Decomposição LU, para qualquer matriz quadrada:

$$C := \begin{pmatrix} 3 & -1 & 2 \\ 1 & 2 & 3 \\ 2 & -2 & -1 \end{pmatrix} \qquad M := \mathrm{lu}(C)$$

decomposiçao LU

$$M = \begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 2 & -2 & -1 \\ 0 & 1 & 0 & 0.5 & 1 & 0 & 0 & 3 & 3.5 \\ 1 & 0 & 0 & 1.5 & 0.667 & 1 & 0 & 0 & 1.167 \end{pmatrix}$$

$P := \mathrm{submatrix}(M, 0, 2, 0, 2)$

$L := \mathrm{submatrix}(M, 0, 2, 3, 5)$ Matriz triangular inferior

$U := \mathrm{submatrix}(M, 0, 2, 6, 8)$ Matriz triangular superior

$$P = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad L = \begin{pmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 1.5 & 0.667 & 1 \end{pmatrix} \qquad U = \begin{pmatrix} 2 & -2 & -1 \\ 0 & 3 & 3.5 \\ 0 & 0 & 1.167 \end{pmatrix}$$

$$P \cdot C = \begin{pmatrix} 2 & -2 & -1 \\ 1 & 2 & 3 \\ 3 & -1 & 2 \end{pmatrix} \qquad L \cdot U = \begin{pmatrix} 2 & -2 & -1 \\ 1 & 2 & 3 \\ 3 & -1 & 2 \end{pmatrix} \qquad P \cdot L \cdot U = \begin{pmatrix} 3 & -1 & 2 \\ 1 & 2 & 3 \\ 2 & -2 & -1 \end{pmatrix}$$

Retorna à matriz C

Método de Choleski - restrito a matrizes simétricas

$$A := \begin{pmatrix} 1 & 2 & 6 \\ 2 & 5 & 15 \\ 6 & 15 & 46 \end{pmatrix} \qquad L := \mathrm{cholesky}(A) \qquad L = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 6 & 3 & 1 \end{pmatrix} \qquad L \cdot L^T = \begin{pmatrix} 1 & 2 & 6 \\ 2 & 5 & 15 \\ 6 & 15 & 46 \end{pmatrix}$$

Retorna à matriz A

```
Triangular_inv(L, n) := | m ← n − 1
                        | for i ∈ 0 .. m
                        |   | x_{i,i} ← 1 / L_{i,i}
                        |   | for j ∈ i + 1 .. m                        if i < m
                        |   |   x_{j,i} ← − [ Σ_{k=i}^{j−1} (L_{j,k} · x_{k,i}) ] / L_{j,j}
                        | x
```

$$x_{i,i} \leftarrow \frac{1}{L_{i,i}} \qquad x_{j,i} \leftarrow -\frac{\left[\sum_{k=i}^{j-1} \left(L_{j,k} \cdot x_{k,i}\right)\right]}{L_{j,j}}$$

$X := \mathrm{Triangular}_{inv}(L, 3)$

$$X = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & -3 & 1 \end{pmatrix} \qquad L \cdot X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad X \cdot L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad X^T \cdot X = \begin{pmatrix} 5 & -2 & 0 \\ -2 & 10 & -3 \\ 0 & -3 & 1 \end{pmatrix} \qquad A^{-1} = \begin{pmatrix} 5 & -2 & 0 \\ -2 & 10 & -3 \\ 0 & -3 & 1 \end{pmatrix}$$

Decomposição QR

$$A := \begin{pmatrix} 1 & 2 & -1 \\ 2.3 & 4 & 4 \\ -2 & 5.1 & 1 \\ 0 & .8 & 6 \end{pmatrix}$$

$M := qr(A)$ A matriz não precisa ser quadrada

$$M = \begin{pmatrix} 0.312 & 0.279 & -0.411 & -0.81 & 3.208 & 0.312 & 1.933 \\ 0.717 & 0.553 & 0.117 & 0.407 & 0 & 6.823 & 3.415 \\ -0.623 & 0.776 & -0.072 & 0.064 & 0 & 0 & 6.213 \\ 0 & 0.117 & 0.901 & -0.417 & 0 & 0 & 0 \end{pmatrix}$$

$Q := submatrix(M, 0, 3, 0, 3)$ $R := submatrix(M, 0, 3, 4, 6)$

$$Q \cdot Q^T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Q é uma matriz ortonormal

$$Q \cdot R = \begin{pmatrix} 1 & 2 & -1 \\ 2.3 & 4 & 4 \\ -2 & 5.1 & 1 \\ 0 & 0.8 & 6 \end{pmatrix}$$

Igual à matriz Original

$$R = \begin{pmatrix} 3.208 & 0.312 & 1.933 \\ 0 & 6.823 & 3.415 \\ 0 & 0 & 6.213 \\ 0 & 0 & 0 \end{pmatrix}$$

Matriz Triangular Superior;

Se A for uma Matriz Quadrada

$$A := \begin{pmatrix} 1 & 3 & 5 \\ 2 & 5 & 15 \\ 6 & 10 & 16 \end{pmatrix}$$

$M := qr(A)$

$$M = \begin{pmatrix} 0.156 & -0.608 & 0.778 & 6.403 & 11.401 & 20.459 \\ 0.312 & -0.717 & -0.623 & 0 & -2.006 & -8.353 \\ 0.937 & 0.34 & 0.078 & 0 & 0 & -4.204 \end{pmatrix}$$

$Q := submatrix(M, 0, 2, 0, 2)$ $R := submatrix(M, 0, 2, 3, 5)$

$$Q \cdot Q^T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Q é uma matriz ortonormal

$$Q \cdot R = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 5 & 15 \\ 6 & 10 & 16 \end{pmatrix}$$

Igual à matriz Original

$$R = \begin{pmatrix} 6.403 & 11.401 & 20.459 \\ 0 & -2.006 & -8.353 \\ 0 & 0 & -4.204 \end{pmatrix}$$

Matriz Triangular Superior;

Decomposição SVD (em valores singulares -que são as raízes quadradas positivas dos calores caracterísiticos da matriz vezes sua transporta):

$$A := \begin{pmatrix} 1 & 2 & -1 \\ 2.3 & 4 & 4 \\ -2 & 5.1 & 1 \\ 0 & 0.8 & 6 \end{pmatrix} \qquad M := \mathrm{svd2}(A)$$

A matriz não precisa ser quadrada

$$B := A \cdot A^T \qquad \lambda := \mathrm{eigenvals}(B)$$

$$\lambda := \sqrt{\lambda} \qquad \lambda := -\mathrm{sort}(-\lambda) \qquad \lambda^T = (25.835 \quad 3.637 \quad 0.575)$$

$$s := M_0 \qquad U := M_1 \qquad V := M_2$$

$$M = \begin{pmatrix} \{3,1\} \\ \{3,3\} \\ \{3,3\} \end{pmatrix} \qquad U = \begin{pmatrix} -0.228 & 0.028 & -0.973 \\ -0.607 & -0.786 & 0.12 \\ -0.761 & 0.618 & 0.196 \end{pmatrix} \qquad V = \begin{pmatrix} -0.233 & -0.439 & -0.868 \\ 0.595 & 0.642 & -0.484 \\ 0.769 & -0.629 & 0.112 \end{pmatrix}$$

$$U \cdot \mathrm{diag}(s) \cdot V = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 5 & 15 \\ 6 & 10 & 16 \end{pmatrix}$$

Retorna à matriz A

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 2.3 & 4 & 4 \\ -2 & 5.1 & 1 \\ 0 & 0.8 & 6 \end{pmatrix}$$

$$s^T = (25.835 \quad 3.637 \quad 0.575)$$

Vetor dos valores singulares

Se A for uma Matriz Quadrada

$$A := \begin{pmatrix} 1 & 3 & 5 \\ 2 & 5 & 15 \\ 6 & 10 & 16 \end{pmatrix} \qquad \lambda := \mathrm{eigenvals}\left(A \cdot A^T\right) \qquad \lambda := \sqrt{\lambda} \qquad \lambda := -\mathrm{sort}(-\lambda)$$

$$M := \mathrm{svd2}(A)$$

$$\lambda^T = (25.835 \quad 3.637 \quad 0.575) \qquad \mathrm{sort}(\mathrm{eigenvals}(A))^T = (-2.614 \quad -0.812 \quad 25.426)$$

$$s := M_0 \qquad U := M_1 \qquad V := M_2$$

$$M = \begin{pmatrix} \{3,1\} \\ \{3,3\} \\ \{3,3\} \end{pmatrix} \qquad U = \begin{pmatrix} -0.228 & 0.028 & -0.973 \\ -0.607 & -0.786 & 0.12 \\ -0.761 & 0.618 & 0.196 \end{pmatrix} \qquad V = \begin{pmatrix} -0.233 & -0.439 & -0.868 \\ 0.595 & 0.642 & -0.484 \\ 0.769 & -0.629 & 0.112 \end{pmatrix} \qquad V \cdot U = \begin{pmatrix} 0.98 & -0.198 & 0.004 \\ -0.157 & -0.787 & -0.597 \\ 0.121 & 0.585 & -0.802 \end{pmatrix}$$

$$U \cdot \mathrm{diag}(s) \cdot V = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 5 & 15 \\ 6 & 10 & 16 \end{pmatrix}$$

Retorna à matriz A

$$\mathrm{cond1}(A) = 92 \qquad \mathrm{cond2}(A) = 44.955$$

$$s^T = (25.835 \quad 3.637 \quad 0.575)$$

$$\lambda^T = (25.835 \quad 3.637 \quad 0.575)$$

Vetor dos valores singulares igual a raiz quadrada positiva dos valores característicos de A vezes sua transporta