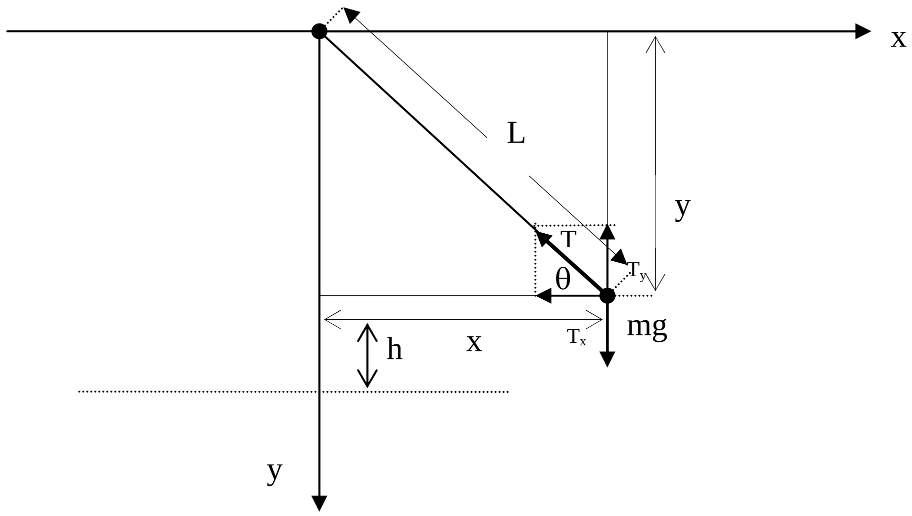
O PÊNDULO SIMPLES



$$\begin{cases} \mathrm{m}\cdot\dfrac{d^2\mathrm{x}(\mathrm{t})}{dt^2} = -\mathrm{T}\cdot\cos(\theta) = -\mathrm{T}\cdot\dfrac{\mathrm{x}(\mathrm{t})}{\mathrm{L}} \\ \mathrm{m}\cdot\dfrac{d^2\mathrm{y}(\mathrm{t})}{dt^2} = -\mathrm{T}\cdot\mathrm{sen}(\theta) + \mathrm{m}\cdot\mathrm{g} = -\mathrm{T}\cdot\dfrac{\mathrm{y}(\mathrm{t})}{\mathrm{L}} + \mathrm{m}\cdot\mathrm{g} \end{cases}$$

$$\mathrm{x}(\mathrm{t})^2 + \mathrm{y}(\mathrm{t})^2 = \mathrm{L}^2$$

Adimensionamento das variáveis:

$x = \dfrac{\mathrm{x}}{\mathrm{L}},\ y = \dfrac{\mathrm{y}}{\mathrm{L}},\ \lambda = \dfrac{\mathrm{T}}{\mathrm{mg}},\ t = \sqrt{\dfrac{\mathrm{g}}{\mathrm{L}}}\cdot\mathrm{t},\ v_x = \dfrac{\mathrm{v_x}}{\sqrt{\mathrm{L}\cdot\mathrm{g}}}$ e $v_y = \dfrac{\mathrm{v_y}}{\sqrt{\mathrm{L}\cdot\mathrm{g}}}$, resulta:

$$\begin{cases} \dfrac{d^2x(t)}{dt^2} = -\lambda(t)\cdot x(t) \\ \dfrac{d^2y(t)}{dt^2} = -\lambda(t)\cdot y(t) + 1 \end{cases} \qquad x(t)^2 + y(t)^2 = 1$$

Ou ainda:

$$\begin{cases} \dfrac{dx(t)}{dt} = v_x(t) \\ \dfrac{dy(t)}{dt} = v_y(t) \\ \dfrac{dv_x(t)}{dt} = -\lambda(t)\cdot x(t) \\ \dfrac{dv_y(t)}{dt} = -\lambda(t)\cdot y(t) + 1 \end{cases} \qquad \boxed{x(t)^2 + y(t)^2 = 1}$$

Derivando em relação a *t* a expressão: $x(t)^2 + y(t)^2 = 1$, resulta:

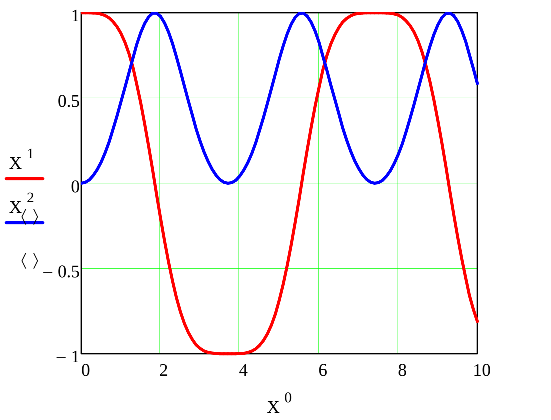
$2\cdot\left[x\cdot v_x + y\cdot v_y\right] = 0$ ou .$\boxed{x(t)\cdot v_x(t) + y(t)\cdot v_y(t) = 0}$

Derivando novamente em relação a *t* esta expressão, resulta:

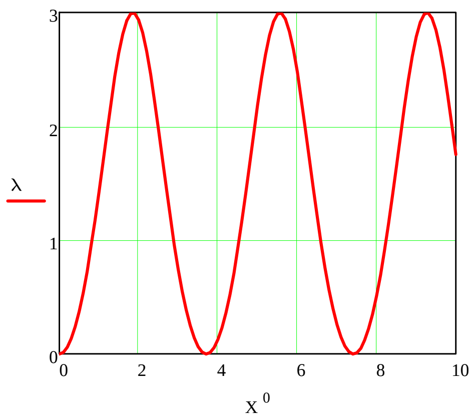
$$v_x^2(t)+x(t)\cdot\left[-\lambda(t)\cdot x(t)\right]+v_y^2(t)+y(t)\cdot\left[-\lambda(t)\cdot y(t)+1\right]=0$$

$$-\lambda(t)\cdot\left[x(t)^2+y(t)^2\right]+v_x^2(t)+v_y^2(t)+y(t)=0$$

Ou seja: $\boxed{\lambda(t)=v_x^2(t)+v_y^2(t)+y(t)}$



$x(t)$ (curva vermelha) e $y(t)$ (curva azul) *versus* tempo



$\lambda(t)$ *versus* tempo

$$\text{Solução para: }\begin{cases} x(0)=1 \\ y(0)=0 \\ v_x(0)=0 \\ v_y(0)=0 \end{cases}$$