = 1.90054\times 10^{10}\cdot\dfrac{erg}{cm^2\cdot sec}$ Fluxo de Energia a 6000 K

Segundo Exercício de integração : O TROCADOR DE CALOR

(a) Projeto do trocador para aquecer o gás carbônico

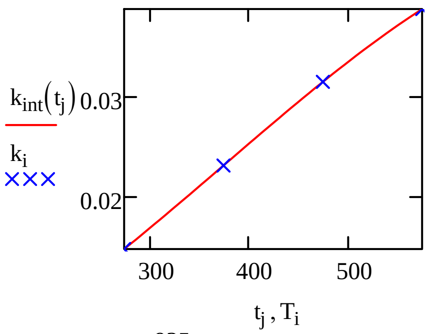
$$T := \begin{pmatrix} 32+460 \\ 212+460 \\ 392+460 \\ 572+460 \end{pmatrix}\cdot R \qquad T = \begin{pmatrix} 492 \\ 672 \\ 852 \\ 1032 \end{pmatrix}\cdot R \qquad T = \begin{pmatrix} 273.333333 \\ 373.333333 \\ 473.333333 \\ 573.333333 \end{pmatrix} K \qquad t := 460\cdot R$$

$$k := \begin{pmatrix} .0085 \\ .0133 \\ .0181 \\ .02228 \end{pmatrix}\cdot\frac{BTU}{hr\cdot ft\cdot R} \quad k = \begin{pmatrix} 0.0085 \\ 0.0133 \\ 0.0181 \\ 0.02228 \end{pmatrix}\cdot\frac{BTU}{hr\cdot ft\cdot R}$$

$$k_{int}(t) := \frac{(t-T_1)\cdot(t-T_2)\cdot(t-T_3)}{(T_0-T_1)\cdot(T_0-T_2)\cdot(T_0-T_3)}\cdot k_0 \ldots + \frac{(t-T_0)\cdot(t-T_2)\cdot(t-T_3)}{(T_1-T_0)\cdot(T_1-T_2)\cdot(T_1-T_3)}\cdot k_1 \ldots + \frac{(t-T_0)\cdot(t-T_1)\cdot(t-T_3)}{(T_2-T_0)\cdot(T_2-T_1)\cdot(T_2-T_3)}\cdot k_2 \ldots + \frac{(t-T_0)\cdot(t-T_1)\cdot(t-T_2)}{(T_3-T_0)\cdot(T_3-T_1)\cdot(T_3-T_2)}\cdot k_3$$

$j := 0..50 \quad t_j := T_0 + (T_3 - T_0)\cdot\frac{j}{50} \quad i := 0..3$

Verificação da Adequação da Interpolação do Coeficiente de Condutividade



$\mu(\theta) := .0032\cdot\left(\frac{\theta}{460\cdot R}\right)^{.935}\cdot\frac{lb}{ft\cdot hr}$ $\quad c_P(t) := .251\cdot\frac{BTU}{lb\cdot R} + 3.46\cdot 10^{-5}\cdot\frac{BTU}{lb\cdot R^2}\cdot(t - 460\cdot R) - \frac{14400\cdot\frac{BTU\cdot R}{lb}}{t^2}$

$W := 22.5\cdot\frac{lb}{hr} \quad D := .495\cdot in \quad Re(t) := \frac{4\cdot W}{\pi\cdot D\cdot\mu(t)} \quad Pr(t) := \frac{\mu(t)\cdot c_P(t)}{k_{int}(t)}$

$h(t) := .023\cdot\frac{k_{int}(t)}{D}\cdot Re(t)^{.8}\cdot Pr(t)^{.4}$

$T_{input} := (60 + 460)\cdot R \quad T_{output} := (500 + 460)\cdot R \quad T_{vapor} := (550 + 460)\cdot R$

$L := \frac{W}{\pi\cdot D}\cdot\int_{T_{input}}^{T_{output}} \frac{c_P(t)}{h(t)\cdot(T_{vapor} - t)}\,dt \quad L = 0.71\cdot m$

(b) Projeto do trocador para aquecer o etileno glicol líquido

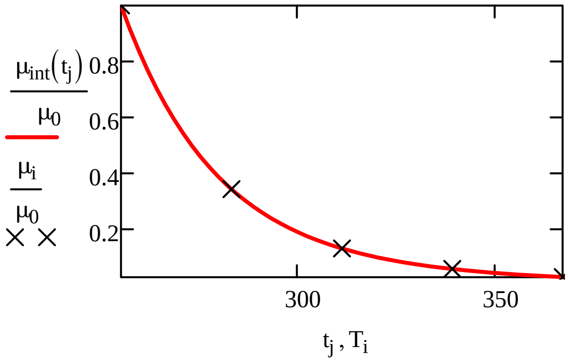
$$T := \begin{pmatrix} 460 \\ 50 + 460 \\ 100 + 460 \\ 150 + 460 \\ 200 + 460 \end{pmatrix}\cdot R \quad \mu := \begin{pmatrix} 242 \\ 82.1 \\ 30.5 \\ 12.6 \\ 5.57 \end{pmatrix}\cdot\frac{lb}{ft\cdot hr} \quad i := 0..4 \quad x_i := \frac{\mu_i}{\mu_0}$$

$$g_1(t) := \frac{(t-T_0)\cdot(t-T_2)\cdot(t-T_3)\cdot(t-T_4)}{(T_1-T_0)\cdot(T_1-T_2)\cdot(T_1-T_3)\cdot(T_1-T_4)} \quad g_2(t) := \frac{(t-T_0)\cdot(t-T_1)\cdot(t-T_3)\cdot(t-T_4)}{(T_2-T_0)\cdot(T_2-T_1)\cdot(T_2-T_3)\cdot(T_2-T_4)}$$

$$g_3(t) := \frac{(t-T_0)\cdot(t-T_1)\cdot(t-T_2)\cdot(t-T_4)}{(T_3-T_0)\cdot(T_3-T_1)\cdot(T_3-T_2)\cdot(T_3-T_4)} \quad g_4(t) := \frac{(t-T_0)\cdot(t-T_1)\cdot(t-T_2)\cdot(t-T_3)}{(T_4-T_0)\cdot(T_4-T_1)\cdot(T_4-T_2)\cdot(T_4-T_3)}$$

$\mu_{int}(t) := (x_1)^{g_1(t)}\cdot(x_2)^{g_2(t)}\cdot(x_3)^{g_3(t)}\cdot(x_4)^{g_4(t)}\cdot\mu_0 \quad j := 0..50 \quad t_j := T_0 + (T_4 - T_0)\cdot\frac{j}{50}$

Verificação da Adequação da Função Interpoladora para a Viscosidade



$k := .153\cdot\frac{BTU}{hr\cdot ft\cdot R}$ $\quad c_P(t) := .53\cdot\frac{BTU}{lb\cdot R} + .00065\cdot\frac{BTU}{lb\cdot R^2}\cdot(t - 460\cdot R)$

$W := 45000\cdot\frac{lb}{hr}$ $\quad D := 1.032\cdot in$ $\quad Re(t) := \frac{4\cdot W}{\pi\cdot D\cdot \mu_{int}(t)}$ $\quad Pr(t) := \frac{\mu_{int}(t)\cdot c_P(t)}{k}$ $\quad h(t) := .023\cdot\frac{k_{int}(t)}{D}\cdot Re(t)^{.8}\cdot Pr(t)^{.4}$

$T_{input} := 460\cdot R$ $\quad T_{output} := (180 + 460)\cdot R$ $\quad T_{vapor} := (250 + 460)\cdot R$

$$L := \frac{W}{\pi\cdot D}\cdot\int_{T_{input}}^{T_{output}} \frac{c_P(t)}{h(t)\cdot(T_{vapor} - t)}\,dt \qquad L = 754.15\cdot m$$

3) O Problema do Foguete

a) Um foguete é lançado do solo sendo sua aceleração registrada nos 80 primeiros segundos após seu lançamento. Estes valores estão tabelados abaixo

t (seg.)	0	10	20	30	40	50	60	70	80
a (m/s^2)	30.00	31.63	33.44	35.47	37.75	40.33	43.29	46.69	50.67

Baseado nos valores tabelados calcule a velocidade e a altura do foguete ao cabo dos 80 s.

$$t := \begin{pmatrix} 0 \\ 10 \\ 20 \\ 30 \\ 40 \\ 50 \\ 60 \\ 70 \\ 80 \end{pmatrix} sec \qquad a := \begin{pmatrix} 30 \\ 31.63 \\ 33.44 \\ 35.47 \\ 37.75 \\ 40.33 \\ 43.29 \\ 46.69 \\ 50.67 \end{pmatrix}\cdot\frac{m}{sec^2} \qquad h := 10\cdot sec$$

$v_0 := 0\cdot\frac{m}{sec}$ $\quad k := 1..4$ $\quad v_k := v_{k-1} + \frac{h}{3}\cdot\left(a_{2\cdot k-2} + 4\cdot a_{2\cdot k-1} + a_{2\cdot k}\right)$

$v^T = (0\ \ 0.6332\ \ 1.343433\ \ 2.1513\ \ 3.087033)\cdot\frac{km}{sec}$ $\quad S_0 := 0\cdot m$ $\quad k := 1..2$ $\quad S_k := S_{k-1} + \frac{2\cdot h}{3}\cdot\left(v_{2\cdot k-2} + 4\cdot v_{2\cdot k-1} + v_{2\cdot k}\right)$

$S^T = (0\ \ 25.841556\ \ 112.746)\cdot km$

Resultados Finais: $\quad v_4 = 3.087033\cdot\frac{km}{sec}$ $\quad S_2 = 112.746\cdot km$

4-) Quadratura Proposta

a) Determine x_1 e x_2 de modo que a fórmula de quadratura abaixo apresente a maior ordem de precisão possível:

$$\int_{-1}^{1} f(x)\cdot dx \cong \frac{1}{3}\cdot[f(-1)+2\cdot f(x_1)+3\cdot f(x_2)]$$

$x_2(x_1) := \dfrac{1-2\cdot x_1}{3}$ $\quad F(x_1) := 2\cdot x_1^2 + 3\cdot x_2(x_1)^2 - 1$

Primeira Solução:

$x_1 := -1$ $\quad x_1 := \text{root}(F(x_1), x_1)$ $\quad x_1 = -0.289898$ $\quad x_2(x_1) = 0.526599$ $\quad \dfrac{1-\sqrt{6}}{5} = -0.289898$

Segunda Solução:

$x_1 := 1$ $\quad x_1 := \text{root}(F(x_1), x_1)$ $\quad x_1 = 0.689898$ $\quad x_2(x_1) = -0.126599$ $\quad \dfrac{1+\sqrt{6}}{5} = 0.689898$

6) Quadratura Gaussiana com peso = $\sqrt{x}$

Determine as abcissas e pesos da fórmula de quadratura tipo

Gauss: $\int_{0}^{1} \sqrt{x}\cdot f(x)\cdot dx \cong W_1\cdot f(x_1) + W_2\cdot f(x_2)$

$I(k) := \dfrac{1}{k+1.5}$ $\quad c := -\text{lsolve}\left[\begin{pmatrix} I(0) & I(1) \\ I(1) & I(2) \end{pmatrix}, \begin{pmatrix} I(2) \\ I(3) \end{pmatrix}\right]$ $\quad c_2 := 1$ $\quad r := \text{polyroots}(c)$ $\quad r^T = (0.289949 \quad 0.821162)$

$W := \text{lsolve}\left[\begin{pmatrix} 1 & 1 \\ r_0 & r_1 \end{pmatrix}, \begin{pmatrix} I(0) \\ I(1) \end{pmatrix}\right]$ $\quad W^T = (0.277556 \quad 0.389111)$

7) Quadratura Gaussiana com peso = $\ln\left(\frac{1}{x}\right)$

$I(k) := \dfrac{1}{(k+1)^2}$ $\quad c := -\text{lsolve}\left[\begin{pmatrix} I(0) & I(1) \\ I(1) & I(2) \end{pmatrix}, \begin{pmatrix} I(2) \\ I(3) \end{pmatrix}\right]$ $\quad c_2 := 1$ $\quad r := \text{polyroots}(c)$ $\quad r^T = (0.112009 \quad 0.602277)$

$W := \text{lsolve}\left[\begin{pmatrix} 1 & 1 \\ r_0 & r_1 \end{pmatrix}, \begin{pmatrix} I(0) \\ I(1) \end{pmatrix}\right]$ $\quad W^T = (0.718539 \quad 0.281461)$

8-) Quadratura Especial com α

Determine os valores de W_1, W_2 e W_3 na fórmula de quadratura:

$\int_{-1}^{1} f(x)\cdot dx \cong W_1\cdot f(-1) + W_2\cdot f(1) + W_3\cdot f(\alpha)$ onde α é um número entre –1 e +1.

Teste seu resultado para a função $f(x) = \sqrt{\dfrac{5\cdot x+13}{2}}$ e α = -0,1.

$k(\alpha) := \text{lsolve}\left[\begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & \alpha \\ 1 & 1 & \alpha^2 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ \frac{2}{3} \end{pmatrix}\right]$ $\quad \alpha := -.1$ $\quad K := k(\alpha)$ $\quad K^T = (0.259259 \quad 0.393939 \quad 1.346801)$

$f(x) := \sqrt{\dfrac{5\cdot x+13}{2}}$ $\quad \int_{-1}^{1} f(x)\,dx = 5.066667$ $\quad K_0\cdot f(-1) + K_1\cdot f(1) + K_2\cdot f(\alpha) = 5.06734$ $\quad 5.066667 - 5.06734 = -6.7\times 10^{-4}$

9-)Quadratura de Lobatto

Deseja-se desenvolver uma fórmula de quadratura do tipo Lobatto:

$$\int_{-1}^{1} f(x)\cdot dx = W_1\cdot[f(-1)+f(1)] + W_2\cdot[f(-\alpha)+f(\alpha)] + W_3\cdot f(0)$$

Calcule o valor da constante $\underline{\alpha}$ e de W_1, W_2 e W_3 de modo que o método apresente a maior ordem de precisão possível.

$$\alpha := \sqrt{\frac{3}{7}} \qquad w := \mathrm{lsolve}\left[\begin{pmatrix} 1 & \frac{3}{7} \\ 1 & \frac{9}{49} \end{pmatrix}, \begin{pmatrix} \frac{1}{3} \\ \frac{1}{5} \end{pmatrix}\right] \qquad w_2 := 2\cdot(1 - w_0 - w_1)$$

$$w^T = (0.1 \quad 0.544444 \quad 0.711111)$$

$$k := 0..8 \qquad I_k := \int_{-1}^{1} x^k\, dx \qquad I_{num_k} := w_0\cdot\left[1 + (-1)^k\right] + w_1\cdot\left[\alpha^k + (-\alpha)^k\right] + w_2\cdot 0^k$$

$$I^T = (2 \quad 0 \quad 0.666667 \quad 0 \quad 0.4 \quad 0 \quad 0.285714 \quad 0 \quad 0.222222)$$

$$I_{num}{}^T = (2 \quad 0 \quad 0.666667 \quad 0 \quad 0.4 \quad 0 \quad 0.285714 \quad 0 \quad 0.236735) \qquad (I - I_{num})^T = (0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad -0.014512)$$

Exercício10-Quadratura de Chebishev

Determine as abcissas e o peso da fórmula de quadratura tipo Chebishev:

$$\int_{0}^{1} \frac{1}{\sqrt{x}}\cdot f(x)\cdot dx \cong W\cdot[f(x_1)+f(x_2)+f(x_3)]$$

$$x := .25 \qquad y := .5 \qquad z := .75 \qquad \frac{3}{7} = 0.428571$$

Given $\qquad x + y + z = 1 \qquad x^2 + y^2 + z^2 = \frac{3}{5} \qquad x^3 + y^3 + z^3 = \frac{3}{7}$

$$\mathrm{Solucao} := \mathrm{Find}(x,y,z) \qquad \mathrm{Solucao} := \mathrm{sort}(\mathrm{Solucao}) \qquad \mathrm{Solucao}^T = (0.071094 \quad 0.178522 \quad 0.750384) \qquad X := \mathrm{Solucao}$$

$$f(x) := \begin{bmatrix} \sum_{i=0}^{2} x_i - 1 \\ \sum_{i=0}^{2} (x_i)^2 - .6 \\ \sum_{i=0}^{2} (x_i)^3 - \frac{3}{7} \end{bmatrix} \qquad |f(X)| = 1.57\times 10^{-16}$$

```
Newton_Raphson(x) := | flag ← 0
                     | while flag = 0
                     |   | y ← f(x)
                     |   | Δ ← lsolve[ [ 1          1          1         ]
                     |   |             [ 2·x_0      2·x_1      2·x_2     ]
                     |   |             [ 3·(x_0)^2  3·(x_1)^2  3·(x_2)^2 ] , y ]
                     |   | x ← x − Δ
                     |   | flag ← 1 if |y| < TOL
                     |   | flag ← 1 if |Δ| < TOL
                     | x
```

$$x := \begin{pmatrix} \frac{1}{6} \\ \frac{1}{3} \\ .5 \end{pmatrix}$$

$$x := \mathrm{Newton\_Raphson}(x) \qquad x := \mathrm{sort}(x) \qquad |f(x)| = 0$$

$x^T = (0.071094 \quad 0.178522 \quad 0.750384)$

$k := 0..4$

$I_{exato_k} := \dfrac{1}{k + \frac{1}{2}} \qquad I_{chebishev_k} := \dfrac{2}{3} \cdot \sum_{i=0}^{2} (x_i)^k \qquad \sum_{i=0}^{2} (x_i)^3 = 0.428571$

$I_{exato}^T = (2 \quad 0.666667 \quad 0.4 \quad 0.285714 \quad 0.222222)$

$I_{chebishev}^T = (2 \quad 0.666667 \quad 0.4 \quad 0.285714 \quad 0.212063) \qquad (I_{exato} - I_{chebishev})^T = (0 \quad 0 \quad 5.55 \times 10^{-17} \quad 0 \quad 0.01)$

11-Determine as abscissas e os pesos das fórmulas de quadratura tipo Radau:

$$\text{(i)} \int_0^1 \frac{1}{\sqrt{x}} \cdot f(x) \cdot dx \cong \omega_0 \cdot f(0) + \omega_1 \cdot f(x_1) + \omega_2 \cdot f(x_2)$$

$$\text{(ii)} \int_0^1 \frac{1}{\sqrt{x}} \cdot f(x) \cdot dx \cong \omega_1 \cdot f(x_1) + \omega_2 \cdot f(x_2) + \omega_3 \cdot f(1)$$

Confronte as precisões das fórmulas de quadratura dos exercícios 10 e 11.

$c := -\text{lsolve}\left[\begin{pmatrix} \frac{1}{3} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{7} \end{pmatrix}, \begin{pmatrix} \frac{1}{7} \\ \frac{1}{9} \end{pmatrix}\right] \qquad c_2 := 1 \qquad raiz := \text{polyroots}(c) \qquad raiz^T = (0.289949 \quad 0.821162)$

$W := \text{lsolve}\left[\begin{pmatrix} 1 & 1 \\ raiz_0 & raiz_1 \end{pmatrix}, \begin{pmatrix} \frac{2}{3} \\ \frac{2}{5} \end{pmatrix}\right] \qquad i := 1..2 \qquad \omega_i := \dfrac{W_{i-1}}{raiz_{i-1}} \qquad \omega_0 := 2 - \omega_1 - \omega_2$

$\omega^T = (0.568889 \quad 0.957257 \quad 0.473854) \qquad raiz_2 := raiz_1 \qquad raiz_1 := raiz_0 \qquad raiz_0 := 0 \qquad raiz^T = (0 \quad 0.289949 \quad 0.821162)$

$g(k) := \dfrac{1}{\left(k - \frac{1}{2}\right) \cdot \left(k + \frac{1}{2}\right)} \qquad d := -\text{lsolve}\left[\begin{pmatrix} g(1) & g(2) \\ g(2) & g(3) \end{pmatrix}, \begin{pmatrix} g(3) \\ g(4) \end{pmatrix}\right] \qquad d_2 := 1$

$Raiz := \text{polyroots}(d) \qquad Raiz_2 := 1 \qquad Raiz^T = (0.081357 \quad 0.58531 \quad 1)$

$\sigma := \text{lsolve}\left[\begin{bmatrix} 1 & 1 & 1 \\ Raiz_0 & Raiz_1 & 1 \\ (Raiz_0)^2 & (Raiz_1)^2 & 1 \end{bmatrix}, \begin{pmatrix} 2 \\ \frac{2}{3} \\ \frac{2}{5} \end{pmatrix}\right] \qquad \sigma^T = (1.109717 \quad 0.75695 \quad 0.133333)$

$k := 0..5$

$R_{k,0} := \dfrac{2}{2 \cdot k + 1} \qquad R_{k,1} := \sum_{i=0}^{2} \left[\omega_i \cdot (raiz_i)^k\right] \qquad R_{k,2} := \sum_{i=0}^{2} \left[\sigma_i \cdot (Raiz_i)^k\right] \qquad R_{k,3} := \dfrac{2}{3} \cdot \sum_{i=0}^{2} (x_i)^k$

$$R = \begin{pmatrix} 2 & 2 & 2 & 2 \\ 0.666667 & 0.666667 & 0.666667 & 0.666667 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.285714 & 0.285714 & 0.285714 & 0.285714 \\ 0.222222 & 0.222222 & 0.222222 & 0.212063 \\ 0.181818 & 0.178886 & 0.185336 & 0.15873 \end{pmatrix}$$

$\left|R^{\langle 0 \rangle} - R^{\langle 1 \rangle}\right| = 0.002932 \qquad \int_0^1 \sqrt{x} \cdot \left[(x - raiz_1) \cdot (x - raiz_2)\right]^2 dx = 0.002932$

$\left|R^{\langle 0 \rangle} - R^{\langle 2 \rangle}\right| = 0.003518 \qquad \int_0^1 \left(\frac{1 - x}{\sqrt{x}}\right) \cdot \left[(x - Raiz_0) \cdot (x - Raiz_1)\right]^2 dx = 0.003518$

$\left|R^{\langle 0 \rangle} - R^{\langle 3 \rangle}\right| = 0.025224$

$\int_0^1 \sqrt{\xi} \cdot (\xi - x_0) \cdot (\xi - x_1) \cdot (\xi - x_2) \, d\xi = 0.010159 \qquad 0.222222 - 0.212063 = 0.010159$

12-Determine as abscissas e pesos da fórmula de quadratura tipo Gauss para o cômputo de integrais duplas:

$$\int_{x=0}^{1}\int_{y=0}^{1} f(x,y)\cdot dy\cdot dx \cong \omega_{1,1}\cdot f(x_1,y_1)+\omega_{1,2}\cdot f(x_1,y_2)+\omega_{2,1}\cdot f(x_2,y_1)+\omega_{2,2}\cdot f(x_2,y_2)$$

$$\lambda := \begin{pmatrix} \dfrac{1-\dfrac{1}{\sqrt{3}}}{2} \\ \dfrac{1+\dfrac{1}{\sqrt{3}}}{2} \end{pmatrix}$$

$m := 0..3$ $\quad n := 0..3$ $\quad Q_{m+n,0} := \dfrac{1}{m+1}\cdot\dfrac{1}{n+1}$ $\quad Q_{m+n,1} := \dfrac{\sum_{i=0}^{1}\sum_{j=0}^{1}\left[(\lambda_i)^m\cdot(\lambda_j)^n\right]}{4}$

$$Q = \begin{pmatrix} 1 & 1 \\ 0.5 & 0.5 \\ 0.333333 & 0.333333 \\ 0.25 & 0.25 \\ 0.125 & 0.125 \\ 0.083333 & 0.083333 \\ 0.0625 & 0.0625 \end{pmatrix} \qquad \left|Q^{\langle 0\rangle} - Q^{\langle 1\rangle}\right| = 1.96\times 10^{-17}$$

$f(x,y) := e^{-x^2}\cdot\left(\dfrac{1-\cos(\pi\cdot y)}{2}\right)$ $\quad \int_0^1\int_0^1 f(x,y)\,dx\,dy = 0.373412$ $\quad \dfrac{\sum_{i=0}^{1}\sum_{j=0}^{1} f(\lambda_i,\lambda_j)}{4} = 0.373297$

13-Calcule a integral imprópria: $\int_0^{\infty}\left(\dfrac{e^{-x^2}}{1+x^2}\right)\cdot dx$ com uma precisão de quatro algarismos significativos.

$$\int_0^{\infty}\frac{e^{-x^2}}{1+x^2}\,dx = 0.671647$$

$J_0 := \dfrac{\sqrt{\pi}}{2}$ $\quad J_1 := .5$ $\quad k := 2..10$ $\quad J_k := \dfrac{k-1}{2}\cdot J_{k-2}$

$c := -\text{lsolve}\left[\begin{pmatrix} J_0 & J_1 \\ J_1 & J_2 \end{pmatrix},\begin{pmatrix} J_2 \\ J_3 \end{pmatrix}\right]$ $\quad \Delta := \sqrt{(c_1)^2 - 4\cdot c_0}$ $\quad q := \begin{bmatrix} -.5\cdot(c_1+\Delta) \\ .5\cdot(\Delta - c_1) \end{bmatrix}$ $\quad q = \begin{pmatrix} 0.300194 \\ 1.252421 \end{pmatrix}$

$\omega := \text{lsolve}\left[\begin{pmatrix} 1 & 1 \\ q_0 & q_1 \end{pmatrix},\begin{pmatrix} J_0 \\ J_1 \end{pmatrix}\right]$ $\quad \omega = \begin{pmatrix} 0.640529 \\ 0.245698 \end{pmatrix}$

$Res_0 := \sum_{i=0}^{1}\left[\dfrac{\omega_i}{1+(q_i)^2}\right]$ $\quad Res_0 = 0.683235$ $\quad 0.671647 - 0.683235 = -1.16\times 10^{-2}$

$c := -\text{lsolve}\left[\begin{pmatrix} J_0 & J_1 & J_2 \\ J_1 & J_2 & J_3 \\ J_2 & J_3 & J_4 \end{pmatrix},\begin{pmatrix} J_3 \\ J_4 \\ J_5 \end{pmatrix}\right]$ $\quad c_3 := 1$ $\quad q := \text{polyroots}(c)$ $\quad q^T = (0.190554 \;\; 0.848252 \;\; 1.799777)$

$$\omega := \text{lsolve}\left[\begin{pmatrix} 1 & 1 & 1 \\ q_0 & q_1 & q_2 \\ (q_0)^2 & (q_1)^2 & (q_2)^2 \end{pmatrix}, \begin{pmatrix} J_0 \\ J_1 \\ J_2 \end{pmatrix}\right] \qquad \omega^T = (0.44603 \quad 0.396468 \quad 0.043729)$$

$$Res_1 := \sum_{i=0}^{2} \left[\frac{\omega_i}{1+(q_i)^2}\right] \qquad Res_1 = 0.671285 \qquad 0.671647 - 0.671285 = 3.62 \times 10^{-4}$$

$$c := -\text{lsolve}\left[\begin{pmatrix} J_0 & J_1 & J_2 & J_3 \\ J_1 & J_2 & J_3 & J_4 \\ J_2 & J_3 & J_4 & J_5 \\ J_3 & J_4 & J_5 & J_6 \end{pmatrix}, \begin{pmatrix} J_4 \\ J_5 \\ J_6 \\ J_7 \end{pmatrix}\right] \qquad c_4 := 1 \qquad q := \text{polyroots}(c) \qquad q^T = (0.133776 \quad 0.624325 \quad 1.342538 \quad 2.262664)$$

$$\omega := \text{lsolve}\left[\begin{pmatrix} 1 & 1 & 1 & 1 \\ q_0 & q_1 & q_2 & q_3 \\ (q_0)^2 & (q_1)^2 & (q_2)^2 & (q_3)^2 \\ (q_0)^3 & (q_1)^3 & (q_2)^3 & (q_3)^3 \end{pmatrix}, \begin{pmatrix} J_0 \\ J_1 \\ J_2 \\ J_3 \end{pmatrix}\right] \qquad \omega^T = (0.325303 \quad 0.421107 \quad 0.133443 \quad 0.006374)$$

$$Res_2 := \sum_{i=0}^{3} \left[\frac{\omega_i}{1+(q_i)^2}\right] \qquad Res_2 = 0.671245 \qquad 0.671647 - 0.671245 = 4.02 \times 10^{-4}$$

$$pp(x) := \frac{e^{-x^2}}{1+x^2}$$

$$RR := \text{Simpson}\left(pp, 0, 100, 20, 10^{-6}\right) \qquad RR = \begin{pmatrix} 0.671647 \\ 0.078125 \end{pmatrix}$$

15-Calcule a integral imprópria: $\int_0^\infty \left(e^{-x} \cdot \ln(x)\right) \cdot dx$ com uma precisão de cinco algarismos significativos.

$$\int_0^{1000} e^{-x} \cdot \ln(x)\, dx = -0.577216 \qquad \gamma = 0.577216$$

$$\int_0^{1000} e^{-x} \cdot x \cdot \ln(x)\, dx - 1 = -0.577216 \qquad \lim_{x \to 0} (x \cdot \ln(x)) \to 0$$

$N := 20$

$i := 0 .. N \quad j := 0 .. N$

$A_{i,j} := (i+j)! \quad b_i := (N+1+i)!$

$c := -\text{lsolve}(A, b) \qquad c_{N+1} := 1 \qquad q := \text{polyroots}(c) \qquad A_{i,j} := (q_j)^i \qquad b_i := i! \qquad \omega := \text{lsolve}(A, b)$

$$I_{quad} := \sum_{i=0}^{N} \left(\omega_i \cdot \ln(q_i)\right) \qquad I_{quad} = -0.560247$$

$f(x) := \text{if}\left(x = 0, 0, e^{-x} \cdot x \cdot \ln(x)\right)$

$RR := \text{Simpson}\left(f, 0, 200, 20, 10^{-9}\right)$ $\quad RR = \begin{pmatrix} 0.422784 \\ 0.000038 \end{pmatrix}$ $\quad RR_0 - 1 = -0.577216$ $\quad -\gamma - \left(RR_0 - 1\right) = -1.12 \times 10^{-10}$

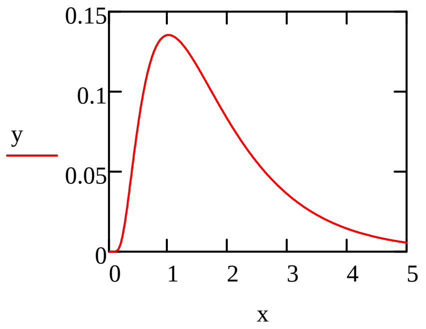
18-Calcule a integral imprópria: $\int_0^{\infty} e^{-\left(x+\frac{1}{x}\right)} \cdot dx$ com uma precisão de quatro algarismos significativos.

$$I := \int_0^{1000} e^{-\left(x+\frac{1}{x}\right)} dx \qquad I = 0.279732$$

$$f(x) := \text{if}\left[x = 0, 0, e^{-\left(x+\frac{1}{x}\right)}\right] \qquad RR := \text{Simpson}\left(f, 0, 200, 20, 10^{-9}\right) \qquad RR = \begin{pmatrix} 0.279732 \\ 0.009766 \end{pmatrix}$$

$$I_{quad} := \sum_{i=0}^{N} \left(\omega_i \cdot e^{-\frac{1}{q_i}} \right) \qquad I_{quad} = 0.285033$$

$$k := 0..500 \qquad x_k := \frac{k}{100} \qquad y_k := f\left(x_k\right)$$



18-A função $Si(x)$ é definida por: $Si(x) = \int_0^x \left(\frac{\text{sen}(\xi)}{\xi} \right) \cdot d\xi$. Calcule, com quatro algarismos significativos, a integral: $\int_0^1 \left(\frac{Si(x) - \text{sen}(x)}{x^3} \right) \cdot dx$.

$$\frac{Si(x) - \sin(x)}{x^3}$$

$$p_7(x) := \frac{x^3}{9} - \frac{x^5}{150} + \frac{x^7}{5880} \qquad p_4(x) := \frac{1}{9} - \frac{x^2}{150} + \frac{x^4}{5880}$$

$$Si(x) := \int_0^x \frac{\sin(\xi)}{\xi} d\xi \qquad Si(1) = 0.946083 \qquad f(x) := \text{if}\left(x = 0, \frac{1}{9}, \frac{Si(x) - \sin(x)}{x^3}\right)$$

$$I := \int_0^1 f(x)\, dx \qquad I = 0.108923$$

$$RR := \text{Simpson}\left(f, 0, 1, 20, 10^{-9}\right) \qquad RR = \begin{pmatrix} 0.108923 \\ 0.0125 \end{pmatrix}$$

$g(x) := \text{if}\left(x = 0, 0, \frac{\text{Si}(x) - \sin(x) - p_7(x)}{x^3}\right)$ $I_{pol} := \frac{1}{9} - \frac{1}{450} + \frac{1}{5 \cdot 5880}$ $I_{pol} = 0.108923$

$RR := \text{Simpson}\left(g, 0, 1, 20, 10^{-9}\right)$ $RR = \begin{pmatrix} -3.474171 \times 10^{-7} \\ 0.0125 \end{pmatrix}$

$\delta := RR_0$ $I_{pol} + \delta = 0.108923$

19-O Método de Monte-Carlo pode ser aplicado para calcular integrais definidas. Tal método aplicado ao cômputo de: $\int_a^b f(x) \cdot dx$ (sendo: para $a \le x \le b \Rightarrow 0 \le f(x) < f_{\max}$) consiste em *sortear simultaneamente* N pares de valores de x entre a e b e de y entre zero e $f_{\max}$. Após os sorteios calcula-se $f(x)$, se $f(x) < y$ faça $k \leftarrow k+1$ (iniciando-se com $k \leftarrow 0$) e parta para novo sorteio; caso contrário, isto é: $f(x) > y$ nada faça e parta para novo sorteio. Ao cabo dos N sorteios calcule: $\int_a^b f(x) \cdot dx \approx \left(\frac{k}{N}\right) \cdot (b-a) \cdot f_{\max}$. Aplique o procedimento para o cálculo da integral: $\int_0^1 e^{-x^2} \cdot dx$, compare o valor obtido com o valor *exato* da integral que é: $\int_0^1 e^{-x^2} \cdot dx = \frac{\sqrt{\pi}}{2} \cdot \text{erf}(1)$ [erf é a função erro].

```
N(n) := | N ← 0
        | for i ∈ 1 .. n
        |    | x ← rnd(1)
        |    | y ← rnd(1)
        |    | N ← N + 1  if  y < e^(-x^2)
        | N
```

$n := 10^6$ $M := N(n)$ $M = 747326$ $\frac{M}{n} = 0.747326$ $\frac{\sqrt{\pi}}{2} \cdot \text{erf}(1) = 0.746824$

$$0.746824 - \frac{M}{n} = -0.000502$$

20-Calcule o valor da integral pelo método de Simpson e pela quadratura do exercício 12:

$$I = \int_{x=0}^{1} \int_{y=0}^{1} e^{-\left(x^2+y^2\right)} \cdot dy \cdot dx$$

$\int_0^1 \int_0^1 e^{-\left(x^2+y^2\right)} \, dx \, dy = 0.557746$ $\left(\frac{\sqrt{\pi}}{2} \cdot \text{erf}(1)\right)^2 = 0.557746$

$f(x,y) := e^{-\left(x^2+y^2\right)}$ $\quad \int_0^1 \int_0^1 f(x,y)\,dx\,dy = 0.557746$ $\quad \dfrac{\sum_{i=0}^{1} \sum_{j=0}^{1} f\left(\lambda_i, \lambda_j\right)}{4} = 0.557404$

$f(x) := e^{-x^2}$

$RR := \text{Simpson}\left(f, 0, 1, 20, 10^{-9}\right)$ $\quad RR = \begin{pmatrix} 0.746824 \\ 0.00625 \end{pmatrix}$ $\quad \left(RR_0\right)^2 = 0.557746$